University of Cape Town
Industrial CFD Research Group, Dept. of Mech. Eng

Eighteen Month Review Meeting
24th - 25th November 2016, DLR - Göttingen

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Full Order CFD

1. SBP-SAT boundary conditionings
   - Improved stability
   - Under-resolved viscous calculation – no-slip boundary conditions
   - Modified SAT for adiabatic no-slip wall

   \[ SAT = \sigma^I A^I (u - u^I_{\text{target}}) + \sigma^W I (u - u^W_{\text{target}}) + \sigma^V I (f^V - f^V_{\text{target}}) \]

   - Equivalent to inviscid on coarse mesh and viscous on fine mesh
   - Stable and accurate BC & interfacing between meshes/methods

2. Implicit flow solution

3. Interface condition to allow nonlinear beam structural model

   - Objective: Carry our CFD analysis of gusts & replacing wind-tunnel data
CRM Model: Mesh creation

- Create suitable finite volume mesh around CRM
- 1\textsuperscript{st} attempt: Delaunay triangulation

<table>
<thead>
<tr>
<th>Type</th>
<th>Nodes</th>
</tr>
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<tbody>
<tr>
<td>Coarse</td>
<td>1M</td>
</tr>
<tr>
<td>Medium</td>
<td>1.9M</td>
</tr>
<tr>
<td>Fine</td>
<td>3M</td>
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<tr>
<td>Extra Fine</td>
<td>3.7M</td>
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</tbody>
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Top of main wing
CRM Mesh creation: Problems Encountered

- 1st order convergence
- Non-smooth Cp plots
- Under predict lift coefficient
CRM Correcting anisotropic mesh
• Mesh to capture high curvature
Correcting unsmooth Cp plots

• Modified SAT for adiabatic no-slip wall

\[ SAT = \sigma^I A^\infty (u - u^\text{target}) + \sigma^w I(u - u^w_{\text{target}}) + \sigma^v I(f^v - f^v_{\text{target}}) \]

• Result
Injection vs SAT boundary conditions

Mach = 0.86  Re = 5M  AoA = 1Deg
ALE: SBP-SAT boundary conditionings

Discretisation at boundary node:

\[ u_t + D_i F_i = \int SAT_{slip} + SAT_{far\ field} \]

\[ SAT_{slip} = \frac{2}{V_{ol}} A_{in} (u - u_{target}) S \]

\[ A_{in} : Jacobian \ of \ incoming \ flux \]

Implicit compressible Flow solver

- Numerical Jacobian Calculation using Finite Difference

\[
\frac{aR_i}{aU_j} = \frac{R_i(EU_j) - R_i(U_j)}{E}
\]

Growth strategy via GMRES

Convergence history of implicit versus explicit for a steady NACA0012 at Ma = 0.5
Beam based Aeroelastic Model

**Linear Interpolation**

**3rd Order Bezier Interpolation**

\[ B(t) = (1-t)^2 P_0 + 2(1-t)t P_1 + t^2 P_2 \]

\[ B(t) = (1-t)^3 P_0 + 3(1-t)^2 t P_1 + 3(1-t)t^2 P_2 + t^3 P_3 \]
Wing Beam: Non-linear Finite Element Analysis

- **Governing equations**
  - Solve Cauchy’s first equation of motion
    \[ V \cdot a + b = pa \]

- **Numerical discretisation and solution**
  - Employ Galerkin FEM method to discretise governing equations
    \[ \sum_c \int_{A_c} [N]^T \{\varepsilon\}_e dA_e - \sum_c \int_{V_c} [B]^T [P] dV_e = \sum_c \int_{V_c} [N]^T [N] dV_e \{\bar{U}\} \]
  - 2 approaches utilised to solve discretised equations
    - Dual-time stepping methodology vs Newton-Raphson method
  - Dual time-stepping
    - Introduce pseudo time-step temporal term
    - Iterate within time-step until convergence
    - Matrix-free solution

\[ \rho_o \gamma_m \frac{d\gamma_m}{dt} = RHS_i \]
\[ \rho_o \frac{d\gamma_i}{d\tau} \gamma_m = RHS_i - \rho_o \frac{d\gamma_i}{dt} \gamma_m \]
Reduced Order Modelling

1. Beam ROM
   - Full order and modal ROM
   - Geometrically non-linear

2. Aerodynamic ROM
   - Unsteady VLM (2D)
   - Calibrated via Kriging interpolation

   - Objective: Acceptable computational cost for gust load calculations
Wing Beam ROM: Quadratic Mode Shape Analysis

- Pragmatic approach to account for higher-order kinematics
- Linear Modal Analysis
  - Determine linear mode shapes, $f_L$
- Transformation into time domain
  - Inclusion of quadratic mode shapes provides non-linearity

$$u(x, t) = \sum_{i} q_i(t) f_i^L + \sum_{i} q_i(t) q_j(t) G^{ij}$$

- Quadratic mode shape components, $G^{ij}$, computed from linear mode shapes

A geometrically non-linear deformed beam
Wing Beam ROM: Verification and Validation

- QMS offers good representation of highly non-linear conditions at reduced cost
- Slight discrepancy in amplitude and frequency ($E < 10\%$)

\[ E = 79\% \]
Wing Beam ROM: Tuned Quadratic Mode Shapes

- Amplitude and frequency of standard QMS conforms to linear model
- Introduce tuning parameter to consider influence of position on response

\[ u(X,t) = I_i q_i(t) e^{RC} + I_j q_j(t) G_{ij} \]

\[ c^C = c_L + G_{ij} \delta_{ij} \]

\[ c^{RC} = y c_L + (1 - y) c_C \]

57% improvement in amplitude

11% improvement in period
Wing Beam ROM: Application to CRM wi

- CRM wing FEM model reduced to beam elements [1]
- Compare linear model to non-linear QMS approach
  - Half-gust load applied at wing tip to elicit large deformations
  - 2% structural damping applied

[1] Condensation performed by Robbie Cook from University of Bristol
ROM Foundation – Incompressible Potential Flow

• Incompressible potential flows (i.e. \( u = Vc/ \)) are governed by the following conservation equations
  • Mass conservation (1):
    \[ \nabla^2 c/ = 0 \]
  • Momentum conservation (2):
    \[ V \left[ p \frac{ac/}{at} + \frac{1}{2} pu \cdot u + P \right] = 0 \]
  • Circulation conservation (3):
    \[ \frac{Dr}{Dt} = 0 \quad \text{where} \quad r = \frac{1}{c} u \cdot dC \]

• Pressure and shear stress can be computed, as follows:
  \[ p \left[ \frac{ac/}{at} + \frac{1}{2} u \cdot u \right] + P = C(t) \]
  \[ T = \mu [Vu + (Vu)^T] \]
ROM Foundation – 2D Unsteady Method (VLM)

- Split $c_f$ into two parts: the body and the wake
  - Body: discretized as distributed vortex panels
  - Wake: discretized as free point vortices
    - propagated by the flow
- Initially the solution is assumed steady
  - Solve for panel strengths using (1)
  - Calculate $r$
- The stepping procedure is:
  - Transport the wake and shed a new vortex
  - Solve for panel and new vortex strengths using (1) and (3)
ROM Details

• 2D ROM has been built for forced oscillation of the following form:

\[ a = a_m + a_o \sin(2k_ct) \quad \text{where} \quad k_c = \frac{w_a \cdot c}{2V} \quad \text{and} \quad t = \frac{V}{c} \cdot t_c \]

• ROM uses VLM calculations of \( C_L \) and \( C_M \) to predict CFD results via Kriging:
  • Run set of compressible CFD simulations
  • Run corresponding VLM cases
  • Build Kriging surrogates of \( C_L^{CFD} \) and \( C_M^{CFD} \):
    • \( C_L^{CFD} = f_{Krig}(a, a', a'', \nu_L^{VLM}, \nu_M^{VLM}) \)
    • \( C_M^{CFD} = g_{Krig}(a, a', a'', \nu_L^{VLM}, \nu_M^{VLM}) \)
ROM Training/Sample Set and Test Cases

- All CFD simulations were inviscid calculations with $M_{oo} = 0.755$
- A NACA0012 was used with $a_m = 0^\circ$ and $a_o = 2.5^\circ$
- $k_c$ was varied as follows
  - Training Cases:
    1. $k_c = 0.0407$
    2. $k_c = 0.0814$
    3. $k_c = 0.1628$
    4. $k_c = 0.3256$
  - Test Cases:
    1. $k_c = 0.0576$
    2. $k_c = 0.1151$
    3. $k_c = 0.2302$

Sectional Coefficient Calculations of all four Training Case for CFD Simulations
Sectional Lift Coefficient

Sectional Lift Coefficient Comparison of the ROM Prediction and CFD Calculation for all three Test Cases
Sectional Moment Coefficient

Sectional Moment Coefficient Comparison of the ROM Prediction and CFD Calculation for all three Test Cases
Concluding Remarks

• Highlights
  • Non-linear structural ROM based on quadratic mode shapes completed
  • Non-linear structural ROM as fast as a normal linear modal calculation
  • 2D transient vortex lattice ROM successfully demonstrated in 2D on transonic flow.

• Lowlights
  • The non-linear beam modelling approach being re-hashes.

• Dissemination:
  • “A Non-Linear Modal Methodology for Cantilever Beams with Application to Aircraft Structures”, to Journal of Aircraft;
  • “A Hybrid Solver for the Investigation of Aircraft Trailing Vortices Under Gust Conditions”, being prepared for submission to Computers & Fluids;
  • “High Resolution Non-linear Aeroelastic Gust model fora full aircraft model” for IFASD’17.