Minimum residual based model order reduction approach for unsteady nonlinear aerodynamic problems

M. Ripepi¹, R. Zimmermann² and S. Görtz¹

¹German Aerospace Center (DLR), Institute of Aerodynamics and Flow Technology, Braunschweig, Germany
²Institute Computational Mathematics, TU Braunschweig, Braunschweig, Germany

Data-driven Model Order Reduction and Machine Learning Workshop
University of Stuttgart, Germany
30 March 2016 – 1 April 2016
Overview

1. Motivation and objectives
2. Model order reduction for unsteady CFD
3. Accelerated greedy MPE procedure
4. Aeronautical application: the LANN wing
5. Summary and outlook
Motivation

- From design to certification of an aircraft many aerodynamic data are needed – for the entire flight envelope –

- All required aerodynamic data can be derived from the pressure and shear stress distribution on the aircraft surface, from steady and unsteady simulations

Prediction of:
- Aerodynamic performance
- "Handling Qualities"
- Structural loads

Input for:
- Aerodynamic design
- Structural design and sizing
- Design of control surfaces and flight control system
- Flight simulators

CFD is mostly limited to the cruise point

Motivation

- From design to certification of an aircraft many aerodynamic data are needed – for the entire flight envelope –

- All required aerodynamic data can be derived from the pressure and shear stress distribution on the aircraft surface, from steady and unsteady simulations

Prediction of:
- Aerodynamic performance
- "Handling Qualities"
- Structural loads

Input for:
- Aerodynamic design
- Structural design and sizing
- Design of control surfaces and flight control system
- Flight simulators

CFD is mostly limited to the cruise point

Motivation

- From design to certification of an aircraft many aerodynamic data are needed – for the entire flight envelope –

- All required aerodynamic data can be derived from the pressure and shear stress distribution on the aircraft surface, from steady and unsteady simulations

Prediction of:
- Aerodynamic performance
- "Handling Qualities"
- Structural loads

Input for:
- Aerodynamic design
- Structural design and sizing
- Design of control surfaces and flight control system
- Flight simulators

CFD is mostly limited to the cruise point
Objectives

Development of a simulation environment, where ALL the necessary aerodynamic data can be determined numerically

- with high accuracy
- with Hi-Fi methods

To reduce the computational complexity, Reduced Order Models based on Hi-Fi CFD should replace (e.g. in the MultiDisciplinary Optimization) high quality models and methods.
Objectives

Development of a simulation environment, where **ALL** the necessary aerodynamic data can be determined numerically

- with high accuracy
- with Hi-Fi methods

ROMs for the prediction of *steady* aerodynamic flows

Model order reduction approach for **unsteady** aerodynamic applications
Model order reduction for unsteady CFD

\[ \tilde{R} \triangleq R(w(t)) + \Omega \frac{\partial w(t)}{\partial t} = 0 \in \mathbb{R}^N \quad \Omega: \text{cell volumes} \]

- **Semi-discrete Navier-Stokes Eqs.**
- Search for an approximated solution \( w = [\rho, \rho v, \rho E^t] \in \mathbb{R}^N \)
  - in the POD subspace \( U \in \mathbb{R}^{N \times r}, r \ll N \)
  - minimizing the **unsteady residual** in the \( L_2 \) norm

\[ w \approx \sum_{i=1}^{r} a_i U_i + \bar{w} = Ua + \bar{w} \]

\[ \min_{a} \| \tilde{R}(Ua + \bar{w}) \|_{L_2}^2 \]

- \( a \): unknown coefficients
- \( \bar{w} = \frac{\sum_{i=1}^{m} w(t_i)}{m} \): average of the snapshots

**Proper Orthogonal Decomposition**

\[ Y = [w(t_1), \ldots, w(t_m)] - \bar{w} \]

- **SVD:** \( Y = U S V^T \)
- **EVD:** \( R = Y Y^T = U S^2 U^T \)
Per each physical time step, find the POD coefficients minimizing the unsteady residual at the time step $n+1$, by solving a nonlinear least squares problem:

$$\mathbf{w}_{\text{rom}}(t_{n+1}) \approx \mathbf{Ua} + \bar{\mathbf{w}}$$

$$\hat{\mathbf{R}} \triangleq \mathbf{R} (\mathbf{w}_{\text{rom}}(t_{n+1})) + \Omega \frac{3\mathbf{w}_{\text{rom}}(t_{n+1}) - 4\mathbf{w}_{\text{rom}}(t_{n}) + \mathbf{w}_{\text{rom}}(t_{n-1})}{2\Delta t}$$

$$(\mathbf{J}^T \mathbf{J} + \lambda \mathbf{I}) \Delta \mathbf{a} = -\mathbf{J}^T \hat{\mathbf{R}}$$

Broyden’s method for updating $\mathbf{J}$

$$\mathbf{J}_{k+1} = \mathbf{J}_k + \frac{\Delta \hat{\mathbf{R}} - \mathbf{J}_k \Delta \mathbf{a}}{\|\Delta \mathbf{a}\|^2} \Delta \mathbf{a}^T$$

Updating pseudo-Hessian matrix $\mathbf{A} \equiv \mathbf{J}^T \mathbf{J} \in \mathbb{R}^{r \times r} \Rightarrow \mathcal{O}(N r^2)$

$$\mathbf{A}_{k+1} \equiv \mathbf{J}_{k+1}^T \mathbf{J}_{k+1}$$

$$= \mathbf{A}_k + \frac{\mathbf{J}_k^T \Delta \hat{\mathbf{R}} - \mathbf{A}_k \Delta \mathbf{a}}{\|\Delta \mathbf{a}\|^2} \Delta \mathbf{a}^T + \Delta \mathbf{a} \frac{\Delta \hat{\mathbf{R}}^T \mathbf{J}_k - \Delta \mathbf{a}^T \mathbf{A}_k}{\|\Delta \mathbf{a}\|^2}$$

$$+ \Delta \mathbf{a} \frac{\Delta \hat{\mathbf{R}}^T \Delta \hat{\mathbf{R}} - \Delta \hat{\mathbf{R}}^T \mathbf{J}_k \Delta \mathbf{a} - \Delta \mathbf{a}^T \mathbf{J}_k^T \Delta \hat{\mathbf{R}} + \Delta \mathbf{a}^T \mathbf{A}_k \Delta \mathbf{a}}{\|\Delta \mathbf{a}\|^4}$$

$$\Rightarrow \mathcal{O}(N r)$$

$a \leftarrow a + \Delta a$
2D Test Case: NACA airfoil

- RANS equations with Spalart-Allmaras turbulence model
- Mach = 0.8, Re = 7.5 \times 10^6
- Training maneuvers exciting up to k = \frac{\omega_c}{V_\infty} = 0.2
- Prediction for periodic motions at different frequencies and amplitudes (using the same ROM)

Training maneuvers

- Schroeder multisine with 0.01 < k < 0.2
- Linear chirp signal with k_{\text{max}} = 0.2
2D Test Case: NACA airfoil
Periodic pitching motion prediction at different oscillation frequencies

ROM built with 50 POD modes
2D Test Case: NACA airfoil
Periodic pitching motion prediction at different oscillation amplitudes

ROM built with 50 POD modes

Online Performances

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Per time step</th>
<th>Speed-up</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ways to improve the performances:
- **Hyper-reduction** (e.g. Gappy POD)
- Different selection of the subspace (e.g. Dynamic Mode Decomposition)
Model order reduction for unsteady CFD
Curse of dimensionality in ROMs of nonlinear systems

\[ \min_a \| \hat{R}(Ua + \overline{w}) \|_{L_2}^2 \]

I. Compute the approx. solution

\[ w(t) \approx Ua(t) + \overline{w} \Rightarrow O(Nr) \]

II. Evaluate the unsteady residual

\[ \hat{R}(w(t)) \overset{def}{=} R(w(t)) + \Omega \frac{\partial w(t)}{\partial t} \]

III. Solve the LS problem

\[ (J^TJ + \lambda I)\Delta a = -J^T\hat{R} \]

The computational cost scales linearly with the dimension \( N \) of the full order model. No significant speedup can be expected when solving the minimum residual ROM.
Hyper-reduction approaches

Complexity reduction by sampling (or compute only a few entries of) the nonlinear unsteady residual vector $\hat{\mathbf{R}}$

Collocation
- omission of many components
- non intrusive

Reconstruction
- approximation of the entire vector, by interpolation or by least-squares projection onto a subspace $\mathbf{V} = \mathbf{U}(\mathbf{U}^T \mathbf{P} \mathbf{P}^T \mathbf{U})^{-1} \mathbf{U}^T \mathbf{P}$

Selecting the subset indices →
- Gappy POD
- Missing Point Estimation
- (Discrete) Empirical Interpolation Method

The complete nonlinear unsteady residual vector $\hat{\mathbf{R}}$ is evaluated,
but only a small subset of its entries are used in the minimization process
Exhaustive greedy missing point estimation procedure

Greedy point selection algorithms minimize an error indicator by sequentially looping over all entries → costly ⇒ > \( \mathcal{O}(Nr^3) \)

Exhaustive greedy MPE → maximize \( \sigma_{\text{min}} \left( P_{s+1}^T U \right) \)

Input: \( U \in \mathbb{R}^{N \times r} \): basis of a \( r \)-dim subspace, \( J_s \in \mathbb{R}^{s \times 1} \): index set, \( P_s \in \mathbb{R}^{N \times s} \): mask matrix with \( s \) indices, where \( s \geq r \).

Output: next index \( J_{s+1} = J_s \cup \{ j_{\text{opt}} \} \), \( P_{s+1} = \begin{bmatrix} P_s \; e_j \end{bmatrix} \), opt

Observation 1)

Observation 2)

The penultimate singular value bounds the growth of \( \sigma_{\text{min}} \)

Accelerated greedy missing point estimation procedure

Idea:

- Select the vectors $v_{j,\text{opt}}$ that feature the largest absolute values in the ultimate component while all other components are comparably small.

- Build fast approximations that sort the set of candidate vectors that induce the rank-one modifications (without solving the modified eigenvalue problem.)

Error bound

$$\lambda_{r-k} < F(\sigma_{r-k}^2, \sigma_{r-k+1}^2, v_i)$$

Fast greedy MPE with target switching based on the growth potential $\Rightarrow \mathcal{O}(N r^2)$

Input: $U \in \mathbb{R}^{N \times r}$: basis of a r-dim subspace, $J_s \in \mathbb{R}^{s \times 1}$: index set, $s \geq r$.

1. $\overline{J}_s = \{1, \ldots, N\} \setminus J_s$
2. while $|J_s| \leq \text{maxpoints}$ do
3. target the largest growth for $\lambda_{\text{min}}$ (i.e. $\lambda_r$) or the next biggest $\lambda_{r-1}$, etc.
4. determine $j_{\text{opt}}$ by sorting $\{v_j, j \in \overline{J}_s\}$ according to the fast estimate
5. update: $J_{s+1} = J_s \cup \{j_{\text{opt}}\}$, $\overline{J}_{s+1} = \overline{J}_s \setminus \{j_{\text{opt}}\}$, $s = s + 1$

Output: index set $J_s$
2D Test Case: NACA airfoil
Nodes selected with the fast greedy MPE

Total number of grid nodes: 21454
2D Test Case: NACA airfoil
Periodic pitching motion prediction at k = 0.16

Maximum lift

Minimum lift

1000 nodes selected by the greedy MPE

< 5% of the total number of grid nodes
2D Test Case: NACA airfoil

Performances

Offline Performances (1 core)
- Compute 400 snapshots by a training unsteady CFD simulation: ~47 min
- POD ROM building: 1min 32s
- Fast greedy MPE (selecting 1000 nodes): ~2 min

<table>
<thead>
<tr>
<th>Online Performances (1 core)</th>
<th>CFD TAU</th>
<th>ROM (built with 50 POD modes)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>w/o MPE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100 nodes</td>
</tr>
<tr>
<td>Total [min]</td>
<td>~18</td>
<td>~9</td>
</tr>
<tr>
<td>per time step [s]</td>
<td>6.98</td>
<td>3.65</td>
</tr>
<tr>
<td>speed-up</td>
<td>1</td>
<td>1.91</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Process</th>
<th>CFD TAU</th>
<th>ROM (built with 50 POD modes)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>w/o MPE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>with MPE (1000 nodes)</td>
</tr>
<tr>
<td>Building the model</td>
<td>—</td>
<td>~49 min</td>
</tr>
<tr>
<td>Simulate 7 parameters</td>
<td>2 h 5 min</td>
<td>1 h 3 min</td>
</tr>
<tr>
<td>(5 reduced frequencies and 2 amplitudes)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3D test case: LANN wing

- RANS equations with SA-neg turbulence model
- $V_\infty = 271.66$ m/s, Mach = 0.82, Re = $7.17 \times 10^6$
- Chirp training maneuver exciting up to $k \equiv \frac{\omega r}{V_\infty} = 0.35$
- Predicted periodic pitching oscillation: $\alpha(\tau) = 2.6^\circ + 0.25 \sin(k \tau), \quad \tau \equiv \frac{V_\infty t}{c_r}$

450560 elements
469213 points

CFD settings
- Min residual: $1e-5$
- Max inner iterations: 100

ROM settings
- 18 POD modes
- 5000 MPE nodes
3D test case: LANN wing

Total number of grid nodes: 469213

5000 nodes selected by the greedy MPE

~1% of the total number of grid nodes
3D test case: LANN wing

Offline Performances (10 cores)
- Compute snapshots: 4d 10h 40min
- POD ROM building: ~5 min
- Fast greedy MPE: ~6 min

Online Performances

<table>
<thead>
<tr>
<th></th>
<th>CFD (10 cores)</th>
<th>ROM (10 cores)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>w/o MPE</td>
</tr>
<tr>
<td>Average Wall Time</td>
<td>total</td>
<td>25 h 50 min</td>
</tr>
<tr>
<td>per time step</td>
<td>~16 min</td>
<td>1 min 36 s</td>
</tr>
<tr>
<td>speed-up</td>
<td>1</td>
<td>9.5</td>
</tr>
</tbody>
</table>

- TAU
- MINRES ROM - 18 PODM
- MPE ROM - 5000 nodes

k=0.3
Summary and Outlook

The effectiveness of reduced order models for unsteady aerodynamic problems have been demonstrated for a NACA airfoil and the LANN wing in transonic flows.

Remarks

- Training maneuvers may be an issue: the quality of the resulting model will reflect the input signal quality
- Further analyses are required (e.g. moving window, full-aircraft config., ...)

Possible improvements

- Different selection of the subspace (e.g. Dynamic Mode Decomposition, Isomap)
- Different ordering of the modes, focusing more on the input-output behavior (e.g. subspace rotation)
- Hybrid strategy by switching in time ROM/CFD
- Sparsity promoting techniques (L1 norm, clustering)
Scenarios

- Multidisciplinary applications
- Real-time applications
- Design and optimization
- Probabilistic applications

ROMs
Acknowledgements

Acknowledgement to the EU:

- Part of this work has been funded from the European Union's Horizon 2020 research and innovation programme under grant agreement No 636053.
- COST Action TD1307: EU-MORNET, Model Reduction Network.

German National Research Projects

- Lufo-IV joint research project AeroStruct

Internal DLR Projects

- DLR multidisciplinary project Digital-X
Bibliography


