Towards Reduced Order Modeling (ROM) for Gust Simulations

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Deutscher Luft- und Raumfahrtkongress 2017
5.-7. September 2017
München
Outline

1. Motivation and objectives

2. Reduced-Order Modeling (ROM) for unsteady nonlinear aerodynamics
   - Unsteady Least-Squares-ROM approach
   - Offline training: building the subspace
   - Online predictions
   - Accelerated greedy missing point estimation (MPE) procedure
   - Results for 2D and 3D test case

3. Summary and outlook
Motivation

- From design to certification of an aircraft many aerodynamic data are needed – for the entire flight envelope –
- **Aerodynamic data** $\rightarrow$ pressure and shear stress distribution on the aircraft surface, from steady and unsteady simulations
**Objective**

**Goal:** based on high-fidelity CFD data, provide predictions of the aerodynamics with lower evaluation time and storage than the original CFD model.

To reduce computational complexity, physics-based ROMs based on hi-fi training data to be used instead of CFD for rapid many queries predictions (maneuver & gust loads).

**Solution($p_1, p_2, ...$)?**

- ROMs for compressible flows
- Structural design and sizing
- MDAO

**Aerodynamic design**

**Result!**

(here: real time!)
Reduced-Order Modeling (ROM) procedure

Collecting snapshots coming from an unsteady simulation → variation in the local effective angle of attack

- Boundary conditions disturbances
- Motion (forced or induced)
- Gust perturbations (here: rigid aircraft, no motion)
Nonlinear unsteady LSQ-ROM approach

Semi-discrete unsteady Navier-Stokes Eqs.
\[ \hat{\mathbf{R}} \overset{\text{def}}{=} \mathbf{R}(\mathbf{w}(t)) + \Omega \frac{\partial \mathbf{w}(t)}{\partial t} = 0 \in \mathbb{R}^N \]
\( \Omega: \) cell volumes

Search for an approximated solution \( \mathbf{w} = [\rho, \rho \mathbf{v}, \rho E^t, v_t] \in \mathbb{R}^N \)
- in the POD subspace \( \mathbf{U}_r \in \mathbb{R}^{N \times r}, r \ll N \)
- minimizing the unsteady residual in the \( L_2 \) norm

\[ \mathbf{w} \approx \sum_{i=1}^{r} a_i \mathbf{U}^i + \bar{\mathbf{w}} = \mathbf{U}_r \mathbf{a} + \bar{\mathbf{w}} \]
\( \mathbf{a}: \) vector of the unknown coefficients \( a_i \)
\( \bar{\mathbf{w}}: \) mean of the snapshots set

\[ \min_{\mathbf{a}} \| \hat{\mathbf{R}}(\mathbf{U}_r \mathbf{a} + \bar{\mathbf{w}}) \|_{L_2}^2 \]

nonlinear least squares problem, solved with parallel Levenberg-Marquart alg.

Physics-based approach
**2D Test Case: NACA airfoil**

- TAU RANS eq., Spalart-Allmaras turbulence model
- Mach = 0.8, Re = 7.5 $10^6$
- Training maneuvers exciting up to $k \leq 0.2$
- Predict aero loads for periodic motions at different frequencies and amplitudes (using the same ROM)

**Training maneuvers**

- Schroeder multisine with $0.01 < k < 0.2$
- Linear chirp signal with $k_{\text{max}} = 0.2$
2D Test Case: NACA airfoil
Periodic pitching motion prediction at different oscillation frequencies $k$

ROM built with 50 POD modes
2D Test Case: NACA airfoil
Periodic pitching motion prediction at different oscillation amplitudes $\alpha_A$

ROM built with 50 POD modes
Curse of dimensionality for ROMs of nonlinear systems

I. compute the approx. solution

\[ w(t) \approx Ua(t) + \bar{w} \Rightarrow O(Nr) \]

II. evaluate the unsteady residual with TAU

\[ \tilde{R}(w(t)) \overset{\text{def}}{=} R(w(t)) + \Omega \frac{\partial w(t)}{\partial t} \Rightarrow O(N) \]

III. solve the LSQ problem

\[
(J^T J + \lambda I) \Delta a = -J^T \tilde{R}
\]

\[ \Rightarrow O(Nr) \]

- Jacobian Matrix \( J \in \mathbb{R}^{N \times r} \)
  - \( N \): order of CFD model (variables x nodes)
  - \( r \): order of the ROM (i.e. number of POD modes)

The computational cost scales linearly with the dimension \( N \) of the full order model. No significant speedup can be expected when solving the minimum residual ROM.
Hyper-reduction approaches

Complexity reduction by sampling (or compute only a few entries of) the nonlinear unsteady residual vector $\hat{\mathbf{R}}$

Collocation
- omission of many components
- non intrusive

Reconstruction
- approximation of the entire vector, by interpolation or by least-squares projection onto a subspace $\mathbf{V} = \mathbf{U}(\mathbf{U}^T \mathbf{P} \mathbf{P}^T \mathbf{U})^{-1} \mathbf{U}^T \mathbf{P}$

Selecting the subset indices $\rightarrow$
- initialize with (Discrete) Empirical Interpolation Method
- select add. points with greedy Missing Point Estimation

- The complete nonlinear unsteady residual vector $\hat{\mathbf{R}}$ is evaluated,
- but only a small subset of its entries are used in the minimization process

Greedy: minimize
$$\|\mathbf{U}(\mathbf{U}^T \mathbf{P} \mathbf{P}^T \mathbf{U})^{-1} \mathbf{U}^T \mathbf{P} \mathbf{P}^T\| = 1/\sigma_{\min}(\mathbf{P}^T \mathbf{U})$$
Exhaustive greedy missing point estimation procedure

Greedy point selection algorithms minimize an error indicator by sequentially looping over all entries → costly ⇒ \( \mathcal{O}(N^{r^3}) \)

Exhaustive greedy MPE for maximizing \( \sigma_{\text{min}}(P_{s+1}^T U) \)

**Input:**
- \( U \in \mathbb{R}^{N \times r} \): basis of a \( r \)-dim subspace,
- \( J_s \in \mathbb{R}^{s \times 1} \): index set,
- \( P_s \in \mathbb{R}^{N \times s} \): mask matrix with \( s \) indices, where \( s \geq r \).

1. \( \sigma_{\text{opt}} = 0, \bar{J}_s = \{1, ..., N\} \setminus J_s \)
2. for \( j \in \bar{J}_s \) do
3. \( \bar{P} = (P_s, e_j) \in \mathbb{R}^{N \times (s+1)} \)
4. Compute \( \sigma_{\text{min}}(\bar{P}^T U) \)
5. if \( \sigma_j > \sigma_{\text{opt}} \) then
6. \( \sigma_{\text{opt}} = \sigma_j, j_{\text{opt}} = j \)

**Output:** next index \( J_{s+1} = J_s \cup \{j_{\text{opt}}\}, P_{s+1}^T = [P_s, e_{j_{\text{opt}}} \]
Accelerated greedy MPE with rank-1 SVD update

Using an (additional) rank-1 SVD update within the iterative greedy step to further accelerate the selection of the grid nodes.

**Computational cost**
- Alg. Ref. [1]: $\mathcal{O}(N r^2 s + r^3 s)$
- Rank-1 SVD update: $\mathcal{O}(N r^2 s + r^3 s)$

- $N$: order of CFD model (variables x nodes)
- $r$: order of the ROM (i.e. number of POD modes)
- $s (> r)$: number of MPE selected nodes

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Maneuver about the steady state LANN wing, $\text{Mach} = 0.82$, $\text{Re} = 7.17 \times 10^6$

LANN wing test case
- 0.47 Mi grid nodes
- 23 POD modes
- 6 cores

Wall-clock time [h]
- Alg. Ref. [1]
- Rank-1 SVD update

# selected nodes / # total nodes [%]
Unsteady loads prediction for the LANN wing

**Online Performances**

- **ROM (10 cores)**
  - w/o MPE: 25 h 50 min
  - with MPE: 2 h 44 min

**Offline Performances (10 cores)**

- Compute snapshots: 4d 10h 40min
- POD ROM building: ~5 min
- Fast greedy MPE: ~6 min

**Average Wall Time**

- Total: 25 h 50 min, 2 h 44 min, 1 h 29 min
- Per time step: ~16 min, 1 min 36 s, 36 s
- Speed-up**: 1, 9.5, 26.7

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**Chart 17**

**TAU**

**MINRES ROM - 18 PODM**

**MPE ROM - 5000 nodes**

**Mach = 0.82, Re = 7.17 \times 10^6**

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[Image of a graph showing lift and moment coefficients over time with annotations for performance metrics.]
Unsteady ROM prediction assessment for a full aircraft

- TAU, RANS equations with SA-neg turbulence model
- $V_\infty = 246$ m/s, Mach = 0.83, $Re = 6.5 \times 10^6$
- Linear chirp training maneuver exciting up to $k_{max} = 3$:
  \[
  \alpha(\tau) = 2.0164^\circ + 2.3266 \sin(k \tau)
  \]
  with $k(\tau) = k_{max} \frac{c_r}{V_\infty} \tau$
- Predicted (1-cos)-like pitching oscillation at $k = 0.33$

### CFD setting (TAU code)
- Dual time stepping
- Min residual: $1e-6$
- Max inner iterations per time step: 5000
- Physical time steps: 550
- Linearly distributed time steps

### ROM settings (SMARTy)
Truncation of the POD modes to 99.999% of their energy content (~100 modes)
Unsteady ROM prediction assessment for a full aircraft

Training maneuver
Chirp pitching oscillation up to reduced frequency $k=3$

Predicted maneuver
(1-cos)–gust like pitching oscillation

Offline: one training maneuver

Online: many ROM predictions possible

- Mach=0.83, Re=6.5 \(10^6\)

$w_g = 10 \text{ m/s}$

$\Delta \alpha \approx 2.3^\circ$

$V_\infty = 246 \text{ m/s}$

ROM run-time (48 cores): 1.3 h
Speed-up: 2.3 (w/o MPE procedure! w/o MKL! FSDM in developer mode)
Unsteady ROM prediction assessment for a full aircraft

Time step @ max C-lift

(1-cos) gust-like pitch oscillation

- Mach=0.83
- Re=6.5 $10^6$
Unsteady ROM prediction assessment for a full aircraft

- Mach=0.83, Re=6.5 $10^6$
Unsteady ROM: accelerated greedy MPE

- minimize a subset of the unsteady residual in the $L_2$ norm
- greedy missing point estimation (MPE) procedure to select the subset
- greedy nodes maximise the "information content" of the POD subspace

Goal: Reduced online cost

Grid
- $N^\circ$ nodes: 3.8 Mi

Rank-1 SVD update
- Greedy MPE
- $N^\circ$ nodes: 0.38 Mi
  (10% of total nodes)

Potential speed-up by using Intel® Math Kernel Library (MKL)
Summary

The use of CFD-based Reduced Order Models has been demonstrated for unsteady aerodynamics, maneuvers and gusts:

- 😃 accurate predictions can be achieved with a proper training snapshot set
- 😞 the selection of the submesh is currently still too expensive (but offline)
- 😞 the achieved speed-up is not yet fully satisfactory (w/o MPE, w/o MKL)
Outlook

- Next: discrete gusts, elastic a/c, parametric a/c
- Include the greedy MPE selection in the ROM prediction for the full aircraft
- Investigate divide-and-conquer algorithm and coarse-grid residual evaluation as an alternative to greedy algorithm
- Apply the nonlinear unsteady least-squares ROM approach to discrete gusts
- What is the best training maneuver? → **ROM challenge** …
- Investigate alternatives to POD (DMD, isomap, clustering, …) for unsteady ROMs
- Consider interpolation-based approaches instead of residual minimization?!

**NEVER FORGET THE PHYSICS**
Nathan Kutz, Washington Uni
Acknowledgements

Acknowledgement to the EU (Grant agreement article 38.1.2):
- Part of the research leading to this work was supported by the AEROGUST project, funded by the European Commission under grant number 636053.

Internal DLR Projects
- DLR multidisciplinary projects Digital-X and VicToria

Collaboration with Ralf Zimmermann
- SDU, Department of Mathematics and Computer Science (IMADA), Odense, Denmark