Contents

15^{th} INTERNATIONAL CONFERENCE ON INTEGRAL METHODS IN SCIENCE AND ENGINEERING

UNIVERSITY OF BRIGHTON

16 - 20 July 2018

Welcome to the University of Brighton.

The University of Brighton has been part of the city of Brighton and Hove since 1859, starting as a school of art in the kitchens of the Royal Pavilion and growing to become the diverse and inclusive institution it is today.

We have more than 21,000 students and 2,800 staff studying and working at our five campuses in Brighton, Eastbourne and Hastings. Subjects taught here range from medicine to engineering, psychology to illustration, sport science to English literature.

Students are at the heart of all that we do and take an active role in their learning experience. They leave us ready to make a difference to the world.

IMSE 2018 will provide an international forum for communicating recent advances in research work that promotes the link between mathematics and the applied sciences and engineering. The conference will also provide an opportunity for delegates to exchange information and ideas that support their work.

One of the objectives is to promote new research tools, methods and procedures beyond the specific realms of Mathematics, emphasizing their applicability in various areas of Exact or Technological Sciences. Thus, the scope of the conference addresses both academic and industrial interests.

Participation is open to all scientists and engineers whose work makes use of analytic and numerical methods, integral equations, ordinary and partial differential equations, asymptotic and perturbation methods, boundary integral techniques, conservation laws, hybrid approaches, vortex methods, signal processing and image analysis.

The conference will be organized by the University of Brighton in association with the University of Tulsa, Oklahoma, USA.

Acknowledgments
The organisers would like to thank the staff at SouthCoast Conferences for all their assistance with the preparations for this conference, and in particular we would like to thank Laura Williams and Richard Huck for all their hard work. We would also like to thank the School of Computing, Engineering and Mathematics for its financial support of the conference.

We would also like to thank Maria Eugenia Pérez and Sergey Mikhailov for all their help with organising the programmes for two mini-symposia.

Paul Harris (Chair)
Dmitry Savostyanov

General Information

Safety
Please familarise yourself with the fire exits from the building. In the unlikely event of a fire or other emergency which requires that the building is evacuated please meet in the car park of the Halls of residence which are on the opposite side of Queensdown School Road from the main entrance to the Huxley Building. All the fire exits are clearly marked.

Smoking
In accordance with current English law, smoking is **not** permitted in any part of the University building.

Conference Sessions
The conference sessions will take place in either the main Lecture Theater or in room 406/7 on the floor above.

Internet Access
The whole of the building is covered by the University wireless network. Details of how to connect to the network are given on a separate sheet. Please note that it is a condition of use that you have up-to-date anti-virus software installed on your computer before you can connect to the network.

Refreshments
Morning and afternoon tea and coffee and the buffet lunches will be served in the social area outside the main Lecture Theater.

Fish and Chip Supper (Tuesday evening)
This will be in the restaurant on Brighton Pier. To get to the pier from the conference venue, walk down Queensdown School Road and cross the main Lewes Road at the pedestrian crossing (traffic lights). Take any route 24, 25, 25X, 28, 29, 29B, 48 or 49 bus and get off at the Old Stein stop. Do not get a route 23 or 50U bus. When you get off the bus, you should be able to see the pier directly in front of you (but please be careful crossing all of the roads). To get to the restaurant you must pass the "Palace of Fun". The restaurant is in the centre of the pier before you get to the fun-fair.

Conference Dinner
This is being held at The Old Ship Hotel on the sea-front. Follow the instructions above for getting to the pier. From the pier, walk west along the sea-front (towards the wrecked West Pier and the I360 viewing platform). The Old Ship Hotel is across the road from the beach, about 0.25 mile from the main pier.

International Steering and Scientific Committee

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Previous Conferences

1985 and 1990: Arlington, Texas, USA.
1993: Sendai, Japan.
1996: Oulu, Finland.
1998: Houghton, Michigan, USA.
2000: Banff, Canada.
2002: Saint-Etienne, France.
2004: Orlando, Florida, USA.
2006: Niagara Falls, Canada.
2008: Santander, Spain.
2010: Brighton, United Kingdom.
2012: Bento Gonalves (RS), Brazil
2014: Karlsruhe, Germany
2016: Padova, Italy
2018: Brighton, United Kingdom.

1 Invited Speakers

Approximation of time domain boundary integral equations

Dugald DUNCAN

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Time domain boundary integral equations (TDBIEs) are used to model time dependent wave scattering from surfaces in an infinite, homogeneous medium. We concentrate on the acoustic scattering case. The solution can be reconstructed anywhere in space and time in terms of a potential which is computed only on the surface. The approximation of the time dependent surface potential by numerical methods is expensive and sensitive to stability issues, but it can be cheaper than solving the wave equation in a very large space domain. We describe a method using a "backward-in-time collocation approximation coupled with Galerkin in space, describe its relationship with Volterra integral equations, and its connections to other approaches for TDBIEs such as full space-time Galerkin and convolution quadrature.

Based on joint work with Penny Davies.

Control Strategies for Multiphase Compact Separators – Development and Implementation

Ram S. MOHAN

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The petroleum industry has relied in the past mainly on conventional, gravity based vessel-type separators to process wellhead production. Since the mid 1990's the industry has shown keen interest in the development and application of compact multiphase cyclonic separators due to their significant advantages such as, simplicity in construction, compactness, low weight, and having low capital and operational costs. Established in 1994, the mission of research consortium, Tulsa University Separation Technology Projects (TUSTP) has been to advance the state-of-the-art of compact multiphase cyclonic separation technology for gas-oil-water-sand flow. To date, there are over 6,500 compact separators deployed in the field in the USA and around the world based on TUSTP R&D.

Because of their small residence time, the performance of compact multiphase separators depends on implementation of stable and robust control strategies appropriate for specific field applications. An overview of the different control strategies that have been developed for performance optimization of compact separators such as Gas-Liquid Cylindrical Cyclone (GLCC©), Liquid-Liquid Cylindrical Cyclone (LLCC©), Liquid-Liquid Hydrocyclone (LLHC) and Compact Multiphase Separation System (CMSS©) will be presented. Detailed mathematical models, which form the basis for compact separator control strategies such as feedback control, predictive control, optimal control, fuzzy logic control, etc. will be discussed.

A dynamic simulation platform (DSP) has been developed based on these control system models in Matlab/Simulink^{TM} for predicting the transient performance of the integrated CMSS©system. This DSP uses a new idea of combining the steady state device models and the transient control system model to predict the performance of the entire system. It has a new droplet-tracking feature, which helps in studying the performance and separation efficiencies of the compact separation devices. A suitable reliability analysis protocol has been developed for a free-water knockout CMSS©system and system reliability has been calculated from simulation data from quantified reliability of components and performance reliability of the system. This protocol has proven to predict performance reliability of new systems, which do not have prior information on failure of components or devices.

Modelling of sprays: recent results and future challenges

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A brief overview of some recent developments and future challenges in spray modelling is presented. The focus is on the comparative analysis of Lagrangian and Fully Lagrangian (Osiptsov) approaches to spray modelling and the applications of the latter approach and its generalisations to the solution of engineering problems, recent developments in modelling the heating and evaporation of spherical and non-spherical droplets and films, and application of the method of integral manifolds in the analysis of spray heating, evaporation and ignition in Internal Combustion (IC) engine-like conditions. Future challenges described in the paper include the generalisation of the Fully Lagrangian approach to enable it to model realistic turbulent sprays, further development of the models of heating and evaporation of deformed droplets, modelling of heating and evaporation of droplets in trance- and super-critical conditions and further development of the integral manifold methods to enable their application to the solution of realistic engineering problems, with particular focus on IC engines. All models are expected to be developed in formats that enable their relatively simple implementation into commercial Computational Fluid Dynamics (CFD) codes.

Keywords: sprays, mathematical modelling, CFD codes

Uncertainty quantification in Engineering

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The numerical manipulation of uncertainty is usually performed by *interval methods, fuzzy variables* or *probabilistic modeling*. Each of these approaches has its particularities, advantages and inconveniences.

This conference concerns the probabilistic approach, which is considered as interesting when some information on the statistical or probabilistic properties of the uncertainties is available. In such a situation, the main goal is to establish a quantitative connection between the uncertainties of the inputs (parameters, external forces, boundary conditions,) and the uncertainties of the outputs, which are generally furnished by numerical calculations. When such a quantitative connection is established, it may be used in order to evaluate risks, reliability, variabilities, robustness , to control or identify a system etc.

The talk presents the standard methods to establish such a connection and illustrates their implementation et behaviour. As a typical situation, we consider the effects of uncertainties in optimal design : indeed, the effects of uncertainties on the solutions of optimization problem may be analyzed from different points of view, generally corresponding to different applications. For instance, let us consider the basic problem

$$x^* = \text{Arg}\min\{F(x) : x \in S\}.$$

where $S \subset V = \mathbb{R}^n$ is the admissible set and F is the objective function. Assume that F or S depend upon a parameter (or a vector of parameters) z. In such a situation, the solution x^* depends upon z and we have $x^* = x(z)$.

The optimal solutions x^* may be sensitive to the variations of the parameters z. When z is affected by uncertainty or present a significant variability, a standard optimization procedure is not enough in order to ensure a desired behavior or a requested safety level within the margins of variation or uncertainty under consideration.

Sensitivity analysis can indicate which parameters and a restriction are more sensitive to variations of the uncertain parameters, but does not furnishes a quantification of the dispersion of the solutions. *Robust optimisation approaches* looks for a solution which is optimal for a range of variation of parameters.

When z is a random variable, for each possible value $z = z(\omega)$, the solution takes the value $x(\omega) = x^*(z(\omega))$, so that x^* becomes a random variable.

In such a situation, an alternative approach is given by *reliability optimisation*, where additional restrictions involving the probabilities of critic events are taken into account. It is possible to determine the dispersion of optimal values $F(x^*)$ (See for instance the works of R.T. Rockafellar on *Value at Risk*). The dispersion analysis of the solutions x^* is a situation about which the literature is not so extensive, since it presents fundamental difficulties.

In order to determine the distribution of x^*, *Montecarlo methods* may be used, but they involve a large number of realizations and, so, a high computational effort. In order to obtain a diminution of the computational cost, we may consider a random variable (or vector) r, which is used to generate a second variable (or vector) $X = \sum_{i=1}^{N} x_i \phi_i(r)$ such that $x^* \approx X$ or, into a more simple way, such that the probability distribution of X is close to the one of x^*. This objective may be attained by different ways:

- If the distribution of the pair (r, z) may be determined (or estimated), it is possible to consider the function $E(F(X, z))$. The minimization of $E(F(X, z))$ furnishes the values of the unknowns $(x_1, , x_N)$.

- When a sample $(r_i, z_i)_{1 \leq i \leq ns}$ of the pair (r, z) may be generated, we may consider $E(F(X, z)) \approx \frac{1}{ns} \sum_{k=1}^{ns} F(X, z_k)$ (approximation of the mean by its estimation on the sample). Then, $(x_1, , x_N)$ is determining by minimizing the estimation of the mean.

- The unknown coefficients may be determined by minimizing a distance or, more generally, a pseudo-distance $d(x, X)$ such as, for instance, $\|x^* - X\|$ or $\|Mp(x^*) - Mp(X)\|$ (with $Mp(u)(= E(u), , E(up))^t$ (vector formed by the moments up to the order p). As in the preceding approach, we may consider a sample $(r_i, z_i)_{1 \leq i \leq ns}$ and the associated values $x^* = (x^*(z_1), , x^*(z_{ns}))$, $X = (X(r_1), , X(r_n s))$, $M_p(u) \approx (\frac{1}{ns} \sum_{i=1}^{ns} u_i, \ldots \frac{1}{ns} \sum_{i=1}^{ns} u_i^p)$. Finally, we may look for a solution such that $\nabla F(X, z) \perp \phi_i(z), i = 1, \ldots, N$.

The choice of the variable r and of the functions ϕ_i has an influence on the quality of the results and has to be analyzed. The simple choice $r = z$ and $\{\phi_i i \in \mathbb{N}\}$ as a total family $L^2(a, b)$, with a and b conveniently chosen has shown to be effective to calculate.

In order to use available optimization codes, these approximations may also be used in the iterations of numerical algorithms, such as, for instance, descent methods.

This approach extends to other situations, such as :

- Linear systems $A(z)x = B(z)$,

- Determination of eigenvalues and eigenvectors $A(z)x = \lambda x$,

- Nonlinear algebraical equations $F(x, z) = 0$,

- Differential equations,
- State estimation,
- Limit cycles.

We shall present numerical examples and compare the different approaches.

Hypercomplex methods in applied analysis.

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Hypercomplex analysis can be seen as some kind of "complex function theory" for higher dimensions, where complex numbers are replaced by quaternions, coquaternions, split quaternions, Clifford numbers, octonions, sedenions etc.. Hyperholomorphic functions play the role of holomophic functions of the complex function theory in the plane. They are zero solutions of higher-dimensional versions of Cauchy-Riemann equations (Riesz system, Fueter system, system of Moisil-Teodorescu, etc.). In this talk we reduce our considerations to quaternion valued functions over 3D-domains. As in the classical function theory also in higher dimenional versions some operators are important: Dirac operator, Teodorescu transform, Cauchy-Fueter operator as well as the orthoprojections on the Bergman space of the Hilbert space (module) and on its complement. For boundary value problems we also need so-called projections of Plemelj type which are connected with the Cauchy-Fueter operator. We use and derive analogs of basic theorems of the plane function theory. For the treatment of (initial) boundary value problems we still need a decomposition theorem of Bergman-Hodge type. We intent to show the advantages and benefits of the hypercomplex access for treatment of boundary value problems of several fluid flow problems.

2 Asymptotic Analysis: Homogenization and Thin Structures Mini-Symposium

On the flow of a non-Newtonian fluid in a thin domain

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We study the steady incompressible flow of a Bingham fluid in a thin T-like shaped domain, under the action of given external forces and with no-slip boundary condition on the whole boundary of the domain. This phenomenon is described by non linear variational inequalities. By letting the parameter describing the thickness of the thin domain tend to zero, we derive two uncoupled problems corresponding to the two branches of the T-like shaped structure. We then analyze and give a physical justification of the limit problem.

Based on joint work with Antonio Gaudiello and Angelo Leopardi.

Keywords: Non-newtonian fluids, junctions, thin structures.

Spectrum of Neumann problems on thin waveguides with strongly corrugated boundary

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We consider a family $\{\Omega^\varepsilon\}_{\varepsilon>0}$ of periodic domains in $\mathbb{R}^2$ with waveguide geometry and analyse spectral properties of the Neumann Laplacian $-\Delta_{\Omega^\varepsilon}$ on Ω^ε. The waveguide Ω^ε is a union of a thin straight strip of the width ε and a family of small protuberances with the so-called "room-and-passage" geometry. The protuberances are attached periodically, with a period ε, along the strip upper boundary. For $\varepsilon \to 0$ we prove a (kind of) resolvent convergence of $-\Delta_{\Omega^\varepsilon}$ to a certain ordinary differential operator. Also we demonstrate Hausdorff convergence of the spectrum. In particular, we conclude that if the sizes of "passages" are appropriately scaled the first spectral gap of $-\Delta_{\Omega^\varepsilon}$ is determined exclusively by geometric properties of the protuberances. The proofs are carried out using methods of homogenization theory.

Based on joint work with Andrii Khrabustovskyi (Technische Universitat Graz).

Keywords: Waveguides, gaps, homogenization.

Dispersive effective behaviour of high-contrast periodic media.

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I will discuss my recent work with Y. Ershova and A. Kiselev, demonstrating that spectral problems for quantum graphs with rapidly oscillating high-contrast weights are asymptotically equivalent to "homogenised" models with energy-dependent interface conditions. We show that these asymptotically equivalent models are directly related (in the sense of Schur-Frobenius duality) to models for time-dispersive media, which in the time domain involve memory, and we characterise the corresponding time convolution kernels explicitly.

Based on joint work with Y. Ershova and A. Kiselev.

A hole close to the boundary

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This talk concerns a Dirichlet problem for the Laplace equation in a domain with a small hole that approaches the outer boundary while shrinking its size. By a functional analytic approach and by a suitable potential theory adapted to the problem we can describe the asymptotic behavior of the solution in terms of real analytic functions of the perturbation parameters. Then, we deduce informations on the asymptotic expansions.

Based on joint work with V. Bonnaillie-Noël, M. Dambrine, and P. Musolino.

Keywords: singularly perturbed perforated domain, real analytic continuation in Banach space, asymptotic expansion.

Homogenization of some quasilinear elliptic problems with nonlinear Robin conditions and L^1 data.

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We present here some recent results concerning the homogenization of a class of quasilinear elliptic problems in a periodically perforated domain Ω_ε, with L^1 data and nonlinear Robin conditions on the boundary of the holes. Since we deal with L^1 data, we cannot have solutions in $H^1(\Omega_\varepsilon)$. Therefore, we use here the convenient notion of renormalized solutions. For the homogenization, we use the Periodic Unfolding Method but we can only apply it to the truncated solutions, which are in $H^1(\Omega_\varepsilon)$. Hence, as a main difficulty, we have to carefully describe the limits of the truncated unfolded solutions and of their gradients. This allow us to prove first that we obtain at the limit an unfolded renormalized problem, then to derive a renormalized homogenized problem in Ω.

Based on joint work with Olivier Guibé (Université de Rouen Normandie) and Alip Oropeza (University of the Philippines Diliman).

Keywords: Renormalized solutions, Robin conditions, periodic unfolding method.

Spectral problems in a T-like structure: high frequencies

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We analyze the behavior of the spectrum of the Laplacian in a planar domain. The thickness of the domain or of one of the components depends on a small parameter ε that we shall make to go to zero. The boundary conditions can be Dirichlet or Neumann depending on the problem. As it is well known, in some of these models, the low frequencies can give rise to vibrations affecting only a part of the structure or ignore, e.g., transverse vibrations. We characterize the asymptotic behavior as $\varepsilon \to 0$ of the high frequencies, in a certain range which depends on the structure. In particular, we look at the behavior of the high frequencies in a T-like planar structure.

Based on joint work with A. Gaudiello and E. Pérez.

Keywords: thin structures, asymptotic analysis, spectral analysis.

Boundary homogenization for polyharmonic operators

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Given a domain Ω in $\mathbb{R}^N$, $N \geq 2$, and a family of domain perturbations Ω_ϵ, $\epsilon > 0$, of Ω, we consider polyharmonic operators of order $2m$ subject to homogeneous boundary conditions on Ω_ϵ and we study the behaviour of the solutions to the corresponding Poisson problems, and the behaviour of the eigenvalues and eigenfunctions as $\epsilon \to 0$. The focus is on problems with intermediate boundary conditions of the type

$$\begin{cases} (-\Delta)^m u + u = f, & \text{in } \Omega_\epsilon, \\ \frac{\partial^l u}{\partial \nu^l} = 0, & \text{on } \partial\Omega_\epsilon, \text{ for all } 0 \leq l \leq m-2, \\ \frac{\partial^m u}{\partial \nu^m} = 0, & \text{on } \partial\Omega_\epsilon, \end{cases}$$

where $f \in L^2(\Omega_\epsilon)$ and ν denotes the unit outer normal. We analyse in particular the case where the boundary of Ω_ϵ is given by a periodic perturbation of a flat part of the boundary of Ω and we study the corresponding homogenization problem identifying the appropriate strange terms appearing in limiting boundary conditions.

Based on joint work with José M. Arrieta and Francesco Ferraresso.

Keywords: Polyharmonic operators, boundary homogenization, strange term.

A functional analytic approach to singular perturbation problems

Massimo LANZA DE CRISTOFORIS

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This talk is dedicated to the analysis of boundary value problems on singularly perturbed domains by an approach which is alternative to those of asymptotic analysis and of homogenization theory.

In particular, we will consider a certain linear or nonlinear boundary value problem on a domain with one or possibly infinitely many holes, whose size is determined by a positive parameter ϵ and we will consider a family of solutions depending on ϵ as ϵ approaches 0. Then we shall represent the dependence on ϵ of the family of solutions, or of corresponding functionals of the solutions such as the energy integral, in terms of possibly singular at 0 but known functions of ϵ such as ϵ^{-1} or $\log \epsilon$, and in terms of possibly unknown real analytic maps.

Boundary value problems on singularly perturbed domains appear in the applications such as those of the analysis of composite materials.

Keywords: singularly perturbed domain, Laplace operator, layer potential method

Homogenization of imperfect transmission problems with weakly converging data

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The aim of the talk is to describe the asymptotic behavior, as $\varepsilon \to 0$, of an elliptic problem, with weakly converging data, in an ε-periodic two component composite with imperfect inclusions of size ε. On the interface we prescribe a jump of the solution that depends on a real parameter γ.
The homogenization results, different according to γ, have as useful application the study of the exact controllability, via HUM method, of a hyperbolic problem set in the same kind of domain and with the same jump condition on the interface.

Based on joint work with Luisa Faella and Carmen Perugia.

Keywords: Homogenization, imperfect interface, weakly converging data.

A nonlinear problem for the Laplace equation with a degenerating Robin condition

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We investigate the behavior of the solutions of a mixed problem for the Laplace equation in a domain Ω. On a connected component of the boundary $\partial\Omega$ we consider a Neumann condition, whereas in another connected component we consider a nonlinear Robin condition which depends on a positive parameter δ in such a way that for $\delta = 0$ it degenerates into a Neumann condition. For δ small and positive, we prove that the boundary value problem has a solution $u(\delta, \cdot)$. We describe what happens to $u(\delta, \cdot)$ as $\delta \to 0$ by means of representation formulas in terms of real analytic maps. Then we confine ourselves to the linear case and we compute explicitly the power series expansion of the solution.

Based on joint work with G. Mishuris.

Keywords: boundary value problems; integral equations methods; singularly perturbed Robin problem.

Analysis of spectral gaps via Floquet-Bloch-Gelfand decomposition

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In this talk, we consider the asymptotic behavior of the spectrum of the Laplacian in a periodically perforated Dirichlet strip. In particular, under certain restrictions on the width of the strip, we show that the continuous spectrum has always has non-empty spectral gaps and their number grows indefinitely when the size of the holes goes to zero. We derive asymptotic formulas for the endpoints of the spectral bands and gaps. In order to derive these bounds we apply Floquet-Bloch-Gelfand transform, techniques of matched asymptotic expansions and of convergence of solutions in homogenization theory, along with techniques from the spectral perturbation theory.

Work in collaboration with M.E. Perez and S. Nazarov.

Keywords: spectral analysis, homogenization, band-gap structure.

Homogenization and dimension reduction for a textile shell

Julia ORLIK

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Textile structure is a network of single yarns, curves cylinders, being in contact. First, a three dimensional elasticity problem for a woven periodic textile plate with contact between yarns is considered. On the microscopic scale, a decomposition of displacement, suggested by Griso in 2008, is used, to obtain Korn-like inequalities for the single yarns depending on the small parameters. From this local estimates we derive global ones for specifically extended fields. To this end an adapted interpolation is introduced only depending on displacements given in the contact-nodes. Finally, we give an unfolding operator modified for the woven textile and its special periodicity-cell and apply the technique of periodic unfolding to obtain the homogenized limit equation. The linear elasticity added by local contact conditions in limit results to a shell model with coupled membrane and bending deformation components and can reason a wrinkling of textiles.

Based on joint work with Georges Griso and Stephan Wackerle.

Keywords: homogenization, thin strucrures, contact.

Asymptotics for spectral problems with mass concentration in small regions near the boundary

Maria Eugenia PÉREZ

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We consider the homogenization of spectral problems for different operators in a bounded domain of $\mathbb{R}^{n+} \equiv \{x \in \mathbb{R}^n : x_n > 0\}$, $n = 2, 3$. The spectral parameter arises on the boundary conditions in small regions, of size $O(\varepsilon)$, periodically placed along $\{x_n = 0\}$. These boundary conditions are of Steklov type, while a Dirichlet condition is imposed in the rest of the boundary. ε is a small parameter that measures the period of the structure, and we look at the asymptotic behavior of the eigenvalues when $\varepsilon \to 0$. We provide an overview of some results for the Laplace operator for the elasticity system from 2006 till 2018, as well some comparisons with results in the case in which the mass concentration affects small regions in volume near the boundary (2009-2018).

Some of these results are based in joint works with D. Gómez, M. Lobo, S.A. Nazarov.

Keywords: singularly perturbed spectral problems, boundary homogenization, Steklov problem.

Asymptotic behaviour of the exact controls for hyperbolic problems with imperfect interface

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The aim of the talk is to study the asymptotic behaviour of an exact controllability problem for a second order linear evolution equation defined in a two-component composite with ε-periodic disconnected inclusions of size ε. On the interface we prescribe a jump of the solution that varies according to a real parameter γ. The case $\gamma = 1$ is the most interesting one, since the homogenized problem is represented by a coupled system of a P.D.E. and an O.D.E., giving rise to a memory effect. Our approach to exact controllability consists in applying the Hilbert Uniqueness Method, introduced by J.-L. Lions, which leads us to the construction of the exact control as the solution of a transposed problem. The main result is that the exact control and the corresponding solution of the ε-problem converge to the exact control of the homogenized problem and to the corresponding solution respectively.

Based on joint work with Luisa Faella and Sara Monsurrò.

Keywords: Homogenization, exact controllability, evolution equations.

Homogenization of a class of singular elliptic problems in perforated domains

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The talk is devoted to present the asymptotic behaviour of a class of quasilinear elliptic problems posed in a domain perforated by ε-periodic holes of ε-size. The quasilinear equation presents a nonlinear singular lower order term, which is the product of a continuous function ζ (singular in zero) and a function f whose summability depends on the growth of ζ near its singularity. We prescribe a nonlinear Robin condition on the boundary of the holes and a homogeneous Dirichlet condition on the exterior boundary.
The main tool for proving the homogenization result is a convergence result stating that the gradient of the solution u_ε behaves like that of the solution of a suitable linear problem associated with a weak cluster point of the sequence $\{u_\varepsilon\}$, as $\varepsilon \to 0$. This idea was originally introduced in the literature for the homogenization of nonlinear problems with quadratic growth with respect to the gradient. In our case, this allows us not only to pass to the limit in the quasilinear term, but also to study the singular term near its singularity, via an accurate a priori estimate.
The difficulties due to the singular term and the boundary nonlinear term are treated using the periodic unfolding method. We also obtain a corrector result for our problem.

Based on joint work with P. Donato and S. Monsurrò.

Keywords: homogenization, singular, perforated.

Asymptotic behavior of structures made of beams

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We consider a periodic structure made of cylinders of length of order ε and which cross section are discs of radius r. Our aim is to give the asymptotic behavior of a solution to the elasticity problem when $\varepsilon \to 0$ and $r \to 0$. In order to obtain apriori estimates for the solution we use displacement decomposition for a beam introduced by G. Griso (G. Griso, Decompositions of displacements of thin structures, J. Math. Pures Appl. 89, 199-223, 2008). The limit problem is obtained using the periodic unfolding method (see, e.g., D. Cioranescu, A. Damlamian, G. Griso, The periodic unfolding method in homogenization, SIAM J. Math. Anal. Vol.40, No.4, 1585-1620, 2008).

Based on joint work with G. Griso (Laboratoire J.-L. Lions, UPMC, Paris IV, France) and Ju. Orlik (Fraunhofer ITWM, Kaiserslautern, Germany).

Keywords: homogenization, thin strucrures, unfolding.

Spectral Laplace problems on periodic and cuspidal domains.

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First, we consider the spectral Dirichlet-Laplace problem on a domain which is formed from a periodic domain $\Omega \subset \mathbb{R}^2$ perturbed by non-compact, non-periodic changes of geometry. We show that the domain perturbation causes an addition to the essential spectrum, which consists of isolated points belonging to the discrete spectrum of a model problem, posed on a domain, which is a compact perturbation of Ω.
Second, we investigate eigenfunctions of the Neumann Laplacian in a bounded domain $D \subset \mathbb{R}^n$, $n > 1$, where a cuspidal singularity is caused by a cavity consisting of two touching balls. We prove for example that if $n > 2$, the gradients of the eigenfunctions have a strong singularity which can be as large as of order $|x - O|^{-2+\sqrt{2}}$ at the point of tangency O.

Based on joint work with Sergey A. Nazarov.

Keywords: periodic domain, non-compact perturbation, essential spectrum.

Optimal design of thin strings with 'gap'

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Results devoted to obtain a measure representation for functionals arising in the context of optimal design problems under non-standard growth conditions and perimeter penalization will be presented. In particular Applications to modelling of strings will be also provided.

Based on joint work with Ana Cristina Barroso.

Keywords: non-standard growth conditions, optimal design, sets of finite perimeter.

3 Boundary-Domain Integral Equations Mini-Symposium

Two-operator boundary-domain integral equations for variable-coefficient Dirichlet BVP with general data

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For a second-order scalar elliptic differential equation with variable coefficient, the Dirichlet BVP with the PDE right-hand side from $H^{-1}(\Omega)$ or $\widetilde{H}^{-1}(\Omega)$ is considered. Applying the two-operator approach in appropriate settings, the problem is reduced to two different systems of two-operator boundary-domain integral equations (BDIEs). It is proved that both BDIE systems are equivalent to original BVP, solvability of BDIE system and invertibility of the boundary-domain integral operators are also proved in the appropriate Sobolev spaces.

Based on joint work with Sergey E. Mikhailov.

Keywords: variable coefficient PDEs, parametrix, analysis of BDIE systems.

Two-operator Boundary-Domain Integral Equations for variable coefficient Dirichlet boundary value problem in 2D

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The Dirichlet boundary value problem (BVP) for the second order "stationary heat transfer" elliptic partial differential equation with variable coefficient is considered in two-dimensional bounded domain. Using an appropriate parametrix (Levi function) and applying the two-operator approach, this problem is reduced to two systems of boundary-domain integral equations (BDIEs). The two-operator BDIEs in 2D have special consideration due to their different equivalence properties as compared to 3D case. Consequently, we need to set conditions on the domain or function spaces for the invertibility of the corresponding layer potentials, and hence the unique solvability of BDIEs. Equivalence of the two operator BDIE systems to the Dirichlet BVP and invertibility of the corresponding boundary domain integral operators are investigated in appropraite Sobolev-Slobodetski (Bessel potential) spaces.

Based on joint work with T.G. Ayele.

Keywords: Two-operator BDIEs, equivalence, invertibility.

Boundary-Domain Integral Equation Systems to the Dirichlet BVP for Stokes Equations with variable viscosity in 2D

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The Dirichlet boundary value problem for the steady-state Stokes system of Partial Differential Equations for an incompressible viscous fluid with variable viscosity coefficient is considered in two dimensional bounded domain. Using an appropriate parametrix, this problem is reduced to a system of direct segregated Boundary-Domain Integral Equations (BDIEs).The BDIEs in the two-dimensional case have special properties in comparison with the three dimension because of the logarithmic term in the parametrix for the associated partial differential equation. Consequently, we need to set conditions on the function spaces for the invertibility of corresponding parametrix-based hydrodaynamic single layer potential and hence the unique solvability of BDIEs. Equivalence of the BDIE systems to the dirichlet BVP and invertibility of the corresponding boundary domain integral operators in appropriate Sobolev (Bessel potential) spaces are shown.

Based on joint work with T.G.Ayele.

Keywords: Boundary-Domain Integral Equations, equivalence, invertibility.

Boundary-Domain Integral Equations to the Mixed BVP for a Compressible Stokes System with Variable Viscosity

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The mixed boundary value problem for a compressible Stokes system of partial differential equations in a bounded domain is reduced to two different systems of segregated direct Boundary Integral Equations (BDIEs) expressed in terms of surface and volume parametrix-based potential type operators. Equivalence of the BDIE systems to the mixed BVP and invertibility of the matrix operators associated with the BDIE systems are proved in appropriate Sobolev spaces.

Based on joint work with Sergey E. Mikhailov.

Keywords: Boundary-Domain Integral Equations,Compressible Stokes System, Variable Viscosity.

Layer potentials and Poisson problems for the Stokes system with nonsmooth coefficients in Sobolev and Besov spaces

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In this talk we present recent well-posedness results for Poisson problems of Dirichlet, or mixed type for the Stokes and Brinkman systems with nonsmooth coefficients and data in L^p-based Sobolev and Besov spaces in Lipschitz domains in Euclidean setting or in compact Riemannian manifolds. First, we focus on a variational approach that reduces boundary problems of transmission, Dirichlet and mixed type for the Stokes or Brinkman system to equivalent mixed variational formulations with data in L^p-based Sobolev and Besov spaces in Lipschitz domains. The isomorphism property of an operator related to the variational formulation of such a boundary value problem in L^p-based Sobolev spaces, with p in a neighborhood of 2, yields to the well-posedness result of the corresponding boundary value problem. Next, the well-posedness results for the analyzed transmission problems allow us to define the layer potentials for the nonsmooth coefficient Stokes or Brinkman system and to obtain their properties in L^p-based Sobolev and Besov spaces. Then the solution of the Poisson problem of Dirichlet type is constructed explicitly in terms of such layer potentials.

Based on joint work with Massimo Lanza de Cristoforis (Padova), Sergey E. Mikhailov (London) and Wolfgang L. Wendland (Stuttgart).

Keywords: Stokes and Brinkman systems with nonsmooth coefficients, variational approach and layer potentials, Sobolev and Besov spaces.

Regularizing properties of periodic layer heat potentials and applications

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As is well known, integral equation methods based on layer heat potentials have been largely exploited for solving corresponding boundary value problems in bounded domains. In this talk we wish to provide analogous tools in order to analyze the behavior of the temperature in an unbounded space-periodic domain.

First we present some regularizing properties for classical layer heat potential, then exploiting these results together with an analysis of the periodic fundamental solution of the heat equation, we can prove some regularizing properties in parabolic Schauder spaces for the periodic case. As an application we consider some initial-boundary value problems for the heat equation in a periodic setting.

Keywords: Heat equation, integral operators in parabolic Schauder spaces, space-periodic layer heat potentials.

Boundary-domain integral equations for variable-viscosity Stokes and Brinkman systems in L_p-based spaces on Lipschitz domains

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In this presentation we show well-posedness results in L_p-based Sobolev spaces for boundary value problems of Robin type for the Stokes and Brinkman systems in a bounded Lipschitz domain in $\mathbb{R}^3$ with the variable viscosity coefficient and data in L_p-based Sobolev and Besov spaces. First, we introduce a parametrix and construct the corresponding parametrix-based variable-coefficient Stokes Newtonian and layer integral potential operators with densities and the viscosity coefficient in L_p-based Sobolev or Besov spaces. Then we generalize various properties of these potentials, known for the Stokes system with constant coefficients, to the case of the Stokes system with variable coefficients. Next, we show that the Robin boundary value problem for the Stokes system with variable coefficients is equivalent to a system of segregated Boundary-Domain Integral Equations (BDIEs). Then we prove that solvability of the variable coefficient system of BDIEs can be reduced to the solvability of a corresponding problem with constant coefficients in L_p-based Sobolev and Besov spaces, which we show to have a unique solution, by exploiting known results for the Robin boundary value problem associated to the Stokes system. Finally, the well-posedness for the Stokes system is used to reduce the Robin problem for the variable coefficient Brinkman system to an equivalent Fredholm equation which is uniquely solvable in L_p-based Sobolev and Besov spaces.

Based on joint work with Mirela Kohr and Massimo Lanza de Cristoforis.

Keywords: Variable-coefficient Brinkman system, Boundary-domain integral equations, L_p-based Sobolev spaces.

Operator perturbation approach for fourth order elliptic equations with variable coefficients

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The homogenization of elliptic divergence-type fourth-order operators with periodic coefficients is studied in a periodic 2-D domain. The aim is to find an operator with constant coefficients and represent the equation through a perturbation around this operator. The resolvent is found as $L^2 \to L^2$ operator using the Neumann series for the periodic fundamental solution of biharmonic operator. Results are based on some auxiliary Lemmas suggested by Bensoussan in 1986, Zhikov in 1991, Yu. Grabovsky and G. Milton in 1998, Pastukhova in 2016. Operators of the type considered in the paper appear in the study of the elastic properties of thin plates. The choice of the operator with constant coefficients is discussed separately. It is chosen in an optimal way w.r.t. the spectral radius and convergence of the Neumann series and uses the known bounds for "homogenized" coefficients. Same idea is usually used in iterative numerical methods with pre-conditioners for finite dimensional problems or discretized PDEs. The presented method is similar to Cholesky factorization transferred to elliptic operators (as in references mentioned above). The method can be applied to non-linear problems.

Based on joint work with H. Andrä and S. Staub (Fraunhofer ITWM, Kaiserslautern, Germany).

Keywords: homogenization for high order equations, Neumann series, preconditioning of elliptic operators.

On analysis of united boundary-domain integro-differential equations for variable coefficient Dirichlet problem with general right hand side

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The Dirichlet boundary value problem for second order elliptic Partial differential equation with variable coefficient is considered. Using an appropriate parametrix (Levi function), this problem is reduced to some united boundary-domain integro-differential equation (BDIDE) or the domain integral equation may be supplemented by the original boundary condition thus constituting boundary-domain integro-differential problem (BDIDP). The PDE right hand side belong to the Sobolev spaces $H^{-1}(\Omega)$ when neither classical nor canonical co-normal derivatives are well defined. Solvability, solution uniqueness, and equivalence of the BDIDE/BDIDP to the original BVP, as well as invertibility of the associated operators are analysed in Sobolev (Bessel potential) spaces.

Based on joint work with Sergey E. Mikhailov (Brunel University London)

Keywords: Integro-differential equations, equivalence, invertibility.

4 Contributed Papers

Transient Convection-Diffusion-Reaction Problems with Variable Velocity Field using DRBEM with Different Radial Basis Functions

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This work describes a novel numerical formulation of the dual reciprocity boundary element method (DRBEM) for two-dimensional transient convection-diffusion-reaction problems with variable velocity. Firstly, the formulation splits the velocity field into an average (constant) and a perturbation (variable) part, with the latter being treated using a dual reciprocity approximation to convert the domain integrals arising in the boundary element formulation into equivalent boundary integrals. The integral representation formula for the convection-diffusion-reaction problem with variable velocity is obtained from the Green's second identity, using the fundamental solution of the corresponding steady-state equation with constant coefficients. Another objective is to discuss the treatment of the convective terms, which involve gradients of the problem variable, and their modeling using DRM. A finite difference method (FDM) is used to simulate the time evolution procedure for solving the resulting system of equations. Numerical experiments are included for two different problems for which analytical solutions are available, to establish the validity of the proposed approach and to demonstrate the efficiency of the proposed technique. Finally, the results obtained show an excellent agreement with the analytical solutions and do not show oscillations or damping of the wave front, as appear in other numerical techniques.

Based on joint work with Luiz C. WROBEL.

Keywords: DRBEM, Transient convection-diffusion-reaction, Variable velocity, Radial basis functions, Time-marching schemes.

q-Calculus formalism for non-extensive particle filter

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A class of sequential Monte Carlo estimation is frequently called particle filter. This filter belongs to the Bayesian strategy for estimation, where a no-linear and no-Gaussian assumptions can be applied. Here, the Tsallis' distribution, from the non-extensive thermo-statistics, is used to design the best likelihood operator. Therefore, no previous likelihood operator is assumed. The new filter formulation will be named as *non-extensive particle filter* (NEx-PF). The distribution estimated by the NEc-PF can compute the standard form of the central limit theorem, as weel as the Levy-Gnedenko central limit theorem. The q-calculus formalism is employed to generalize some definitions and properties.

Based on joint work with Helaine C. M. FURTADO* and Haroldo F. de CAMPOS VELHO+
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Keywords: q-Calculus, non-extensive entropy, particle filter.

Alternative propositions of Cross Wavelet Analysis for use in non synchronized transient detection problem

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Since the definitions of the Wavelet Power Spectrum (WPS), the Wavelet Cross Spectrum (WCS) and Wavelet Coherence Analysis (WCA), it is an established practice to obtain these from point-by-point multiplication of the continuous wavelet transform (CWT) of the time series. However, once transients appear at different times and for different realizations from processes, the estimators of WPS and WCS will be inconclusive, due to the occurrence of as many local maxima as there are distinct transients in the time series. Therefore, they cannot be properly used for detection of these non synchronized transients. Furthermore, the estimator of wavelet coherency WCO has the inherent pitfall of being identical to unity as documented in the literature already decades ago.

Hence, the objective of this work is to propose an alternative definition for WPS, WCS and WCA in a similar way as the Wiener-Khintchine relations. Thus, as the present discussion shows these definitions circumvent the detection of non synchronized transients in a Gaussian noise and thus are more adequate for this type of analysis. To exemplify this, a detection problem is shown with simulated signals where the classical and new definitions are used. A comparison between the results is then performed, pointing out relevant aspects of each technique.

After all, it would be desirable to develop a more complete theoretical framework, which we postpone to future discussions, but with our alternative definition we believe to have opened a new pathway that does not have the drawback of the traditional analysis. Moreover, we are completely aware of the fact, that with the present work we proposed an alternative only, nevertheless pointing towards the possibilities that arise from such a new definition.

Based on joint work with Adalberto SCHUCK.

Keywords: Wavelet Power Spectrum, Wavelet Cross Spectrum, Wavelet Coherence

Approximations of spectra of Schrödinger operators with complex potentials on $\mathbb{R}^d$

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We study spectral convergence for sequences of unbounded linear operators T_n, $n \in \mathbb{N}$, that converge to some operator T in strong resolvent sense. It is well known that, even in the case of purely discrete spectra, the eigenvalues of T_n may accumulate at a point that is not an eigenvalue of T. In addition to the occurrence of such spurious eigenvalues, for non-selfadjoint operators not every eigenvalue of T may be approximated. We present sufficient conditions and perturbation results that prevent the occurrence of these two unwanted phenomena. The results are applied to non-selfadjoint Schrödinger operators in $L^2(\mathbb{R}^d)$ that are truncated to bounded but expanding domains in $\mathbb{R}^d$.

Based on joint work with P. Siegl and C. Tretter.

Keywords: non-selfadjoint Schrödinger operators, domain truncation, approximation of eigenvalues.

Parametric solutions in phase space of the neutron transport equation by Hermite polynomials

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This work uses a Monte Carlo simulator to compute the tracking and interaction of neutrons in a nuclear reactor. This simulator utilizes continuous energy cross-section functions, and the simulation of the transport equation is continuous in all seven dimensions of phase space. A simulation was performed for 100000 neutrons in a homogeneous reactor with uniform temperature, and only the stationary case was considered. The purpose of this work is to present approximative parametric solutions for the stationary Boltzmann neutron transport equation. To this end the simulation data is analyzed in order to interpolate projections in phase space. The probability distributions that represent the neutron population can then be obtained by fitting Hermite polynomials to the projections of the simulation data.

Based on joint work with Bardo BODMANN, Sérgio BOGADO LEITE and Marco Túlio VILHENA.

Keywords: Monte Carlo simulation, neutron transport equation, probability distribution functions.

Bending of Plates with Transverse Shear Deformation: The Robin Problem.

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The Robin problem in a finite domain for the system of equations describing the bending of elastic plates with transverse shear deformation is solved by means of a generalized Fourier series method based on the structure of the governing system. Numerical illustration of the results, obtained by three different computational methods, is also provided.

The adjoint spectral Green's function method applied to direct and inverse neutral particle source–detector problems

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In direct source–detector problems the use of the adjoint technique allows to obtain the detector response due to multiple sources by a single solution to the adjoint problem in each energy group. On the other hand, in inverse source–detector problems it is possible to calculate the intensity of the source in each energy group given its location and the detector response. This work is based on the application of the adjoint spectral Green's function method (SGF†) for solving direct and inverse source–detector transport problems in the energy multigroup discrete ordinates formulation with arbitrary $L'th$–order of scattering anisotropy. The offered SGF† method along with the one–region block inversion iterative scheme generates numerical solutions that are completely free from spatial truncation errors; therefore a spatial reconstruction scheme is developed to analytically determine the detector response in direct problems and source intensities in inverse problems.

Based on joint work with Odair P. da Silva and Ricardo C. Barros.

Keywords: multigroup adjoint discrete ordinates, source–detector problems, inverse source–detector problems

Image processing for UAV autonomous navigation applying self-configuring neural network

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Application and development of Unmanned Aerial Vehicles (UAV) have had a rapid growth. The flight control of these aircarfts can be performed remotely or autonomously. There are different strategies for the UAV autonomous navigation. The positioning estimation can be done by using inertial sensors and General Navigation Satellite Systems (GNSS). The use of the GNSS signal can present some difficulties: natural or not natural interference. An alternative for positioning adjustment is to use a data fusion from different sensors by a Kalman filter. A supervised artificial network (ANN) is trained to emulate the filter for reducing the computational effort. An automatic best topology for the neural network is obtained by minimizing a functional by a new meta-heurisc called Multi-Particle Collision Algorithm (MPCA). Our results show similar accuracy between the ANN and the Kalman filter, with better processing performance to the neural network.

Based on joint work with Haroldo F. de CAMPOS VELHO and Elcio H. SHIGUEMORI

Keywords: Unmanned aerial vehicles, Kalman filter, artificial neural networks.

On the LTS_N nodal solution for the angular neutron flux in a two-dimensional Cartesian domain: A new approach to evaluate the exiting flux at the boundary

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In literature, a great variety of solutions are found for the neutron nodal S_N equation in a rectangular domain, whose main idea consists of the transverse integration of the S_N equation. This procedure results in coupled one-dimensional S_N equations, by additional unknown functions, that is, the angular flux at the border. In order to solve these problems using classical methods for the S_N problem in a sheet, we must presuppose the outgoing angular flow at the border. In the literature, usually, the outgoing angular flux is considered as a constant or exponential decreasing function. At this point, it is noteworthy to point out that the solutions found with this method present some incorrect physical results to the angular flux at the border (negative fluxes), whereas the scalar flux is semi-positive definite. To overcome this disadvantage and also to facilitate the application of boundary conditions, a new approach is proposed in this work. The problem in the rectangular domain is covered by a finite discrete set of rectangular sub-domains, that are narrow when compared to their lengths, so that in each rectangle of the coverage the problem of the transport of neutrons is approached by a one-dimensional problem. Thus, by applying the LTS_N method, combined with the DNI technique in order to interpolate the directions of the two-dimensional problem by means of one-dimensional directions, it is possible to obtain the angular flow at the border with the one-dimensional LTS_N solution on the desired points. Numerical simulations and comparisons with the results found in literature are presented.

Based on joint work with Cynthia F. SEGATTO and Bardo BODMANN.

Keywords: LTS_N method, angular neutron flux, reconstruction of exiting flux

Reconstruction of small particles with whispering gallery resonators

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Optical micro-cavities that confine light at selected frequencies have been recently proved as detection mechanisms able to discern particles down to the single molecule level. We present a mathematical and numerical framework for the reconstruction of small particles from the perturbation of whispering gallery modes which arise in optical resonators. These reconstruction techniques are based on some classic results in eigenvalue perturbation theory and on some new insights. In particular we review some recent results regarding the asymptotic behavior of eigenvalues of elliptic operators under small singular domain perturbations and new directions for the analysis of quasinormal modes for open resonators.

Keywords: Eigenmode, resonance, reconstruction.

A comparison between two models for suspension bridges, introducing the convexification of the cables

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Inspired by the Melan equation we propose two isolated models for suspension bridges with two cables linked to a deck through hangers, supposed inextensible in a case and possibly slackened in the second one. The aim is to study the torsional stability of the system, since suspension bridges are subject to such phenomena.
The slackening of the hangers is obtained through a two DOF model applying a geometrical procedure that maintains the convexity of the cables shape even if the deck moves upwards; this fact produces an involved system of nonlinear and nonlocal hyperbolic PDE. We prove existence and uniqueness of a weak solution and we perform some numerical experiments on it.
It turns out that the slackening mechanism hidden in the convexification of the cables yields energy thresholds of instability significantly smaller than in models where slackening is neglected.

Based on joint work with Graziano CRASTA and Filippo GAZZOLA.

Keywords: instability, slackening, convexification.

On Multi-group Transport Equation in Planar Geometry: Solution for Anisotropic Scattering.

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In this work we present a hierarchical solution for the anisotropic multi-group neutrons transport equation starting from the knowledge of the solution for the isotropic problem. The basic idea consists in the application of the idea of decomposition method to construct a recursive system of equation which has the main feature. The first equation is a multi-group isotropic neutron transport having the boundary conditions of the considered problem. On the other hand, the sources in the remaining equations carries the information of the anisotropic scattering. These sources are know because are evaluated from the knowledge of the solution in the previous equation od this system. Further, these equations fulfills until boundary condition. The size of the recursive system determines the desired accuracy for the results. We also report numerical results and comparisons against the ones of literature.

Based on joint work with Fernando OLIVEIRA, Marco VILHENA and Bardo BODMANN.

Keywords: Transport Theory, Nuclear Reactor, Anisotropic Scattering Neutron, Laplace and Fourier Transforms.

Singularity subtraction approximation for nonlinear weakly singular Fredholm integral equations of the second kind

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The well known singularity subtraction numerical method for linear Fredholm weakly singular integral equations of the second kind is generalized to the case of nonlinear equations. Convergence is proven under rather standard hypotheses on the nonlinearity, and using both the fundamental Inverse Function theorem and the notion of a ν-convergent sequence of bounded linear operators in the real Banach space of real-valued continuous functions defined on a compact interval of the real line, equipped with the uniform convergence norm, a notion introduced by M. Ahues, one of the authors of this presentation, in a co-authored book with A. Largillier and B.V. Limaye, about spectral approximation for bounded operators in 2001. Numerical experiments confirm the theoretical results with a Hammerstein equation.

Based on joint work with Mario Ahues, Filomena Dias d'Almeida, and Paulo Vasconcelos.

Keywords: Singularity, nonlinear, integrals.

A cohesive zone approach analysing time dependent effects in viscoelastic materials

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A history-dependent cohesive zone model approach is used to study the crack behaviour in visco-elasto-plastic materials. The cohesive (yield) stress at the cohesive zone points is related to the nonlinear normalised equivalent stress functional over the stress history at these points, and is expressed in the form of an Abel-type (fractional) integral. We analyse the cohesive zone length evolution in time and the crack tip opening during the stationary crack stage as well as during the propagating crack stage. We consider the external load increasing linearly with time and compare the solution with the case of the constant load. We obtain the solution numerically and analyse the influence of the viscoelasticity by comparing with the case of purely elastic behaviour of the bulk of the material.

Based on joint work with Professor Sergey Mikhailov, Brunel University London.

Keywords: Cohesive zone, Time dependent load, Abel integral equation.

A combined boundary element and finite element model of cell motion due to chemotaxis

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Chemotaxis is the biological process whereby a cell moves in the direction in which the concentration of a chemical in the fluid medium surrounding the cell is increasing. In some cases of chemotaxis, cells secrete the chemical in order to create a concentration gradient that will attract other nearby cells to form clusters. When the cell secreting the chemical is stationary the linear diffusion equation can be used to model the concentration of the chemical as it spreads out into the surrounding fluid medium. However, if the cell is moving then its motion, and the resulting motion of the surrounding fluid, need to be taken into account in any model of how the chemical spreads out. In the case of a single, circular cell it is possible to express the fluid velocity which results from the motion of the cell in terms of a dipole located at the centre of the cell. However if the cell is not circular and/or there is more than one cell in the fluid, a more sophisticated method of determining the fluid velocity is needed.

This paper presents a mathematical model for simulating the concentrations of chemical secreted into the surrounding fluid medium from a moving cell. The boundary integral method is used to determine the velocity of the fluid due to the motion of the cell. The concentration of the chemical in the fluid is modelled by the convection-diffusion equation where the fluid velocity term is that given by the boundary integral equation. The resulting differential equation is then solved using the finite element method.

The method is illustrated with a number of typical examples.

Keywords: Boundary element, finite element, chemotaxis.

Numerical Calculation by quadruple precision higher order Taylor series method of The Pythagorean problem of three bodies

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The Pythagorean problem of three bodies(Baurrau's problem) is studied by C.Burrau in 1913. By Szebehely, Yale University in 1967, using Levi-Civita transformation, it was solved by numerical computation.

In this talk, it is shown that it's possible to get a highly precise calculation result with higher order Taylor series method of high precision (24th order, the quadruple precision) without special transformations.

Keywords: higher order Taylor series method, ordinary differential equation, Pythagorean problem of three bodies.

Scattering by an Open Tubular Waveguide in R^3 with a Period Refractive Index

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Scattering of time-harmonic waves from periodic structures at some fixed real-valued wave number becomes analytically difficult whenever there arise surface waves: These non-zero solutions to the homogeneous scattering problem physically correspond to modes propagating along the periodic structure and clearly imply non-uniqueness of any solution to the scattering problem. In this paper, we consider a medium that is defined by a space dependent contrast function which is periodic with respect to the x_3-axis and constant outside of a cylinder in x_3-direction. We prove that there is a so-called limiting absorption solution to the associated scattering problem. By definition, such a solution is the limit of a sequence of unique solutions for artificial complex-valued wave numbers tending to the above-mentioned real-valued wave number. Our method of proof seems to be new: By the Floquet-Bloch transform we first reduce the scattering problem to a finite-dimensional one that is set in the linear space spanned by all surface waves. In this space, we then compute explicitly which modes propagate along the periodic structure to the left or to the right. This finally yields a representation for our limiting absorption solution which leads to a proper radiation condition. Finally, we show how to prove uniqueness and existence of a solution under this radiation condition (the latter without making use of the limiting absorption principle).

Keywords: scattering, open wave guide, radiation condition.

Shape optimization for interior Neumann and transmission eigenvalues

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Shape optimization problems for interior eigenvalues is a very challenging task since already the computation of interior eigenvalues for a given shape is far from trivial.

For example, a maximizer with respect to shapes of fixed area is theoretically established only for the first two non-trivial Neumann eigenvalues. The existence of such a maximizer for higher Neumann eigenvalues is still unknown. Hence, the problem should be addressed numerically. Better numerical results are achieved for the maximization of some Neumann eigenvalues using boundary integral equations for a simplified parametrization of the boundary in combination with a non-linear eigenvalue solver.

Shape optimization for interior transmission eigenvalues is even more complicated since the corresponding transmission problem is non-self-adjoint and non-elliptic. For the first time numerical results are presented for the minimization of interior transmission eigenvalues for which no single theoretical result is yet available.

Keywords: shape optimization, interior transmission eigenvalues, boundary integral equations

Integro-differential Radiative Conductive Transfer Equation: Modified Decomposition Method.

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In this work we present a solution for the radiative conductive transfer equation in spherical geometry. We discuss a semi-analytical approach to the non-linear S_N problem, where the solution is constructed by Laplace transform and a decomposition method. In the present discussion, we report on arithmetic stability issues for the recursive scheme. In order to stabilise convergence by virtue of limited arithmetic precision, we propose a modification of the Adomian decomposition method by splitting responsible terms for instability, i.e. the source terms constructed from the non-linearity and regrouping them in a finite number of source terms. This procedure results then in a finite number of modified recursion steps controlled by a splitting parameter α, whereas all recursion steps beyond the usual recursion scheme is used. Finally, we report on some case studies with numerical results for the solutions and convergence behaviour.

Based on joint work with Bardo E.J. BODMANN, Marco Túllio DE VILHENA.

Keywords: Radiative conductive transfer equation, spherical geometry, semi-analytical solution, modified decomposition method.

A new class of Volterra-type integral equations from relativistic quantum physics

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In this talk, I present a new kind of linear integral equations for a relativistic quantum-mechanical two-particle wave function $\psi(x_1, x_2)$, where x_1, x_2 are spacetime points. In the case of retarded interaction, these integral equations feature a Volterra structure in the time variables. They are interesting not only in view of their applications in physics, but also because of the following mathematical features: (a) time and space variables are more interrelated than in normal time-dependent problems, (b) the integral kernels are singular, and the structure of these singularities is non-trivial, (c) they feature time delay. I will give some examples of such equations and sketch how to prove the existence and uniqueness of solutions for them.

Based on joint work with Roderich Tumulka.

Keywords: multi-dimensional Volterra integral equations, time delay, singular integral equations

Generalized Jacobi polynomials for the Cauchy integral equation of the second kind

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In this work, a projection method is formulated and applied to numerically solve the Cauchy integral equation

$$x(s) - \frac{1}{\pi} \oint_{-1}^{1} \frac{x(t)}{s-t} dt = f(s), \quad -1 < s < 1, \tag{1}$$

where the integral is a Cauchy principal value, based on the generalized Jacobi polynomials.

The convergence analysis and associated theorems are presented and proved.

Moreover, numerical examples illustrate the theoretical results and show the effectiveness of the method.

Keywords: Cauchy kernel, generalized Jacobi polynomials, integral equation.

Existence of the solutions for a singularly perturbed nonlinear non-autonomous transmission problem

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In this talk we analyse a boundary value problem for the Laplace equation with a nonlinear non-autonomous transmission condition on the boundary of a small inclusion of size ϵ. We show that the problem has solutions for ϵ small enough and we investigate the dependence of a specific family of solutions upon ϵ. By adopting a functional analytic approach we prove that the map which takes ϵ to (suitable restrictions of) the corresponding solution can be represented in terms of real analytic functions.

Keywords: nonlinear non-autonomous transmission problem, singularly perturbed perforated domain, Laplace operator

On The Derivation of The Solution of The Neutron Diffusion Kinetic Equation in Cylindrical Geometry Starting From The Cartesian Geometry Solution

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In this work we construct a hierarchical solution of the neutron diffusion kinetic equation in cylindrical geometry starting from the knowledge of the solution in cartesian geometry, which plays the role of the recursive inisialisation, we specialize the solution derivation for one-dimensional problem. For such, following the idea of decomposition method we construct a recursive system of neutron diffusion kinetic equations such that the source terms of all subsequents recursive steps carries the information of the cylindrical curvature. The main feature relies in the fact that the first equation is a neutron diffusion kinetic equation, meanwhile, the remaining equations have a known source that scales the cylindrical curvature. In fact, the source is evaluated from the knowledge of the solution of the previous equation. The suitable choice of the length of this system allows us to determine solution with a prescribed accuracy. Finally, we report numerical comparisons against ones of the literature for cylindrical geometry.

Based on joint work with Julio Fernandes, Bardo E. J. Bodmann and Marco T. de Vilhena.

Keywords: Neutron transport.

Extension of the Fully Lagrangian approach for the integration of the droplet number density on caustic formations.

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The spatial structure of agglomeration regions in dispersed flows is studied. The Fully Lagrangian Approach (FLA) has been extended to account for the Hessian of the deformed dispersed phase continuum defined for the transformation of the Lagrangian to the Eulerian variables. The Hessian is calculated by integrating an initial value problem along the droplet trajectory. The initial value problem for the Hessian is similar to the initial value problem used for the calculation of the Jacobian of the transformation as introduced in the standard FLA. This second order description of the structure for the dispersed phase distribution allows for the calculation of the spatially filtered number density.

Keywords: Fully Lagrangian Approach, second order structure, caustics

A framework to determine black-hole distribution for large redshift

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Massive and super-massive black-holes (BH) can be find in the center of all galaxies, according to the opinion from the scientific community. However, an open question is to know the origin of such BHs. One explanation is consider such objects coming from the stars with large redshifts – very old stars colapsed during the ancient eras (population-III stars: the *first-born* stars created in the universe). A framework to investigate this scenario is to derive a mathematical model describing the evolution of the black hole distribution. From the distribution observed nowadays, the problem is formulated as an inverse problem to determine a possible disrtibution for large redshift, where the cost function is given by the square difference between the observation and the model data under association with a regularization operator. The regularized inverse solution can be computed using an optimizer to identify the best solution for the functional to be minimized. Preliminar results are shown using synthetic observational data.

Based on joint work with Pedro A. SANTOS* and Haroldo F. de CAMPOS VELHO*
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Keywords: Massive and supermassive black holes, inverse problem, regularized solution.

Decompositions of solutions of the wave equation in Poincaré wavelets

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We present integral representations of solutions of an initial-boundary value problem for the wave equation in a half-plane. In doing so, we decompose solutions in terms of known localized solutions from a wide class. Our constructions are based on the Poincaré affine wavelet analysis. We prove the convergence of integrals in these representations. We study two cases: an exact representation in a homogeneous medium and an asymptotic one in an inhomogeneous medium. As examples of localized solutions in the case of a homogeneous medium, we use exact solutions known as "Gaussian wave packets". In the case of an inhomogeneous medium, we use asymptotic solutions called "quasiphotons". We provide an example of the implementation of the developed techniques for a problem of seismic migration.

Based on joint work with Eugeny Gorodnitskiy.

Keywords: wave equation, Poincaré wavelet analysis, localized solutions

The method of fundamental solutions and interior transmission eigenvalues

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Interior transmission eigenvalues arise in the study of inverse scattering problems as a prerequisite to make conventional qualitative methods work - at least from a theoretical perspective. Typical areas of practical applications are non-destructive testing or medical imaging. Their accurate and fast calculation is therefore a desired but challenging task due to the non-elliptic, non-self-adjoint and non-linear structure of the underlying eigenproblem.

We present an improved solver for the efficient calculation of approximate interior transmission eigenvalues both for anisotropic and inhomogeneous media based on the method of fundamental solutions. It distinguishes from most of the currently utilized methods since it is mesh- and integration free. As a discretized compact boundary integral operator equation, the main issue to cope with was the ill-conditioning of the resulting matrices which we could tame by a clever orthogonalization procedure. A variety of numerical examples proves our approach to be particularly effective for regular scattering objects.

Based on joint work with Andreas Kleefeld.

Keywords: interior transmission eigenvalues, method of fundamental solutions, anisotropic and inhomogeneous media.

Semi Analytical Pressure Solution for Oil Displacement by Miscible Gas Injection in a Homogeneous Porous Media

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In an Enhanced Oil Recovery (EOR) project, materials not present in the reservoir are injected to improve the final oil recovery. Historically, gas-flooding has been the second most applied EOR method. Recently, carbon dioxide injection has become more attractive because is also environmental friendly. In this work we present a solution for oil displacement by miscible gas injection at constant rate. Our model considers a three-component, two-phase, 1-D incompressible homogeneous isothermal system. Dispersion, gravity and capillary effects are neglected. Moreover, it is assumed that Amagat's law is valid and viscosity depends on the phase composition only. This problem is governed by a system of two hyperbolic equations that is solved by the method of characteristics for saturation and concentrations. Then, the pressure profile is obtained by integrating Darcy's law over the spatial domain. This general solution is applied to a typical set of rock and fluid data.

Based on joint work with Luana CANTAGESSO, Luara SOUSA and Alvaro PERES

Keywords: pressure transient analysis, miscible flooding, Enhanced Oil Recovery.

Solution of a homogeneous version of the Love type integral equation in different asymptotic regimes

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We consider one-dimensional convolution integral equation on an interval of Fredholm second kind whose particular non-homogeneous versions are known as Love, Gaudin and Lieb-Liniger equation arising in rotationally symmetric electrostatic/fluid dynamics problems and quantum-mechanical statistics of Fermi/Bose gases, respectively. Despite seemingly simplicity of the kernel function, this equation evades applicability of majority of the known constructive technics. After discussing general spectral structure of the integral operator and basic properties of eigenfunctions, I present new methods for their asymptotic approximations in regimes of the small and large size of the interval (or, alternatively, the parameter value in the kernel function). I point out connections with prolate spheroidal wave functions and spectrum of the simplest hypersingular integral operator, and then explain asymptotic reducibility of the problem to an equation of Wiener-Hopf type and an elementary second order ODE, both amenable to explicit solution construction. Asymptotic analysis leads to nearly (but not exactly) trigonometric structure of eigenfunctions which is confirmed by numerical simulations.

Based on joint work with Juliette Leblond and Laurent Baratchart.

Keywords: Convolution integral equations on an interval, Love equation, Gaudin/Lieb-Liniger equation.

Stress Concentrations Around Inclusions in Micropolar Elasticity

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We apply the real boundary integral equation method to obtain the solution of inclusion boundary-value problems arising in plane theories of elasticity with significant microstructure. We find the solutions in the form of integral potentials and employ the boundary element method to derive the approximate representation for the corresponding integral density. Finally we consider a number of examples of inclusions of various shapes with a homogeneously imperfect interface and find the distribution of shear stresses along the inclusion interface to demonstrate the effectiveness of the method.

Keywords: micropolar elasticity, boundary integral equation method, inhomogeneities.

Series expansion for the effective conductivity of ideal and non-ideal two-phase dilute composites

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In this talk, we discuss the asymptotic behavior of the effective conductivity in a two-phase dilute composite. The composite is obtained by introducing into an infinite homogeneous matrix a periodic set of inclusions of a different material. The diameter of each inclusion is proportional to a real positive parameter ϵ. We consider the effective conductivity for two cases of contact conditions at a two-phase interface: an ideal contact (the normal component of the heat flux and the temperature field are continuous), a non-ideal contact (the normal component of the heat flux is continuous but the temperature field displays a jump proportional to the normal heat flux).

We will show that the effective conductivity can be represented as a convergent power series of ϵ and a constructive method to compute explicitly the coefficients of such series by solving recursive systems of integral equations.

Based on joint work with M. Dalla Riva and P. Musolino.

Keywords: effective conductivity, transmission problem, singularly perturbed domain.

Approximation of the atomic wave function for perturbed potentials

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The atomic wave function is important because it provides the eigenstates and energy spectrum of the atom. Although the wave function is generally obtained by solving the Schrödinger equation, for many atomic systems, specially those subjected to external perturbations, is not possible to find an analytical solution to said equation. I present an approximation of the atomic wave function for hydrogen-like atoms that experience small local changes in their potential. The calculated wave function is presented as a linear combination of eigenstates of the unperturbed potential, providing a direct relation between the magnitude of the perturbation and the internal degrees of freedom of the atom. The results are relevant for quantum metrology of weak interactions.

Keywords: Atomic wave function, Perturbed potential, Quantum metrology.

Modified projection method for second kind Fredholm integral equations with trignometric polynomials

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Approximation methods that are based on using functions that are globally smooth(over entire domain of integration) are quite often rapidly convergent than are methods based on using piecewise polynomial functions. Let $X = L^2[0, 2\pi]$. Consider the integral operator

$$(Tx)(s) = \int_0^{2\pi} k(s,t)x(t)dt,\ s \in [0, 2\pi],$$

with $k(\cdot,\cdot) \in C([0, 2\pi] \times [0, 2\pi])$ and 2π periodic in both the variables. We are interested in the approximate solution of second kind Fredholm integral equation

$$u - Tu = f$$

Classical methods such as the Galerkin and iterated Galerkin have been studied for approximate solutions of above equations. In the research literature, a modified projection method and its iterated version is proposed in the context of piecewise polynomial functions. It has proved to be better than the classical Galerkin and the iterated Galerkin method. We study the modified projection method and its iterated version, when the approximating space is the space of trigonometric polynomials. We obtain better orders of convergence of the solutions compared to the Galerkin and the iterated Galerkin one. We also expect better results in the collocation methods with trigonometric polynomials.

Keywords: Fredholm integral equations, modified projection method, trigonometric polynomials.

Eliminating chromatic aberration using simple lenses

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Creating an optical system that handles polychromatic radiations has been a challenge in optical engineering. Since different colors moves with different speeds inside a material this leads to focusing error called chromatic aberration. This aberration is corrected using several lenses. In this talk we study the existence of simple lenses that eliminate chromatic aberration at the target.

Bi-chromatic rays, with two colors, are emitted from infinity (far field) or from a point source (near field). We will study the existence of a lens composed of two surfaces such that all the colors exit the upper face with the same direction.

In the far field case, we use a fixed point argument and prove nonexistence of such lenses. The near field problem requires the study of a functional system of differential equations of the form

$$Z'(t) = H(t; Z(t), Z(z_1(t)); Z'(t), Z'(z_1(t))), \qquad Z(0) = 0.$$

We show that under appropriate initial conditions such a system has a solution.

Based on joint work with Cristian E. Gutiérrez.

Keywords: Differential Equations, Chromatic Aberration, Geometric Optics.

Epidemic Genetic Algorithm for solving inverse problems: parallel algorithms

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Parallel Genetic Algorithm (PGA) is employed to solve inverse problems. The PGA is codified considering the island model (individuals are free to migrate to any other processor, subjected to specific rules); and stepping-stone model (migration is allowed only for closest processors). The parallel code is generated using calls to the message passing communication library MPI (Message Passing Interface). In the our GA approach, a new genetic operator, named *epidemic*, is applied. The technique is employed to solve an inverse heat conduction problem of determining the initial temperature from the transient noisy temperature profile at a given time. This ill-posed problem requires the use of a regularization technique.

Based on joint work with Haroldo F. de Campos Velho* and Leonardo D. Chiwiacowsky+.
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Keywords: Genetic algorithm, parallel computing, inverse problem.

Tensor–product interpolation for high–dimensional integrals

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High–dimensional integrals appear in problems with uncertainty and noise and are common in stochastic calculus, mathematical finance, quantum physics, etc. Textbook quadratures applied to d–dimensional integrals require n^d function evaluations, that is unfeasible for $d \gtrsim 10$ (this is notoriously known as curse of dimensionality). Typically such integrals are treated using Monte Carlo algorithm (random sampling) or quasi-MC (adapted lattices).

As an alternative to this approach, we propose the method based on the cross interpolation of tensors. The given function is interpolated on the adaptively selected set of nodes, and represented using the tensor–product format, a discrete version of separation of variables. Some functions, like $1/(x+y)$, $x, y \in [0,1]$, can be accurately represented by tensor–product format; others, like $1/(x-y)$, $x, y \in [0,1]$, can not.

Currently we have both theoretical and experimental evidence that tensor approximation converges faster than Monte-Carlo for certain stochastic PDEs. In this talk we present the cross interpolation algorithm and demonstrate its applications to evaluation of high–dimensional integrals in different contexts. We also discuss challenges in development of a parallel version of this algorithms and the ways they can be overcome.

Keywords: high–dimensional integrals, tensor–product algorithms, cross interpolation

A Chemical Kinectics Extension by NOx and SO2 to the Advection Diffusion Equation

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The chemical reactions of the pollutants after their release from power plants represent a sink or source term that must be considered in the study of the time dependent chemical composition of the atmosphere. In this work the chemical kinectics of nitrogen oxides (NOx) and sulfur dioxide (SO2) are introduced into the advection diffusion equation. The contribution of each pollutant and its dispersion after release into the atmospheric boundary layer was validated using the experimental data from Hanford, for a stable case, and from Prairie Grass, for an unstable case. A comparison was also made using the proposed model against a simulation using the CALPUFF Modelling System, with and without considering the chemical reaction in both models. The results show that the chemical reactions have to be considered in the atmospheric models since they have a significant contribution to the reduction of the time dependent concentrations of the pollutants.

Based on joint work with Bardo BODMANN.

Keywords: Chemical Kinectics, Dispersion Model, Advection-Diffusion

A Simple Nonlinear Transfer Function for a Wiener-Hammerstein Model of Guitar Distortion and Overdrive Effects

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In music, the holy grail for guitarists, bassist and keyboard players is the valve (or tube) amplifiers. This preference is justified by the kind of distorted sound they can provide at high volumes, which many musicians appreciate. This happens because of the particular way valves distort the sound when they are overdriven, a clear particular Non-linear behaviour. Initially, in order to reproduce the same kind of distortion caused by overdriven valves, many electronic devices based on diodes' limiters were made, the so called pedal effects of Distortion and Overdrive. More recently, the valve amplifiers have been modulated using digital signal processing techniques. One of these is the the Wiener-Hammersetin cell, an extension of the Volterra-Wiener theories for modelling non-linear systems (NLS). One necessary step in modelling the NLS with the Wiener-Hammerstein cell is to identify the Nonlinear Static Transfer Function (NLTF) in some way. With the objective of modelling audio distortion pedal effects almost based on PN junction limiter circuits, we propose a NLTF model derived from a modification of the Shockley equation.

In order to test the model proposed, 3 limiter circuits with different kind of diodes, namely silicon, germanium and LED diodes, were mounted in a breadboard and evaluated using a voltage sine wave of 10 Hz and amplitude sufficient to provide 10 mA of input current to the circuits. About 10 seconds of both input and output signals were digitalized by an oscilloscope Agilent Infinium 54833 D MSO, with a sample rate of 100 kSamples/s. The algorithm used to fit the model to the data was the Levenberg-Marquardt nonlinear least square fit. The model worked well, providing a root mean square standard error between the data and fitting models of about 10^{-4}, except for the LED limiter circuit. This difference in behaviour is caused by constructive aspects of these diodes, and the Shockley equation does not model correctly its Input current vs. Tension characteristic curve.

Based on joint work with Luiz Fernando FERREIRA, Ronaldo HUSEMANN and Bardo BODMANN.

Keywords: Wiener-Hammersetin cell, Modified Shockley equation, Nonlinear Static Transfer Function

On the application of the piecewise linear approximation and collocation method for a hypersingular integral equation

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We consider the hypersingular integral equation for a function $g(y)$ on the simple smooth surface Σ, that can be can be closed or opened with the boundary $\partial\Sigma$:

$$(I\,g)(x) = \int_\Sigma g(y) \frac{\partial^2 F(x-y)}{\partial n_x \partial n_y}\, d\sigma_y = f(x),\ x \in \Sigma, \quad F(x-y) = \frac{1}{|x-y|},$$

where $\partial/\partial n_x$ is the derivative along the unit outward normal vector $\vec{n}(x)$ and it is calculated as a partial derivative of a function depending on x. The integral is considered in the sense of Hadamard finite value. This equation arises if one solves the Neumann boundary value problem for the Laplace equation using double layer potential representation of the solution.

We develop the numerical method for that equation using the surface triangulation, piecewise linear approximation of the unknown function on the cells and collocation method. For the particular class of closed surfaces we prove the uniform convergence of the numerical solution to the accurate one on the grid as the triangulation size tends to zero. The method is tested using specific examples.

Based on joint work with A.V. Setukha, setuhaav@rambler.ru.

Keywords: numerical methods, boundary integral equation method, hypersingular integrals.

Design and Performance of a Multiphase Flow Manifold

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(Presented by Ram MOHAN)

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An experimental and theoretical study is conducted on the design and performance of multiphase flow manifold. The objective of a manifold is to dissipate incoming slugs from the inlet pipelines, promoting stratification to enable equal splitting of the gas and the liquid into downstream processing facilities. An experimental multiphase flow manifold test section is designed and constructed. Sixteen experimental runs are carried out to investigate slug dissipation in the manifold. A mechanistic model is developed for the prediction of the diameter and length of the manifold, based on a modified slug-tracking model. Comparison between the developed model predictions and the acquired experimental data shows less than 8 percent average absolute relative error.

Based on joint work with Mobina Mohammadikharkeshi, Asad Molayeri, Ramin Dabirian and Ram S. Mohan.

Keywords: Multiphase Flow, Manifold, Slug Dissipation.

Analytical approach for solving beam equation

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In my talk, my focus will be on existence results of several nonlinear problems arising mainly in civil engineering. Specifically the existence of solution of boundary value problems such as Beam equation and the existence of solution of integral equation is highlighted in my presentation. For the same, fixed point approach is employed. Some nontrivial examples are elaborated with computer simulation. Furthermore, in this talk it is intended to discuss some open problems for the future aspects.

Keywords: Boundary value problem, Beam equation, Integral equation, fixed point.

Existence of solutions of nonlinear integral equations arising from science and engineering problems

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The main motive of this presentation is to highlight the hegemony of fixed point theory and its applications to various nonlinear problems, primarily the applications for the existence of the solutions of various integral equations, boundary value problems (BVPs), equations arising in oscillation of a spring, equations representing LCR circuits, simple harmonic motion, application to conversion of solar energy to electrical energy etc. Keeping these utilizations of fixed point theory, some novel results are established along with applications. Ingenious examples will embroider the results, for the same computer simulation is done. We also suggest some open problems whose solutions may be derived from our results.

Keywords: Fixed point, Integral equations, Differential equations, BVP.

Operators, equations, and boundary value problems

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We study elliptic pseudo-differential equations on a manifold with a non-smooth boundary. These operators and equations arise in various problems of mathematical physics. Using the local principle we first consider a local situation for each point of a manifold, and add some boundary conditions if it is necessary. Such boundary conditions appear if an index of special symbol factorization is not vanishing. For the latter case we consider simplest non-smooth domain like a multidimensional cone and describe some boundary conditions for which we can construct the solution of the boundary value problem in Sobolev–Slobodetskii spaces.

Keywords: pseudo-differential equation, elliptic symbol, wave factorization

On Neumann's method and double layer potentials

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Neumann's classical integral equations with the double layer boundary potential is considered on different spaces of boundary charges such as continuous data, L^2 and energy trace spaces on the domain's boundary for interior and exterior boundary value problems of elliptic partial differential equations. Corresponding known results for different classes of boundaries are discussed in view of collocation and Galerkin boundary element methods.

Double-negative electromagnetic metamaterials due to chirality

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In this talk we focus on building a double-negative metamaterial out of plasmonic nanoparticles in chiral materials and provide a mathematical theory for understanding the mechanism behind the double-negative refractive index phenomenon in chiral materials. We rely on media that consist of only a single type of dielectric resonant element, and show how the chirality of the background medium induces double-negative refractive index metamaterial, which refracts waves negatively, hence acting as a superlens. Using plasmonic dielectric particles, we prove that both the effective electric permittivity and the magnetic permeability can be negative near some resonant frequencies. We provide a rigorous justification of the approximation of a plasmonic particle in a chiral medium by the sum of a resonant electric dipole and a resonant magnetic dipole. Moreover, we characterize the set of resonant frequencies. For an appropriate volume fraction of plasmonic particles with certain conditions on their configuration, we obtain a double-negative effective medium when the frequency is near one of the resonant frequencies.

Based on joint work with Prof. Dr. Habib Ammari and Dr. Sanghyeon Yu.

Keywords: Plasmonic nanoparticles, chiral materials, double-negative metamaterials.

Fixed point theorems and their applications

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In this presentation, we introduce Kannan type F-contraction and invoke this contraction to establish the existence of fixed point of the mapping involved. In particular, we give some criteria of the usual functional type for the convergence of iterations generated by a Kannan type mapping to a fixed point of the mapping. As an application, we apply our results to establish the existence of solution of nonlinear integral equation of the mixed Hammerstein type. Some examples will spiff up the established results and the corresponding computer simulation will elaborate the theoretical treatment. At the end, we suggest some open problems in terms of integral equations arising from real world problems.

Keywords: Integral equations, Boundary value problem, Fixed point, F-contraction.

Error bounds for time-continuous schemes applied to nonlinear integro-differential equations

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We investigate approximate solutions to systems of strongly joint nonlinear integro-differential equations determined via dynamic iterations and constructed to transform the given systems into linear systems of disjoint, simple ordinary differential equations that can be easily integrated in parallel computing environments through the use of arbitrary implicit time integrators. The focus is on the error bounds for the successive iterates and on the analysis of their rates of convergence to the exact solutions of the model equations.

In special cases, we obtain sharp error bounds for arbitrarily long time intervals. Extensions to general cases will also be considered, where the corresponding error bounds manifest that the smaller the length of the time interval, the faster the convergence rate. Examples are used to illustrate the theoretical results. These examples arise from practical applications and demonstrate fast convergence rates that are independent from the length of the time intervals.

Keywords: strongly joint nonlinear equations, error bounds, convergence over long time intervals.