A novel aerodynamics reduced order model (ROM) is proposed for gust load calculations on two- and three-dimensional wing geometries at transonic flow conditions. Force estimates from sectional potential flow solutions are taken as inputs to kriging models that predict the aerodynamic loads. The kriging models are trained with coupled aeroelastic CFD calculations of two gust lengths (viz. short and long) which define a range in which the ROM is applicable. An interface is developed to handle the coupling of fluid and structure in the training simulations. The interface ensures conservative transfer of forces from the fluid to the structure and reconstruction of the fluid mesh subject to structural motion. Two test cases are put forward as a means to evaluate ROM: the FFAST Crank aerofoil and the NASA Common Research Model (CRM). In the former case, individual run cost decreased by an order of magnitude and error in the aeroelastic response never exceeded 8%. At the time of writing, the CFD calculations for the CRM case are underway.

I. Introduction

Non-linear aeroelasticity is a field of increasing interest as the volume of transonic commercial flights continues to grow. Aviation regulations require that a substantial set of design load cases be investigated and thus the need for an efficient non-linear aeroelastic modelling solution is pressing. the H2020 project Aerogust, of which this research is a result, focuses on aeroelastic reduced order models (ROMs). It aims at improving on the computational cost of traditional computational fluid dynamics (CFD) models for gust loads calculation.

Much progress has been made on the development of aerodynamic ROMs for loads calculation. The range of theory applied to the problem is wide and includes approaches based on eigen realisation algorithms (ERAs) [1,2], Proper Orthogonal Decomposition (POD) [3], Volterra theory [4], and surrogates [5,6]. In potential flow, recent efforts have been focussed on the doublet lattice method [7,8]. Kriging [9] – a particularly useful tool in the field – is often used as the basis for aerodynamic surrogates [10,11].

This article reports on the development of a general aeroelastic ROM methodology for transient aerodynamic gust load applications. The method involves the coupling of an aerodynamics ROM to a structural model which represents the wing structure. The aerodynamics ROM uses sectional potential flow solutions, computed with an unsteady vortex panel method (UVPM) [14], at multiple stations along the wing to estimate the load distribution. Kriging is then used to correct the estimates by establishing a relationship between the force coefficients of the UVPM sections and the forces applied to structure – as calculated by CFD. The coupled aeroelastic CFD simulations are facilitated by a fluid structure interface which handles force transfer and surface reconstruction.

For the purpose of assessing the proposed aerodynamic ROM, two test cases have been selected. In both cases, trained ROMs are coupled to structures and used to predict the aeroelastic response of untried gusts. The results are evaluated, in terms of accuracy and computational cost, with respect to coupled CFD calculations of the same cases.
II. Governing Equations and Modelling Approach

A. Euler Equations

The principal objective of this work is to accurately model the aeroelastic response of an aircraft subject to gust loading in transonic flow regimes. Thus an arbitrary Lagrangian-Eulerian (ALE) formulation of the Euler equations is used for the full order model [15]. While this set of equations neglects viscous effects, it still captures the key physics of concern. The transient gust propagating through the domain is applied via the split velocity method (SVM) [16]. This approach treats the fluid velocity \( \mathbf{u} \) as the sum of a prescribed (gust) component \( \mathbf{u}_g \) and a response component \( \tilde{\mathbf{u}} \). SVM operates directly on the governing equations by introducing source terms that prevent the dissipation of the gust as it moves towards the aircraft from the far-field boundary. The SVM ALE Euler equations are

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \left[ \rho (\tilde{\mathbf{u}} + \mathbf{u}_g - \mathbf{v}) \right] = 0, \tag{1}
\]

\[
\frac{\partial (\rho \tilde{\mathbf{u}})}{\partial t} + \nabla \cdot \left[ \rho \tilde{\mathbf{u}} (\tilde{\mathbf{u}} + \mathbf{u}_g - \mathbf{v}) + P \mathbf{I} \right] + s_m = 0, \tag{2}
\]

\[
\frac{\partial (\rho \tilde{E})}{\partial t} + \nabla \cdot \left[ \rho \tilde{E} (\tilde{\mathbf{u}} + \mathbf{u}_g - \mathbf{v}) + P \tilde{\mathbf{u}} \right] + s_e = 0. \tag{3}
\]

Here \( \rho \) is density, \( \tilde{E} \) is total response energy, \( P \) is pressure, \( s_m \) is the momentum source term, \( s_e \) is the energy source term, \( \mathbf{v} \) is mesh velocity, and \( \mathbf{I} \) is the identity matrix. \( P \) (subject to the assumption of a calorifically perfect ideal gas), \( s_m \), and \( s_e \) are given respectively by

\[
P = (\gamma - 1) \rho \left( \tilde{E} - \frac{\tilde{\mathbf{u}} \cdot \tilde{\mathbf{u}}}{2} \right), \tag{4}
\]

\[
s_m = \rho \left[ \frac{\partial \mathbf{u}_g}{\partial t} + \nabla \mathbf{u}_g \cdot (\tilde{\mathbf{u}} + \mathbf{u}_g - \mathbf{v}) \right], \tag{5}
\]

\[
s_e = s_m \cdot \tilde{\mathbf{u}} + P \nabla \cdot \mathbf{u}_g. \tag{6}
\]

where \( \gamma \) is the ratio of specific heats and taken as 1.4 throughout this work. In order to model Equations (1)-(3), the vertex-centred finite volume method (FVM) solver Elemental is used. Notional second order accuracy is achieved using the Monotonic Upwind for Conservation Laws (MUSCL) scheme [17] with the symmetric van Albada limiter [18]. The face fluxes are calculated with the approximate Riemann solver of Roe [19]. Time integration is performed with a second-order accurate backward difference in conjunction with dual time stepping [20]. The dual time stepping uses a four-stage Runge-Kutta scheme, with coefficients as described by Lallemand [21], to iterate towards a converged solution for the time step.

B. Incompressible Potential Flow

A tradiof must be made between computational efficiency and solution accuracy when developing a reduced order model (ROM). In the case of this work, the tradiof is made by modelling only a subset of the physics. To this end, the ROM under development is based on incompressible potential flow. In potential flow, the flow velocity \( \mathbf{u} \) is assumed to be the gradient of a scalar potential \( \phi \) (i.e. \( \mathbf{u} = \nabla \phi \) [22]. In conjunction with the assumption of incompressibility, this velocity definition leads to the following mass and momentum conservation relations:

\[
\nabla^2 \phi = 0, \tag{7}
\]

\[
\nabla \left[ \rho \left( \frac{\partial \phi}{\partial t} + \frac{\nabla \phi \cdot \nabla \phi}{2} \right) + P \right] = 0. \tag{8}
\]

Here \( \phi \) may be solved for – subject to boundary conditions – as the sum of discrete solutions to Equation (7) owing to the linearity of \( \phi \) therein. Equation (8) is integrated in space to yield Bernoulli’s equation for incompressible potential flows:
\[ \rho \left( \frac{\partial \phi}{\partial t} + \nabla \phi \cdot \nabla \phi \right) + P = f(t) \quad (9) \]

where \( f(t) \) is some function of time. With regard to aircraft configurations, it is fair to assume that the left-hand side of Equation (9) tends toward constant as distance approaches infinity. As a result, \( f(t) \) is effectively a constant reference pressure \( P_0 \) and \( P \) may be computed directly from the already known potential.

For the purposes of the ROM, \( \phi \) is solved for – in discrete sections – using a two-dimensional unsteady vortex panel method (UVPM) \[14\]. The aerofoil surface is discretised as vortex panels with linear strength functions. The strengths (which are continuous everywhere except at the trailing edge) are solved for such that the surface is a slip boundary. The Kutta condition is applied at the trailing edge from which the vortex panel wake sheds. As Equation (7) contains no transience and Equation (8) is used to calculate \( P \), Kelvin’s circulation theorem governs the time evolution of the state variables:

\[ \frac{D\Gamma}{Dt} = 0, \text{ where } \Gamma = \oint_c \mathbf{u} \cdot d\mathbf{c} \quad (10) \]

Here \( \frac{D}{Dt} (\cdot) \) denotes the Lagrangian derivative with respect to time. \( \Gamma \) is circulation and \( c \) is an arbitrary curve enclosing the aerofoil and its wake. Equation (10) is used to determine the amount of vorticity shed into the wake. This functionality was implemented into Elemental®.

### C. Structure

The structures dealt with in this study undergo displacements within the range of linear elasticity. As such, a linearised governing equation is used to model the resulting motion:

\[ \mathbf{M} \cdot \ddot{\mathbf{x}} + \mathbf{C} \cdot \dot{\mathbf{x}} + \mathbf{K} \cdot \mathbf{x} = \mathbf{f} \quad (11) \]

Here \( \mathbf{x} \) and \( \mathbf{f} \) are the vectors containing the displacements of and forces applied to each degree of freedom respectively. \( \mathbf{M} \), \( \mathbf{C} \), and \( \mathbf{K} \) are the mass, damping, and stiffness matrices respectively. \( \dot{\mathbf{x}} \) and \( \ddot{\mathbf{x}} \) denote the first and second derivatives of displacement with respect to time respectively (i.e. velocity and acceleration). Time integration of Equation (11) is handled implicitly with Newmark’s Method \[23\] where

\[ \ddot{\mathbf{x}}_{i+1} = \dot{\mathbf{x}}_i + \Delta t \left[ (1 - \xi) \ddot{\mathbf{x}}_i + \xi \ddot{\mathbf{x}}_{i+1} \right] \text{ where } 0 \leq \xi \leq 1, \quad (12) \]

\[ \mathbf{x}_{i+1} = \mathbf{x}_i + \Delta t \dot{\mathbf{x}}_i + \frac{\Delta t^2}{2} \left[ (1 - 2\beta) \ddot{\mathbf{x}}_i + 2\beta \ddot{\mathbf{x}}_{i+1} \right] \text{ where } 0 \leq 2\beta \leq 1. \quad (13) \]

Here \( \Delta t \) is the time step duration and the subscript \( i \) denotes an arbitrary time step. The values of \( \xi \) and \( \beta \) were taken as 0.5 and 0.25 respectively which yields the constant average acceleration method. This method is second order accurate and strongly stable.

### III. Reduced Order Modelling Strategy

The aerodynamics ROM under development is intended to replace the transonic CFD calculations. In so doing, it is also desirable that the ROM obfuscate the need for a fluid surface interface (as discussed below). As such, the ROM has been designed to accommodate the CFD modelling requirements as well as the inputs and outputs of the structural model. While the application of this study is specific to gusts, specific effort has been made to develop an approach that generalises to arbitrary velocity-based disturbances.

Figure [1] shows that the general ROM consists of two sets of UVPM models and a set of kriging models. While only one UVPM set sees the gust \( U \), both receive \( \mathbf{x} \) and \( \dot{\mathbf{x}} \) from the structure as well as the vector \( \mathbf{l} \). \( \mathbf{l} \) contains the positions of each beam node along the length of the undeformed beam. Therefore, in this work all beam coordinates are fed into the ROM. From this information, each UVPM estimates coefficients of drag, lift, and moment which are then compiled into the respective coefficient vectors \( \mathbf{C}_D \), \( \mathbf{C}_L \), and \( \mathbf{C}_M \). Coefficient vectors without a superscript include the effect of the gust while those with the \( ng \) superscript do not.

These coefficients as well as \( \mathbf{x}, \ddot{\mathbf{x}}, \) and \( \mathbf{l} \) are passed to the kriging models. Here the inputs are used to predict the forces acting at each beam node which are then compiled into the vector \( \mathbf{f} \) and passed to the structure. The specific details of the UVPM models, kriging models, and training approach are discussed in the sections to follow.
A. UVPM Models

The key function of the UVPM models is to provide a physical underpinning to the predictions of the ROM. Being a two-dimensional approach, the UVPM is not directly applicable to the three-dimensional wings of interest. This problem is overcome by following a process similar in nature to that work of van Rooyen [24]. A set number of vertical cross sections (parallel to the freestream direction) are taken at discrete points along the length of the wing as illustrated in Figure 2. Each section is then treated as an independent aerofoil and modelled with two UVPMs (with and without gust). This is so that the kriging models are able to isolate the effect of the gust as the behaviour of the wing changes substantially under the influence thereof. By providing coefficient sets with and without the gust contribution, the kriging models have a means of determining the strength and location of the gust. While both UVPMs are constrained to move as the beam dictates, only one sees the gust being applied to the system.

In order to determine the displacement and velocity of the cross sections, cubic splines – much like those discussed in section IV subsection C – are fitted to $x$ and $\dot{x}$. Only translations and rotation in the plane of the section are considered as the UVPM approach is two-dimensional. Thus the cross sections are limited to three degrees of freedom: two translations and one rotation. Given the computed motion, each UVPM computes drag, lift, and moment coefficients. These coefficients inform the kriging models of both the magnitude and distribution of load along the wing.

The number of sections used is itself a parameter that requires optimisation. While more sections along the wing may give a better description of the load distribution, there is a point of diminishing returns (which is case specific). Increasing the number of sections increases simulation time and the number of variables (six per additional section) the kriging models must be trained for. This in turn requires larger training sets and more training time. In the interest of...
B. Kriging Models

As discussed previously, a set of kriging models is used to compute the force vector applied to the structure. The set consists of six models – three for each of the Cartesian forces \( (f_x, f_y, \text{and} f_z) \) and three for the corresponding moments \( (m_x, m_y, \text{and} m_z) \). All models take identical inputs and use a combination of anisotropic Matérn covariance \([25]\) and a linear regression function for predictions. In contrast, each model is trained independently as the magnitude and distribution of the forces and moments differ substantially.

The nodal displacement \( x_i \) and velocity vectors \( \dot{x}_i \) (containing six variables each) are taken as inputs in conjunction with \( l_i \), \( l_i \) (obtained from \( l \)) is the nodal position along the undeformed beam length and is necessary because it provides context for \( x_i \) and \( \dot{x}_i \). In general, large displacements near the root of the wing indicate far greater forces than similar displacements at the tip. Thus, including the coefficient vectors of the UVPM models, the kriging models assume the general form

\[
q = g(l_i, x_i, \dot{x}_i, \vec{C}_D, \vec{C}_L, \vec{C}_M, \vec{C}^{ng}_D, \vec{C}^{ng}_L, \vec{C}^{ng}_M)
\]

(14)

where \( q \) is an arbitrary force or moment. As is evident in Equation (14), each kriging model is essentially a function that predicts the force acting at some location along the wing subject to its displacement and distribution of load.

C. Training Approach

The decision to use kriging was informed by two properties of the method. First, data points in the training set are exactly recovered. Thus, by including steady state data points, the ROM can be trained to recognise and maintain the steady state condition. Second, the error distribution of the kriging surface is controlled by the choice of data points. In other words, unnecessarily high error in a particular region can be reduced by adding local data points during training. As a result of these properties, the accuracy of the ROM hinges on the selection of the training set.

The first step in training the ROM is to generate data points. For this purpose, two gust lengths are chosen as the bounds of interpolation and simulated using the coupled CFD model. The \( x, \dot{x}, \text{and} \gamma \) vectors are recorded at each time step of the simulations. The UVPM models are then run with the same gusts while being constrained to move as per \( x \) and \( \dot{x} \). This is so that the corresponding UVPM force coefficients \( \vec{C}_D, \vec{C}_L, \vec{C}_M, \vec{C}^{ng}_D, \vec{C}^{ng}_L, \text{and} \vec{C}^{ng}_M \) can be computed. The generated data for both gusts is then stored as a single pool of training points. This pool is used by an automated procedure to train each kriging model. Training refers to the optimisation of the anisotropic weights and covariance parameters that define the fit of the response surface.

To begin with, a small set of data points (evenly spaced in time) is taken from each gust. The number of points taken is the minimum required to fit the response surface and depends on the number of UVPM sections used. A restricted maximum likelihood method (REML) \([25]\) is used to compute the necessary weights and parameters with the initial training set. The result is used to calculate the error at every point in the training pool. Error is taken as absolute deviation from the CFD predicted value. The error is scaled relative to the peak absolute value of the force or moment under consideration.

From here an iterative procedure begins. The training set is augmented with the data point having the largest measured error. The REML method is then repeated and new errors are calculated. Points are added until the mean and peak error drop below set tolerances or a size limit is reached. Though each model is trained independently, all models are required to have the same training set size. This avoids a bias towards a particular force or moment.

D. Two-dimensional Configuration

The ROM is also capable of handling two-dimensional aerofoils. This is achieved by simplifying the configuration thereof. As the aerofoil is exactly one cross section, the ROM needs only one UVPM that sees the gust and one that does not. As a consequence, the coefficient vectors of the UVPMs become individual scalars and there is no longer a need for cubic splines. Similarly, the number of kriging models is reduced to three – two for the vertical lift and drag forces, and one for the pitching moment.

Alterations are made to the inputs of the ROM as well. Since the aerofoil is essentially a wing with no depth, there is no longer a need for \( l \). This change also means that vectors \( x \) and \( \dot{x} \) contain information for exactly one beam node. In particular, they contain at most three degrees of freedom – two translations and one rotation. As a result, \( x \) and \( \dot{x} \) are now fed directly into the kriging models and the general form, in this configuration, becomes
\[ q = g(\mathbf{x}, \mathbf{x}', \hat{C}_D, \hat{C}_L, \hat{C}_M, \bar{C}_D^{\text{ng}}, \bar{C}_L^{\text{ng}}, \bar{C}_M^{\text{ng}}) \]  

(15)

Here \( \hat{C}_D, \hat{C}_L, \) and \( \hat{C}_M \) are drag, lift, and moment coefficients respectively of the UVPM that sees the gust. \( \bar{C}_D^{\text{ng}}, \bar{C}_L^{\text{ng}}, \) and \( \bar{C}_M^{\text{ng}} \) are the corresponding coefficients for the other UVPM.

IV. Fluid Surface Interface

For the three-dimensional case, a fluid surface interface is needed to ensure dynamic continuity between the fluid and structure at every time step. The interface must perform two key functions. First, conservative decomposition of the aerodynamic surface forces onto the beam nodes which are thereafter displaced by the structural code. Second, reconstruction of the wing surface subject to the computed displacements. In order to facilitate these functions, a mapping between the surface and beam nodes must be established. This mapping is complicated by the large disparity in resolution of the wing surface and underlying beam mesh.

A. Wing Surface Partitioning

The process of establishing the required mapping is based on partitioning the wing surface into segments corresponding to the underlying beam elements in the undeformed configuration. Each segment is defined by two planes, placed at the start and end points of the beam element, whose normals are aligned with the axis thereof. A skin node will be mapped to a particular beam element if it lies within the region between the associated planes. This simple procedure, while intuitive, leaves three special cases which require careful treatment: the wing tip, regions of overlap, and regions of exclusion.

![Fig. 3 Illustration of the wing surface partitioning with regions of overlap and exclusion shown](image)

Figure 3 presents an example of a partitioned wing where blue lines indicate beam elements, black dashed lines represent the dividing planes, and regions of overlap and exclusion are highlighted in green and red respectively. It can be seen that the wing tip extends beyond the end of the last beam element. Any skin node on the tip would not be assigned by the standard partitioning procedure. To avoid this problem, any nodes beyond the last beam node are assigned to the last beam element. The force decomposition and surface reconstruction procedures are unchanged for these nodes with the latter representing an extrapolation.

Regions of overlap and exclusion arise because the beam elements, rather than all being collinear, fall into distinct collinear sections. Within a region of overlap, two beam elements compete for control of the skin nodes. In contrast, there is no obvious choice of beam element in a region of exclusion so the non-collinear pair creating the region must be referenced instead. Both cases, however, require that the surface forces be conservatively divided and the surface reconstruction be blended between the two referenced beam elements. In order to make decisions in this regard, reasonable metrics must be developed.

Consider the case illustrated in Figure 4 where red indicates a region of exclusion and green that of overlap. First – labelling the axes of the left-hand and right-hand side beam elements as \( \mathbf{n}_l \) and \( \mathbf{n}_r \) respectively – the common plane (that which is associated with both elements) is defined by the intercept of the two elements and the normal \( \mathbf{n}_l \times \mathbf{n}_r \). Second, consider two points, one in the exclusion region and the other in the overlap region, whose projections into the common plane (relative to the intercept) are denoted \( \mathbf{p}_e \) and \( \mathbf{p}_o \) respectively. Third, measure the angles of \( \mathbf{p}_e \) and \( \mathbf{p}_o \) to the end plane of the left element (\( \psi_l \) and \( \phi_l \) respectively) and the start plane of the right element (\( \psi_r \) and \( \phi_r \) respectively).

For the purpose of force decomposition, it is sufficient to bisect the regions of exclusion and overlap. Those points falling on the left-hand side of the bisection are assigned to the left-hand beam element and vice-versa. In the exclusion
region, \( p_e \) is on the left-hand side if \( \psi_l < \psi_r \). Conversely, in the overlap region, \( p_o \) is on the left-hand side if \( \varphi_r < \varphi_l \).

In the case of surface reconstruction, however, bisection is inadequate as it would cause shearing of the surface along the division line. Rather, to recover a smooth surface, the positions predicted by the two beam elements must be blended based on the relative position of the point. To this end, movement weights for the left-hand (\( \chi_l \)) and right-hand side (\( \chi_r \)) are defined as

\[
\chi_l = \begin{cases} 
\psi_r \\
\psi_l + \psi_r, \text{ region of exclusion} \\
\varphi_l \\
\varphi_l + \varphi_r, \text{ region of overlap} 
\end{cases}
\] (16)

\[
\chi_r = 1 - \chi_l. 
\] (17)

The reconstructed position of points \( p_e \) and \( p_o \) becomes the weighted sum of those calculated by each beam element. Because the movement weights always sum to one, the blending is smooth and conservative.

**B. Aerodynamic Force Decomposition**

For the purpose of describing the force decomposition process, consider a wing skin node with a force \( f_{aero} \) acting in an arbitrary direction. This skin node and the beam element to which it has been assigned are shown in Figure 5. The vector \( h \) is the perpendicular distance (or height) vector from the beam axis – which runs from node 0 to node 1 – to the position of the skin node. The scalars \( a \) and \( b \) are the respective distances from node 0 and node 1 to the projection of the skin node onto the beam axis. The length of the element is denoted \( L \). Force \( f_{aero} \) can be decomposed into components parallel (\( f_\parallel \)) and perpendicular (\( f_\perp \)) to the beam element. Likewise, the associate aerodynamic moment \( m_{aero} \) (defined as \( h \times f_{aero} \)) also consists of parallel (\( m_\parallel \)) and perpendicular (\( m_\perp \)) components. Given these moments, the forces can now be applied at the skin node projection.

Due to the discrete nature of the beam solver, the bending forces and moments are next decomposed onto the beam nodes according to the methodology developed by McGuire and Gallagher [26]. The approaches for splitting \( f_\perp \) and \( m_\perp \) are given in Figures 6 and 7 respectively. The parallel components \( f_\parallel \) and \( m_\parallel \) (or torsion) are decomposed onto the beam nodes according to the distance ratios \( \frac{a}{L} \) and \( \frac{b}{L} \). This is illustrated for the torque \( m_\parallel \) in Figure 8. The decomposed forces and moments are then passed to the structural module that will compute the deformation of the beam and subsequently the new beam coordinates.
C. Wing Surface Reconstruction

Owing to the low resolution of the beam mesh, reconstruction directly from the linear beam elements would result in surface wrinkles near the beam nodes. To this end, a cubic spline \( [27] \) (having \( C^2 \) continuity) is fitted to each of the originally collinear beam sections to ensure a smooth wing skin. The nodes of the section are taken as the knots of the spline – which is defined parametrically in terms of undeformed length – making each beam element a piecewise polynomial. Therefore, the deformed position of a point along an undeformed beam element can be computed with its distance from the start of said element.

Now the wing surface can be accurately reconstructed with a three step procedure that accounts for the translation, twisting, and bending of the beam. First, with reference to Figure 5 the projection of the skin node onto the deformed beam element is found with the above position splines using distance \( a \). Second, the twist at the projection is computed using a similarly defined spline and applied to \( h \). Here twist is taken as the component of the rotational displacement parallel to the undeformed beam element. Third, the bending of the beam is computed by finding the rotation which maps the undeformed axis to the deformed axis – which is simply the first derivative of the position spline. This rotation is applied to the \( h \) vector subsequent to the twist giving \( h' \). Adding \( h' \) to the projection computed in the first step yields the new position of the skin node.

V. Case Studies

Two test cases have been selected to evaluate the performance of the ROM in terms of accuracy and computational efficiency. Any ROM can only be considered successful if it performs well in both these areas. The first case is a two-dimensional study based around the FFAST Crank aerofoil \([1]\). It serves to test the core premise of the proposed ROM without requiring substantial simulation time. The second case study focusses on the NASA Common Research Model (CRM) \([28]\) and is intended as a general proof of concept for the ROM. In both cases, discrete 1-cosine waveform gusts are simulated. The vertical gust velocity \( U \) is given by
A. FFAST Crank Aerofoil

In this test case, the two-dimensional configuration of the ROM is applied to the FFAST Crank aerofoil. The flight conditions used for this study are those prescribed by the Aerogust project and are summarised in Table 1. Under these transonic conditions, a strong standing shock is present on both the upper and lower surfaces of the aerofoil.

Table 1 Flight conditions for the FFAST Crank Aerofoil Case

<table>
<thead>
<tr>
<th>Altitude ( z ) (m)</th>
<th>Density ( \rho_{\infty} ) (kg.m(^{-3}))</th>
<th>Pressure ( P_{\infty} ) (kPa)</th>
<th>Mach number ( M_{\infty} )</th>
<th>Flow velocity ( U_{\infty} ) (m.s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>10668</td>
<td>0.3806</td>
<td>23.92</td>
<td>0.86</td>
<td>255.1</td>
</tr>
</tbody>
</table>

1. CFD model

A Delaunay mesh consisting of triangles and containing 13,403 vertices was used for the purpose of running the CFD calculations. In this mesh, the far-field boundary was placed 15 chord lengths away from the aerofoil surface. The velocity, density, and pressure at the far-field boundary were specified – as per Table 1. A slip boundary condition was imposed on the aerofoil surface where
\[ \mathbf{u} \cdot \mathbf{n} = \mathbf{v} \cdot \mathbf{n}. \] (19)

Here \( \mathbf{n} \) is the surface normal. Each transient simulation was started with the gust at the far-field boundary and the aerofoil at its neutral position. The neutral position was found to be at \( h = -0.1003 \) m and \( \alpha = 0.1726^\circ \). Figure 9 shows the steady state solution of pressure around the FFAST crank aerofoil. The mesh is overlain on the solution field with black lines to indicate its fineness near the aerofoil surface.

![Steady-state pressure around the FFAST Crank Aerofoil with mesh included](image)

**Fig. 9** Steady-state pressure around the FFAST Crank Aerofoil with mesh included

2. **Structural model**

A pitch-plunge model is used to represent the aeroelastic response of the structure. In this model, the aerofoil is free to rotate (pitch) and translate in the vertical direction (plunge) but its horizontal motion is constrained. Taking clockwise rotation as positive pitch and negative vertical displacement as positive plunge, the motion of the aerofoil is governed by

\[
\begin{bmatrix}
   m & S_{\alpha} \\
   S_{\alpha} & I_{\alpha}
\end{bmatrix}
\begin{bmatrix}
   \ddot{h} \\
   \ddot{\alpha}
\end{bmatrix}
+ \begin{bmatrix}
   K_h & 0 \\
   0 & K_{\alpha}
\end{bmatrix}
\begin{bmatrix}
   \dot{h} \\
   \dot{\alpha}
\end{bmatrix}
= \begin{bmatrix}
   -L \\
   M_{\alpha}
\end{bmatrix}
\] (20)

where \( h \) is plunge of the elastic centre, \( \alpha \) is pitch about the elastic centre, \( L \) is lift, and \( M_{\alpha} \) is pitching moment. The structural properties \( m, S_{\alpha}, I_{\alpha}, K_h, \) and \( K_{\alpha} \) are mass, static imbalance, mass moment of inertia, plunge stiffness, and pitch stiffness respectively and the values thereof are summarised in Table 2. The elastic centre – through which all forces act – is located on the chord line, 31% of the chord away from the leading edge. The aerofoil has a chord length of 8 m.

<table>
<thead>
<tr>
<th>Mass ( m ) (kg)</th>
<th>Static imbalance ( S_{\alpha} ) (kg.m)</th>
<th>Mass moment of inertia ( I_{\alpha} ) (kg.m²)</th>
<th>Plunge stiffness ( K_h ) (N.m⁻¹)</th>
<th>Pitch stiffness ( K_{\alpha} ) (N.rad⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>824.8</td>
<td>-1944</td>
<td>10 602</td>
<td>136 000</td>
<td>1 800 000</td>
</tr>
</tbody>
</table>

3. **Gust selection and ROM training**

Four gust half lengths were chosen for this test case: 70 ft, 150 ft, 250 ft, and 350 ft. The corresponding reference velocities are listed in Table 3 and the values of \( A_k \) and \( F_g \) were taken as 0.74662 and 1.0 respectively. Training of
the ROM was effected using the CFD responses to 70 ft and 350 ft gusts. Referring to section [III] subsection [C], the mean and peak error tolerances were set at 0.1% and 0.5% respectively for the training of the kriging models. These tolerances were met for both kriging models (lift and pitching moment) once the training sets contained 200 data points.

Table 3 Gusts simulated for the FFAST Crank Aerofoil Case

<table>
<thead>
<tr>
<th>Gust half length $H$ (ft)</th>
<th>Reference gust velocity $U_g$ (m.s$^{-1}$)</th>
<th>EAS</th>
<th>TAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>13.05</td>
<td>23.42</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>14.82</td>
<td>26.59</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>16.14</td>
<td>28.95</td>
<td></td>
</tr>
<tr>
<td>350</td>
<td>17.07</td>
<td>30.62</td>
<td></td>
</tr>
</tbody>
</table>

Once trained, the aerodynamics ROM was coupled to the structure (in place of the CFD). The combination was used to predict the response of the aerofoil to the 150 ft and 250 ft gusts.

4. Aeroelastic Results

Comparative plots of the CFD and ROM predicted responses to the 150 ft and 250 ft gusts are presented in Figures [10] and [11] respectively. The plots show generally good agreement between the CFD and ROM predictions for both gusts. It is, however, apparent that the ROM follows the trends of the longer gust more closely. Similarly, the ROM predicts the plunge responses to both gusts with greater accuracy than the pitch responses. It is clear that agreement is worst during actuation of the gust.

Table 4 Error metrics of the displacement histories predicted by the aeroelastic ROM

| Gust half length $H$ (ft) | Absolute maximum $|\epsilon|$ (% abs. max.) | Mean | Maximum |
|--------------------------|---------------------------------------------|-----|---------|
| Plunge $h$               | 150                                         | 0.481 m | 0.901 | 3.89 |
|                          | 250                                         | 0.583 m | 0.597 | 1.71 |
| Pitch $\alpha$           | 150                                         | 2.99$^\circ$ | 1.85 | 8.05 |
|                          | 250                                         | 3.76$^\circ$ | 0.862 | 3.62 |

Table 4 contains the error metrics of each curve scaled relative to the corresponding absolute maximum displacements. In this regard, error is taken as the absolute deviation of the ROM prediction from the CFD values at a given time. The metrics in question are in agreement with the observations from Figures [10] and [11]. Examining the error more closely, the ratio of maximum error to mean error was found to be less than 5 in all cases. This is explicable as ratio of the maximum and mean error tolerances used during training was similarly 5. With the largest error of the ROM being below 10% – and substantially lower mean errors on each gust – the ROM is sufficiently accurate for engineering purposes.

5. Computational Cost

It remains to evaluate the computational efficiency of the ROM. For this purpose, the time taken to run a single gust case with the CFD model (summed over all parallel threads) is used as one unit of computational cost. The corresponding ROM run cost 0.026 units which amounts to a speed up factor of 39 per run. This factor cannot be viewed in isolation as the ROM requires two full CFD calculations to train. Thus the total cost required to run all four gust cases is a more industrially relevant comparison. Including the 0.003 units used to train the kriging models, the setup cost of the ROM amounts to 2.003 units. As the response to two of the gusts is computed during setup, the ROM need only be run on the remaining two cases. The addition of two ROM runs brings the total cost up to 2.055 units.
This represents a 49% reduction in computational cost over the use of traditional CFD. The cost reduction would further increase if more gusts within the range of the ROM need to be simulated.

Fig. 10 Displacement comparisons of CFD and ROM predictions for H = 150 ft gust.
Fig. 11 Displacement comparisons of CFD and ROM predictions for H = 250 ft gust.

B. NASA Common Research Model

The complete three-dimensional ROM is to be tested using the CRM. The wing/body/tail configuration of the CRM – which lacks the nacelles, pylons, and vertical tail of the complete geometry – is used for this purpose. This is deemed appropriate as the key area of interest is the wing. As in the previous test case, the Aerogust project advises a set of transonic flight conditions. These conditions are listed in Table 5.
Table 5  Flight conditions for the NASA CRM Case

<table>
<thead>
<tr>
<th>Altitude z (m)</th>
<th>Density $\rho_{\infty}$ (kg.m$^{-3}$)</th>
<th>Pressure $P_{\infty}$ (Pa)</th>
<th>Mach number $M_{\infty}$</th>
<th>Flow velocity $U_{\infty}$ (m.s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9142</td>
<td>0.4593</td>
<td>30.17</td>
<td>0.86</td>
<td>260.8</td>
</tr>
</tbody>
</table>

1. CFD model
Specific effort has been made to generate a mesh that accurately resolves the flow around the wing. First, a refinement block is included in front of the leading edge to assist in capturing the flow split. Second, a baffle is inserted around the wing tip so that flow roll-up is correctly identified. Third, surface elements are concentrated at the edges of the wing to ensure edge effects are sufficiently resolved. These three adaptations are highlighted in Figure 12. The resulting unstructured mesh contained 1.5 million vertices is used to simulate the CRM. Owing to the symmetric nature of the test case, the mesh has a symmetry plane to halve the size of the domain. The far-field boundary of the mesh is a hemisphere with a diameter of 1400 m (which equates to 200 reference chords or approximately 24 spans).

![Refinement block in front of the leading edge](image1)
![Refinement baffle around the wing tip](image2)
![Surface elements concentrated at the edges of the wing](image3)

Fig. 12  Mesh refinements focussed on capturing key flow phenomena around the wing

A slip boundary condition – as per Equation (19) – is applied on the aircraft surface. The flight conditions of Table 5 are imposed on the far-field boundary. All simulations are performed with the geometry at a 0° angle of attack and begin with the gust at the far-field boundary.

2. Structural model
For the purposes of this test case, only the motion of the wing is considered. The fuselage is thus rigidly fixed in all directions and rotations. The structural representation of the wing is adapted from a condensation of the Maximum Take Off Weight (MTOW) configuration FERMAT FEM model developed by Klimmek [30]. The condensation process, performed by Cook et al. [31], involved reducing the FEM structure to a beam model. This was done by finding an equivalent neutral axis and sectional properties that yield similar modal frequencies.

3. Gust selection and ROM training
Three gust half lengths have been selected for this test case: 30 ft, 70 ft, 150 ft. Values of 0.781364 and 0.7785 are used for $A_s$ and $F_g$ respectively. The references velocities for the three gust can be found in Table 6.

The 30 ft and
150 ft gust are intended as the training cases of the ROM while the 70 ft gust will be used to evaluate the performance thereof. Figure 13 shows the velocity magnitude distribution across the geometry in response to the 30 ft gust which is shown on the symmetry plane.

<table>
<thead>
<tr>
<th>Gust half length</th>
<th>Reference gust velocity $U_g$ (m.s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$ (ft)</td>
<td>EAS (m)</td>
</tr>
<tr>
<td>30</td>
<td>11.33</td>
</tr>
<tr>
<td>70</td>
<td>13.05</td>
</tr>
<tr>
<td>150</td>
<td>14.82</td>
</tr>
</tbody>
</table>

VI. Conclusion

In this work, a novel aerodynamics ROM approach has been proposed for use in aeroelastic gust calculations of two- and three-dimensional wing geometries. The ROM makes use of sectional potential flow solutions, solved via an unsteady vortex panel method, to estimate the aerodynamic loads at stations along the length of the aircraft wing. The load estimates are fed into kriging models which predict the aerodynamic forces to be applied to the structure. In conjunction with the ROM, a fluid surface interface has been developed for the coupled three-dimensional aeroelastic calculations needed to train and evaluate the ROM.

Two test cases were put forward as a means of evaluating the ROM. In the first case, a two-dimensional ROM was constructed to predict the aerodynamic loads of the FFAST Crank aerofoil under transonic flow conditions at Mach 0.86. To this end, gusts of half-length 70 ft and 350 ft were used as the training boundaries of the ROM. The aeroelastic response of the aerofoil was computed for gust half-lengths of 150 ft and 250 ft using the trained ROM for load estimates. The results showed strong agreement with the correspond CFD-based calculations having worst mean and maximum deviations of 1.85% and 8.05% respectively. In terms of computational efficiency, individual ROM runs were 39 times faster than the corresponding CFD calculations. Accounting for setup costs, the ROM reduced the computational cost of calculating the four gust responses by 49%.

The second test case studies the performance of proposed three-dimensional ROM with the NASA CRM travelling at Mach 0.86. The kriging models are trained with 30 ft and 150 ft half-length gusts and the ROM is evaluated using a gust of 70 ft half-length. At the time of writing, this test case is incomplete as the necessary CFD calculations are underway. Despite the need for three-dimensional validation, it can be concluded that the proposed approach is viable. The accuracy shown in the two-dimensional the test case is acceptable for engineering purposes – especially so in the context of the substantial speed-up the ROM achieved.
Acknowledgements

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References


