

# Minimum residual based model order reduction approach for unsteady nonlinear aerodynamic problems

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The advent and development of large-scale high-fidelity computational fluid dynamics (CFD) in aircraft design is requiring, more and more, procedures and techniques aimed at reducing its computational cost in order to afford accurate but fast simulations of, e.g., the aerodynamic loads. The adoption of reduced order modeling techniques in CFD represents a promising approach to achieve this goal. Several methods have been developed to obtain reduced order models (ROMs) for the prediction of steady aerodynamic flows using low-dimensional linear subspaces (cf. [4]) as well as non-linear manifolds (cf. [1]), whose performances may be further improved by applying hyper-reduction procedures (cf. [3]).

In this talk, a model order reduction approach (cf. [2]) for unsteady aerodynamic applications is presented. Following the idea of Ref. [4], the problem of finding the CFD ROM is formulated as a non-linear least-squares optimization problem, by searching in a subspace for an approximate flow solution having a minimum norm least-squares solution for the corresponding unsteady (flow solver) residual. The reduced basis for representing the reduced-order solutions of the governing equations is obtained through a Proper Orthogonal Decomposition (POD) applied to a given set of solutions of the full-order model at different time steps.

The arising nonlinear least-squares problem for the POD coefficients is solved by using a Levenberg-Marquardt (LM) algorithm. A Broyden update procedure is employed to approximate the Jacobian of the reduced-order system of equations, and it is further exploited to reduce the computational costs to generate the approximate Hessian matrix for the LM procedure. In addition, the potential of masked projection approaches, such as the missing point estimation (MPE) (cf.[5]), is going to be investigated.

The proposed approach is demonstrated for the Navier-Stokes equations by modeling the transonic flow around the LANN wing oscillating in pitch at different reduced frequencies.

## References

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