Reduced Order Modelling for gusts
Motivation

- From design to certification of an aircraft many aerodynamic data are needed – for the entire flight envelope –
- Aerodynamic data \(\rightarrow\) pressure and shear stress distributions as well as global coefficients from steady and unsteady simulations

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- From design to certification of an aircraft many aerodynamic data are needed – for the entire flight envelope –
- Aerodynamic data \(\rightarrow\) pressure and shear stress distributions as well as global coefficients from steady and unsteady simulations

![Diagram](image-url)

**Motivation Diagram**

- 50 flight conditions
- \(\mathcal{O}(10^6)\) simulations
- 4 control laws
- 100 mass configs.
- 5 manoeuvres + 25 gust lengths
- Idealized drag coefficient
- Lift coefficient
- \(1\)-cos gust
- Performances
- Handling qualities

**Diagram Details**

1. Aerodynamic data
2. Pressure and shear stress distributions
3. Global coefficients from steady and unsteady simulations
4. Flight envelope
5. Control laws
6. Mass configurations

**Diagram Elements**

- Aerodynamic data flow
- Pressure and shear stress distributions
- Global coefficients
- Flight envelope
- Control laws
- Mass configurations
- Idealized drag coefficient
- Lift coefficient
- 1-cos gust
- Performances
- Handling qualities

**Diagram Notes**

- \(\mathcal{O}(10^6)\) simulations
- 4 control laws
- 100 mass configs.
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- Idealized drag coefficient
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Overview

Various different reduced order models (ROMs) techniques have been proposed for various topics

This presentation will focus on methods developed and applied within the AeroGust project:

- Frequency domain based methods
  - Linearised frequency domain
  - Harmonic balance
- Proper orthogonal decomposition based methods
  - Residual minimisation
  - Subspace projection
  - Zonal POD
- Corrected surrogate methods
Nonlinear unsteady LSQ-ROM approach

- Physics-based ROM using high-fidelity data for rapid many queries unsteady predictions, like manoeuvre & gust loads

**OFFLINE**
- gust simulations

**TRAINING INPUT**
- Gust Amplitude $A_g$
- Gust Length $L_g$

**TRAINING OUTPUT**
- $C_L$
- $(L_g, A_g)_x$
- $(L_g, A_g)_1$
- $(L_g, A_g)_2$
- $(L_g, A_g)_3$

**FLOW FIELD SNAPSHOTS**
- $w(t_i)$
- $w(t_j)$

**POD**
- $(L_g, A_g)_x$

**ROM PREDICTION**
- Flow field output time history!
- $w(t_i)$
- $w(t_j)$

Evaluate the ROM performance and compare results to FOM and LFD results
I. compute the approx. solution
\[ \mathbf{w}(t) \approx U \mathbf{a}(t) + \overline{\mathbf{w}} \Rightarrow O(Nr) \]

II. evaluate the unsteady residual
\[ \hat{R}(\mathbf{w}(t)) \overset{\text{def}}{=} R(\mathbf{w}(t)) + \Omega \frac{\partial \mathbf{w}(t)}{\partial t} \Rightarrow O(N) \]

III. solve the LS problem
\[ (J^T J + \lambda I) \Delta \mathbf{a} = -J^T \hat{R} \Rightarrow O(Nr) \]

The computational cost scales linearly with the dimension \( N \) of the full order model. No significant speedup can be expected when solving the minimum residual ROM and hyperreductions are necessary.
Theory of Nonlinear unsteady LSQ-ROM

Complexity reduction by sampling (or compute only a few entries of) the nonlinear unsteady residual vector \( \hat{\mathbf{R}} \)

- omission of many components
- non intrusive
- approximation of the entire vector, by interpolation or by least-squares projection onto a subspace

Selecting the subset indices \( \rightarrow \)

- (Discrete) Empirical Interpolation Method
- Missing Point Estimation

**Greedy: minimize**

\[
\| \mathbf{U} \left( \mathbf{U}^T \mathbf{P} \mathbf{P}^T \mathbf{U} \right)^{-1} \mathbf{U}^T \mathbf{P} \mathbf{P}^T \| = \frac{1}{\sigma_{\min}(\mathbf{P}^T \mathbf{U})}
\]

- The complete nonlinear unsteady residual vector \( \hat{\mathbf{R}} \) is evaluated,
- but only a small subset of its entries are used in the minimization process
2D Test Case: FFAST crank airfoil

- RANS equations with SA-neg turbulence model
- \(V_\infty = 223\) m/s, Mach = 0.754, Re = 6.9 \(10^5\)
- Two 1-cos gusts as training signals
- Amplitudes of training signals based on CS25
- 3 greedy missing point estimation levels:
  - All points, 6000 points, 3000 points

**CFD settings**
- Dual time stepping
- Min residual: 1e-4
- Max inner iterations: 300
- Physical time steps: 500
- Linearly distributed time steps

**ROM setting**
Truncation of the POD modes to 99.999% of their energy content (~100 modes)

This work was carried out by DLR
2D Test Case: FFAST crank airfoil

Maximum lift coefficient vs gust length

Surface pressure at max. lift coefficient

Lift coefficient response over time

Gust length: 92m / $c_{ref}$

Gust amplitude: CS25
3D Test Case: NASA CRM

- RANS equations with SA-neg turbulence model
- $V_\infty = 261 \text{ m/s}, \text{ Mach} = 0.86, H = 9142\text{m}$
- Using 1-cos gust with $L_g = 213 \text{ m}$ and CS25 certification gust amplitude as training signal
- Reconstruction of the training signal

**CFD setting**
- Dual time stepping
- Min unsteady residual: $1e-6$
- Max inner iterations: 500
- Physical time steps: 300
- Linearly distributed time steps

**ROM setting**
Truncation of the POD modes to 99.9999% of their energy content (~100 modes)
3D Test Case: NASA CRM

Response behavior for lift and pitching moment coefficient

Differences in surface pressure distribution at max. lift coeff

Gust length: 213 m
Gust amplitude: CS25

This work was carried out by DLR
REDUCED ORDER MODELS FROM FREQUENCY DOMAIN SIMULATION

• FOM : O(100 million) d.o.f. plus time OK for “one-off” simulations
• LFD / HB : O(100 million) d.o.f. OK for “routine” simulations
• ROM : O(100) d.o.f. “towards real time” simulations
Basic Idea of Frequency-Domain Methods – some maths

How many CFD linear solves are needed to predict 1-cos gust responses of flexible aircraft (say, m=99 structural modes) for sufficient number of gust gradients?

\[
(A_{ss} - i\omega I) - A_{sf} (A_{ff} - i\omega I)^{-1} A_{fs} \hat{w}_s = -A_{sf} (A_{ff} - i\omega I)^{-1} \hat{b}_f
\]

\[
(A_{ff} - i\omega I) \hat{w}_f = \hat{b}_f - A_{fs} \hat{w}_s \quad \text{m+1 per } \omega
\]

\[
\begin{pmatrix}
(A_{ff} - i\omega I) & A_{fs} \\
A_{sf} & (A_{ss} - i\omega I)
\end{pmatrix}
\begin{pmatrix}
\hat{w}_f \\
\hat{w}_s
\end{pmatrix} =
\begin{pmatrix}
\hat{b}_f \\
0
\end{pmatrix}
\quad \text{1 per } \omega
\]

\[
(A_{ff} - i\omega I) - A_{fs} (A_{ss} - i\omega I)^{-1} A_{sf} \hat{w}_f = \hat{b}_f \quad \text{1 per } \omega
\]

\[
(A_{ss} - i\omega I) \hat{w}_s = -A_{sf} \hat{w}_f
\]

This work was carried out at The University of Liverpool
‘Modal Decomposition and Projection’ Reduced Order Model

• Eigenmode Basis

$O(100 \text{ million}) \times O(100 \text{ million})$

$\Psi^H A \Phi = \Lambda$

$O(100) \times O(100)$
‘Modal Decomposition and Projection’ Reduced Order Model

- Proper Orthogonal Decomposition Basis

\[ \Phi^H A \Phi = \tilde{A} \]

\[ O(100 \text{ million}) \times O(100 \text{ million}) \]

\[ O(100) \times O(100) \]

POD mode

‘energy’ content
Linear Aerodynamics POD ROM – NACA 0012 Aerofoil

M=0.3, Re=10m, α=0.0 deg.

M=0.8, Re=10m, α=0.0 deg.

M=0.8, Re=10m, α=3.0 deg.
“... but certification gust amplitudes are not small!”  Idea!
Frequency-Domain Non-linear Gust Response Computation

• Calculate steady-state solution
• Compute LFD solutions covering the relevant frequency range
• Reconstruct time-domain response for small to medium gust lengths

• For each ‘non-linear’ gust length:
  • Choose a base frequency and harmonics for HB method
  • Solve HB equation: \[ 0 = \omega D W_{hb} + R_{hb} \]
  • Add LFD solutions for frequencies that are not covered by HB
  • Reconstruct time-domain response

\[
W_{hb} = \begin{pmatrix}
W(t_0 + \Delta t) \\
W(t_0 + 2\Delta t) \\
\vdots \\
W(t_0 + T)
\end{pmatrix}, \quad R_{hb} = \begin{pmatrix}
R(t_0 + \Delta t) \\
R(t_0 + 2\Delta t) \\
\vdots \\
R(t_0 + T)
\end{pmatrix}
\]

\[
D_{i,j} = \frac{2}{N_T} \sum_{k=1}^{N_H} k \sin(2\pi k(j - i)/N_T)
\]
Non-linear Frequency-Domain Results – NACA0012 (M=0.75, Re=10m, $\alpha=0$ deg)
Non-linear Frequency-Domain Results – NACA0012 (M=0.75, Re=10m, $\alpha=0$ deg)
### Non-linear Frequency-Domain Results – Cost

<table>
<thead>
<tr>
<th>Method</th>
<th>TD</th>
<th>LFD</th>
<th>HB(4)</th>
<th>HB(6)</th>
<th>HB(8)</th>
<th>HB(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run-time [h]</td>
<td>26.3</td>
<td>0.6</td>
<td>5.0</td>
<td>12.5</td>
<td>28.0</td>
<td>48</td>
</tr>
<tr>
<td>time-saving factor</td>
<td>1.0</td>
<td>44</td>
<td>5.3</td>
<td>2.1</td>
<td>0.9</td>
<td>0.4</td>
</tr>
</tbody>
</table>

*) LFD cost for computing 40 sinusoidal gusts used for reconstruction of all ‘1-cos’

**) TD and HB state cost per ‘1-cos’
Non-linear Frequency-Domain Results – Aeroelastic Aerofoil, 6% gust amplitude

![Graph 1: ΔC-my / v_gz vs. non-dim. time](image1)

- **TD**
- **LFD**
- **HB Nh = 1**
- **HB Nh = 4**

![Graph 2: Δpitch / v_gz vs. non-dim. time](image2)

- **TD**
- **LFD**
- **HB Nh = 1**
- **HB Nh = 4**
Non-linear Frequency-Domain Results – Aeroelastic Aerofoil, 12% gust amplitude
Aeroelastic ROM for gusts

The Aeroelastic ROM is constructed based on the Non-Linear Harmonic method (NLH) that has been developed by Numeca for turbomachinery configurations.

This method has first been applied to external aerodynamic configurations by Debrabandere [2014] in the scope of the FFAST project. It has been extended to 2-way coupling by González Horcas [2016] for wind turbine applications.

It is hereafter proposed to extend the NLH-based ROM with 2-way coupling to gust simulations on un-structured meshes.

Q-criterion iso-surfaces for a value of 0.5
Aeroelastic ROM for gusts

The NLH method

The instantaneous conservative flow variables $U = (\rho, \rho v_x, \rho v_y, \rho v_x, \rho E)$ are decomposed into a time-averaged value $\bar{U}$ and a sum of unsteady perturbations $U''_n$, assumed to be periodic:

$$U (\vec{x}, t) = \bar{U} (\vec{x}) + \sum_n U''_n (\vec{x}, t)$$

A Fourier decomposition is applied to each of the periodic perturbations. Hence, every perturbation $U''_n$ can be written as a finite sum of $N_h$ time harmonics:

$$U''_n (\vec{x}, t) = \sum_{h=1}^{N_h} \left[ \tilde{U}_h (\vec{x}) e^{i \omega_n t} + \tilde{U}_{-h} (\vec{x}) e^{-i \omega_n t} \right] = 2 \sum_{h=1}^{N_h} \left[ \tilde{U}_h^{Re} \cos (\omega_n t) - \tilde{U}_h^{Im} \sin (\omega_n t) \right]$$

where the harmonic amplitudes $\tilde{U}_h$ and $\tilde{U}_{-h}$ are complex conjugates related to the $n^{th}$ perturbation and defined by the real part $\tilde{U}_h^{Re}$ and the imaginary part $\tilde{U}_h^{Im}$.

The value of $N_h$ is an input of the method, driving both the frequency and space resolution of the computed flow unsteadiness.

For the particular case of rotating machinery applications, the fundamental harmonic frequency $\omega_1$ can be related to the Blade Passing Frequency (BPF).
Aeroelastic ROM for gusts

The NLH formulation is derived by introducing this variable decomposition into the unsteady Navier-Stokes equations. A new set of equations is obtained, associated to the time-mean contribution of every conservation law and the corresponding harmonics. In finite-volume compact formulation:

\[
\frac{\partial \bar{U}}{\partial \Omega} \Omega + \sum_{\text{cell faces}} \left( \vec{F}_C - \vec{F}_V \right) \cdot \vec{S} = \vec{Q} \Omega
\]

\[
\frac{\partial \widetilde{U} |_h}{\partial \Omega} \Omega + I \omega_h \widetilde{U} |_h \Omega + \sum_{\text{cell faces}} \left( \vec{F}_C |_h \right) \cdot \vec{S} - \sum_{\text{cell faces}} \left( \vec{F}_V |_h \right) \cdot \vec{S} = \vec{Q} |_h \Omega
\]

Obtained equations are only space-dependent, their resolution is performed through the pseudo-time derivative. This justifies the computational time saving of the NLH method. The harmonic solution can be reconstructed in time to perform a more comprehensive post-processing of the unsteady flow.
Aeroelastic ROM for gusts

Structure modeling: modal approach

Aeroelastic equilibrium based on linear dynamics equation:

\[ M \frac{\partial^2 \ddot{u}}{\partial t^2} + C \frac{\partial \dot{u}}{\partial t} + K \ddot{u} = f_s \]

A modal basis of the structure can be obtained by solving the eigen value problem when assuming no loading and no damping:

\[ M \frac{\partial^2 \ddot{u}}{\partial t^2} + K \ddot{u} = 0 \]

\[ \omega_k = \sqrt{\lambda_k} \quad \text{Natural frequencies (related to eigen values)} \]

\[ \phi_k \quad \text{Mode shapes (i.e. eigen vectors)} \]

Obtained prior to the FSI computation

The linear dynamics equation can be expressed based on this basis. Normalizing the mode shapes by the mass, assuming Rayleigh damping \( C = \alpha M + \beta K \), and a frequency-independent stiffness, it can be uncoupled for every mode \( k \) as:

\[ \frac{\partial^2 q_k}{\partial t^2} + 2\xi_k \omega_k \frac{\partial q_k}{\partial t} + \omega_k^2 q_k = \phi_k^T \tau_{k,s} f_s \]

With the generalized displacements \( q_k \) that can be related to the deformations as: \( \ddot{u} = \sum q_k \phi_k \)
Aeroelastic ROM for gusts
In order to obtain a formulation adapted to the NLH solver, variables are decomposed into time-averaged and harmonic contributions:

\[ q_k(\vec{x}, t) = \bar{q}_k(\vec{x}) + \sum_n q''_{k,n}(\vec{x}, t) \]

\[ \tilde{f}_S(\vec{x}, t) = \bar{f}_S(\vec{x}) + \sum_n \tilde{f}''_{S,n}(\vec{x}, t) \]

Introducing this decomposition into the linear dynamics equations and simplifying the set of perturbations by \( q''_k \) and \( \tilde{f}''_S \):

\[
\frac{\partial^2}{\partial t^2} (\bar{q}_k + q''_k) + 2\xi_k\omega_k \frac{\partial}{\partial t} (\bar{q}_k + q''_k) + \omega_k^2 (\bar{q}_k + q''_k) = \phi_k^T \left( \bar{f}_S + \tilde{f}''_S \right)
\]

### Table: Time-mean equation

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{q}_k )</td>
<td>( \frac{\phi_k^T \bar{f}_S}{\omega_k^2} )</td>
</tr>
<tr>
<td>( \bar{u} )</td>
<td>( \sum_{k=1}^{N_k} \bar{q}_k \phi_k )</td>
</tr>
</tbody>
</table>

### Table: Harmonic equation

<table>
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<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{q}_k</td>
<td>_h )</td>
</tr>
<tr>
<td>( \tilde{u}</td>
<td>_h )</td>
</tr>
</tbody>
</table>
Application of Aeroelastic ROM for gust modelling to FFAST Crank airfoil

Final test case description has been defined: FFAST crank airfoil with flow condition “D”
- Altitude: 35,000 ft
- Free stream Mach number = 0.754
- Chord length: 8 m
- Three gust scenarios

Three key numerical parameters are investigated for this configuration
- Gust windowing: Repetition number set to 10
- Spectrum truncation: 16 Harmonics
- Space discretization: 20 mesh points to represent harmonic 15
Application of Aeroelastic ROM for gust modelling to FFAST Crank airfoil
Time evolution of lift and torsion moment for three different gust scenarios.
Application of Aeroelastic ROM for gust modelling to UAV wing

Time evolution of wing deflection due to gust passage (H30)
Application of Aeroelastic ROM for gust modelling to UAV wing

Maximum deformed wing (Gust 30ft)
Time-mean deformed wing
Un-deformed wing

Vertical Displacement [m]

This work was carried out by NUMECA
Development of Reduced Order Models for accelerating CFD predictions

- **Perform several full CFD simulations** for different values of the parameters
- **Build a Proper Orthogonal Decomposition (POD) basis** with these simulations
- **Predict** the behavior of the system for a new value of the parameters by using low cost hybrid simulations

Possible approaches for Hybrid simulations

**Domain decomposition**

A CFD solver is used to compute the solution in a small region which is strongly influenced by the effects of the change in the parameter space

General purpose approach (compressible/incompressible equations, steady-unsteady problems,...)

**Poisson POD solver**

One of the most time consuming functions in CFD solver for incompressible flows is the iterative solver which is used for the Poisson problem in the correction step.

Use a POD basis to get a cheap solution of the Poisson problem: do not alter the other functions in the CFD tool
Domain decomposition: dynamic coupling between CFD and POD

Goal: Reduce simulation cost by exploiting available HiFi solutions through POD-based ROMS

But: Certain non-linearities are difficult to describe by POD

• Use CFD where POD-ROM may fail
• The POD model is used to define the boundary conditions for a reduced [space : time] CFD domain

• Introduction of static forcing terms for gust simulations:
  \[ U_{ROM}(x,t) = U_{avg}(x) + U_g(x,t) + \sum a_i(t)\Phi_i(x) \]
• POD accounts for corrections due to gust/mean field interaction

• The coefficients of the POD expansion \(a_i\) are computed by minimizing the distance between the CFD solution and the POD expansion in the overlapping region at each time step:
  \[ \min (\|U_{CFD} - U_{ROM}\|_{\Omega_o}) \]
Development of hybrid technology in Aerogust

Application to:
✓ an airfoil-vortex interaction problem
✓ gust in transonic conditions

Usability:
? How to chose high-fidelity region [Pb1]
? How to chose overlapping region [Pb2]
? How to relate to parameter space sampling [Pb3]
Pb1 & PB2: How to parameterize space?

Leave-One-Out Cross Correlation

- Mask #0
- Mask #1
- Mask #N-1

Data base

This work was carried out by INRIA and Optimad
Pb1 & PB2: How to parameterize space?

This work was carried out by INRIA and Optimad
Pb1 : How to choose the HiFi region?

Use HiFi solver if POD basis cannot reproduce the solution:

\[ \Omega_{CFD} = \{ x \cdot e(x) > \sigma^{MIN} \} \]

with \( \sigma^{MIN} \) being the allowable error

\[ \sigma^{MIN} = 0.01 \quad \sigma^{MIN} = 0.001 \]
Pb2: How to choose the overlapping region?

Take as many points as possible as “sensors”, but avoid misleading sensing due to low correlations:

**Leave-one-out strategy (2)**

- Create a database of high-fidelity simulations
- For \(i = 1, N_{sim}\) {
  - Remove the \(i\)-th simulation from the database
  - Use the remaining simulations to build a POD basis
  - For \([\text{MIN}, \text{MAX}]\) {
    - Choose a threshold and define the corresponding overlapping region
    - Use the obtained POD basis to perform an hybrid simulation and to evaluate the prediction error on a given goal function
  }
- Identify the overlapping region which gives (in average) the minimum prediction error
Pb2: How to choose the overlapping region?

Predictive simulation: $C_l$ vs time for different $\sigma_0$
Pb3: relation to available data

1) Fixing the prediction error, the size of the CFD domain can be reduced

2) Fixing the size of the domain, the prediction errors decrease
Pb3: Increase in efficiency as database is enriched
Non-Linear Aeroelastic ROM

1. Beam ROM: Non-linear Modal ROM
2. Aerodynamic ROM: Corrected Unsteady Potential Flow
Wing Beam Modal ROM

Beam Modal ROM

Real Beam

This work was carried out by University of Cape Town
Wing Beam Quadratic ROM

- Pragmatic approach to account for higher-order kinematics
- Linear Modal Analysis
  - Determine linear mode shapes, $\phi^i_{L}$
- Transformation into time domain
  - Inclusion of quadratic mode shapes for non-linearity
    $$u(x, t) = \sum_i q_i(t)\phi^i_{L} + \sum_i \sum_j q_i(t)q_j(t)G^{ij}$$
  - Quadratic mode shape components $G^{ij}$
Wing Beam Quadratic ROM: Validation

• Cantilever beam undergoing geometrically non-linear deflection
• Tip vertical displacement

\[ \varepsilon = 7\% \]

\[ \frac{w}{L} \text{ of Typical wing deflection} \]

Current Analysis
Wing Beam Tuned Quadratic ROM

- Amplitude and frequency of standard QMS conforms to linear model
- Introduce tuning parameter to consider influence of position on response

\[
u(x, t) = \sum_i q_i(t) \phi^T_C + \sum_i \sum_j q_i(t) q_j(t) G_{ij}
\]

\[
\phi^C = \phi_L + G_{ij} \delta_{ij}
\]

\[
\phi^{TC} = \gamma \phi_L + (1 - \gamma) \phi_C
\]

57% improvement in amplitude

11% improvement in period

This work was carried out by University of Cape Town
Wing Beam Tuned Quadratic ROM: Validation

- CRM wing FEM model reduced to beam elements [1]
- Compare linear modal to tuned QMS approach
  - Half-gust load applied at wing tip
  - 2% structural damping applied
- IFASD conference paper 2017

[1] Condensation performed by Robbie Cook from University of Bristol
Novel Non-linear Aeroelastic ROM

- Kriging Corrected Unsteady Vortex Panel Method (UVPM) for Aero
- Speed: Mach 0.85 (0.99 on swept wing)
- Wing structure: Pitch-Plunge
- $1 - \text{cosine gust}$:
  - $U = A_s f_g \frac{U_g}{2} \left[1 - \cos \left(\frac{\pi l}{H}\right)\right]
- Training cases: Mach 0.85
  - 70 ft & 350 ft
- Untried cases: Mach 0.85
  - 150 ft & 250 ft
Novel Non-linear Aeroelastic ROM

- Kriging Corrected aerodynamic UVPM
- Inputs:
  - Structure position & velocity
  - Transient load (i.e. gust)
- Two UVPM models:
  - w/ and w/o gust
  - Get UVPM estimates of lift and moment
- Two Kriging models:
  - Lift and Moment
  - Correct UVPM estimates

\[
\hat{C}_L = f(h, \alpha, \dot{h}, \dot{\alpha}, \tilde{C}_L, \tilde{C}_M, \tilde{C}^{ng}_L, \tilde{C}^{ng}_M)
\]

\[
\hat{C}_M = g(h, \alpha, \dot{h}, \dot{\alpha}, \tilde{C}_L, \tilde{C}_M, \tilde{C}^{ng}_L, \tilde{C}^{ng}_M)
\]
Aerodynamic ROM Training

1. Run two transonic CFD aeroelastic gust cases (boundaries)
2. Run UVPM models with the CFD response histories
3. Select an initial training data set
4. Add points to the training set iteratively
   1. Use the current set to predict all sample points
   2. Add the sample with highest error to the training set
5. Iterate until max. error < 0.5% peak (entire pool)

NB: $C_L$ and $C_M$ have different training sets (same number of points)
Aerodynamic Surrogate Alternative

- Replace UVPM models with $\bar{U}$ and $\zeta$
- Lacked robustness, required numerous attempts
- Introduced more training constraints:
  - Mean error < 0.1% peak
  - Mean and max criteria must be satisfied for 4 consecutive iterations
- ROM already satisfied new constraints

\[
\bar{U} = \frac{\int U(x)dx}{c}
\]

\[
\zeta = \frac{2}{\pi} \arctan \left( \frac{x_g - x_m}{H} \right)
\]

\[
\hat{C}_L = f(h, \alpha, \dot{h}, \dot{\alpha}, \bar{U}, \zeta)
\]

\[
\hat{C}_M = g(h, \alpha, \dot{h}, \dot{\alpha}, \bar{U}, \zeta)
\]
Aerodynamic Predictions on Training Cases

For 70 ft:

- CFD
- Surrogate
- ROM

For 350 ft:

- CFD
- Surrogate
- ROM
Aeroelastic ROM Evaluation

• Untried Case: H=250ft
• CPU speed-up vs. full-order: 40 times

• Journal paper (2019)
• AIAA conference paper (2019)

ROM Max. Error: 1.71%
Surrogate Max. Error: 8.45%

ROM Max. Error: 3.62%
Surrogate Max. Error: 15.3%
SUMMARY

• Reduced order modelling offers potential computational savings of an order of magnitude for gusts

• Although certification gusts are not periodic, frequency domain based methods can still be applied
  • Linearised frequency domain good accuracy for many certification gusts
  • Harmonic balance can be used when non-linearities become significant

• Proper orthogonal decomposition based methods have proved very effective and efficient in modelling Gusts

• A different approach of correcting a simple model with high fidelity data produces models with a wide span of applicability
The research leading to this work has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement number 636053.