Application of Harmonic Balance Method for Non-linear Gust Responses

Reik Thormann and Sebastian Timme

University of Liverpool, School of Engineering

SciTech – Structural Dynamics
8th - 12th January 2018, Kissimmee, Florida, USA
Motivation

- Gust analysis one challenge in certification
- Covering a large parameter space
- Linear potential methods (DLM) fail in transonic regime
- Non-linear RANS equations coupled to structure and flight dynamics computationally too expensive
- Linearised RANS methods retain RANS accuracy at significantly reduced cost
Linearised Frequency-Domain: A Short Introduction

• Starting with spatially discretised RANS equations
• Separate variables in steady mean state and small time-dependent perturbation
• Linearise non-linear residual function around steady flow-field
• Transform equation into frequency domain
• Obtain a large, but sparse system of linear equations

\[
\left( \frac{\partial R}{\partial W} - j \omega I \right) \hat{W} = - \frac{\partial R}{\partial v_g} \hat{v}_g
\]
Frequency-Domain Non-linear Gust Response Computation

Motivation

• CS 25: gust amplitude increases with gust length
• Linearised frequency domain (LFD) accurate for infinitesimally small amplitudes
• Impact on accuracy considering certification amplitudes?
Frequency-Domain Non-linear Gust Response Computation

Motivation

- CS 25: gust amplitude increases with gust length
- Linearised frequency domain (LFD) accurate for infinitesimally small amplitudes
- Impact on accuracy considering certification amplitudes?
  - Compare LFD to non-linear time-domain simulations
  - Shown is max. lift response and $\int_0^T \Delta c_L^2$ for NACA0012 test case
  - Good agreement till non-dim. gust length 20
    (for a typical aircraft case: about 120m)
Frequency-Domain Non-linear Gust Response Computation

Motivation

- Detailed analysis of largest gust length of 35
Frequency-Domain Non-linear Gust Response Computation

**Idea**

- Use Harmonic Balance (HB) method to enhance accuracy at low frequencies
Frequency-Domain Non-linear Gust Response Computation

Idea

• Use Harmonic Balance (HB) method to enhance accuracy at low frequencies
• Not an amplitude non-linearity “per frequency” (minor effect)
• Reduction in magnitude due to coupling between the harmonics of excitation and response
  → HB must be used for 1-cos gust, not single frequency sinusoidal
• Observation made for an aerofoil, but can we see a similar result for an aircraft case?
Frequency-Domain Non-linear Gust Response Computation

Idea

same for full aircraft case at cruise flight

Instantaneous at $c_L$-max
Frequency-Domain Non-linear Gust Response Computation

Approach

1. Calculate steady-state solution
2. Compute LFD solutions covering the relevant frequency range
3. Reconstruct time-domain response for small and medium gust lengths
4. For each “non-linear” gust length:
   1. Choose a base frequency and number of harmonics for Harmonic Balance method
   2. Solve HB equation
   3. Add LFD solutions for frequencies that are not covered by HB
   4. Reconstruct time-domain response
Frequency-Domain Non-linear Gust Response Computation

**HB Approach**

1. $2N_H + 1$ solution vectors equidistantly distributed over a period
2. Compute at each time-slice the residual vector
3. Transform into frequency-domain
4. Compute update via pseudo-time integration

\[
\frac{dW_{HB}}{d\tau} = \omega_b D W_{HB} + R_{HB}
\]

\[
D_{ik} = \frac{2}{2N_H - 1} \sum_{m=1}^{N_H} m \sin \frac{2\pi (k - i)m}{2N_H + 1}
\]
Frequency-Domain Non-linear Gust Response Computation

Results: NACA0012

- Mach 0.75, AoA = 0 deg., Re = 10 million
- Weak transonic case
Frequency-Domain Non-linear Gust Response Computation

Results: NACA0012 – HB-LFD with 3 harmonics

- Mach 0.75, AoA = 0 deg., Re = 10 million
- Weak transonic case
- Harmonic Balance with 3 harmonics
- Significant improvement in both norms
- Small deviations remain at highest gust lengths
- About 5x faster than TD per gust simulation
Frequency-Domain Non-linear Gust Response Computation

Results: NACA0012 – HB-LFD with 3 harmonics

Gust length = 21

Gust length = 35
Frequency-Domain Non-linear Gust Response Computation

Results: NACA0012 – HB-LFD with 4 harmonics

Gust length = 21

Gust length = 35

Frequency-Domain Non-linear Gust Response Computation

Results: NACA0012 – HB-LFD best fit

Gust length = 21, 6 harmonics

Gust length = 35, 10 harmonics
Intermediate conclusion

- Aerodynamic responses of gust encounter compared between linearised frequency domain and non-linear time-domain simulations using CS-25 gust definitions
  - Good agreement for small and medium gust lengths for NACA0012 aerofoil
  - Lift response over-estimated by LFD for larger gust lengths and amplitudes
- Applying Harmonic Balance method with a small number of harmonics combined with LFD results for higher frequencies yields improvement for NACA0012

Next step:
- Compute gust response of fluid-structure coupled configuration using Harmonic Balance and LFD
LFD4Gust with FSI

- Rearrange structural equation in system of 1st order ODE
- Augmented LFD system

\[
\begin{pmatrix}
A_{ff} & A_{fs} \\
A_{sf} & A_{ss}
\end{pmatrix} - j \omega I \begin{bmatrix}
w_f \\
w_s
\end{bmatrix} = \begin{bmatrix}
b_f \\
0
\end{bmatrix}
\]

with subscripts \( f \) and \( s \) denoting fluid or structural DoF, respectively
- Right-hand-side vector defined by field-velocity method
HB4Gust with FSI

- Similar to LFD, the system of equations and the vector of unknowns is augmented with their structural part
- Thus, HB solves for $W_f$ and $W_s$ at each time slice
- Corresponding fluid and structural residuals are computed
  - Involves updating grid point locations and velocities according to structural motion for each time slice
  - Grid movement can be realised using deformation or here rigid-body motion (pitch-plunge aerofoil)
- The rest is usual HB approach
- For implicit solution scheme, coupled Jacobians are used (see LFD solver)
HB4Gust with FSI

- Previous test case extended by pitch-plunge structure
- In-vacuum reduced frequencies of 0.34 for heave and 1.0 for pitch
- Sinusoidal gust encounter with wave length of 21 chord lengths and two gust amplitudes
- TD Signal recorded after 20 periods

$v_{gz} = 6\%$ free-stream velocity

$v_{gz} = 12\%$ free-stream velocity

Lift response
HB4Gust with FSI

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Conclusion

• LFD and HB solver extended to compute response of fluid-structure coupled systems due to gust encounter
• Demonstrated for sinusoidal gusts
  • Good agreement between HB(4) and TD reference
  • Lift and heave response dynamically linear $\rightarrow$ LFD sufficient
  • Contributions of higher harmonics for moment and pitch response
  • Nonlinearities captured well by HB method

Future steps:
• Application to 1-cos gusts
• Apply symbiotic approach of HB-LFD to coupled system
First results: 1-cos gust

- 1-cos response of longest gust: \( L_g = 35.5, v_{gz} = 6.6\% \)
- Lift and heave response is over-predicted by LFD while pitch response is under-predicted
- HB(14)(!) improves the prediction at the peak
Thank you!

The research leading to this work has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement number 636053.