Nonlinear Unsteady Reduced Order Models based on Computational Fluid Dynamics for Gust Loads Predictions

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AIAA AVIATION, 25–29 June 2018

Hyatt Regency Atlanta, Atlanta, Georgia
Outline

▪ Motivation
▪ Theoretical Approach
▪ Results
  ▪ Supercritical Airfoil
  ▪ NASA CRM
▪ Conclusions and Outlook
Motivation

- From design to certification of an aircraft many aerodynamic data are needed – for the entire flight envelope –
- Aerodynamic data → pressure, shear stress distributions and global coefficients from steady and unsteady simulations

Focus on CFD loads:
- Unsteady, nonlinear ROM for gust loads
- Parametric with respect to gust parameters
- Based on URANS
- Transonic conditions with shocks and separation
Theoretical Approach | Overview

**Gust Amplitude $v_g$**

- **Training input**
- **OFFLINE**
  - $C_L(L_g, v_g)_1$
  - $C_L(L_g, v_g)_2$
  - $C_L(L_g, v_g)_3$

**Flow field snapshots**

**FOM simulations**

**Training output**

- $(L_g, v_g)_1$
- $(L_g, v_g)_2$
- $(L_g, v_g)_3$

**Flow field output time history**

**ROM predictions**

**ONLINE**

$CL(L_g, v_g)_X$

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Funded by the European Union
Theoretical Approach | nonlinear ROM Idea

**Physics-based** reduced order model which minimizes the unsteady nonlinear residual to find POD coefficients

1. **Input:**
   Flow field snapshots
   \( S = [\mathbf{w}(t_1), ..., \mathbf{w}(t_m)] \)

2. **Basis computation**
   Compute POD basis with \( m \) modes:
   \( \mathbf{w}(t) \approx \mathbf{U} \mathbf{a}(t) + \bar{\mathbf{w}} \)

3. **Order reduction**
   Truncate Basis to \( r \) modes using relative information content

4. **Hyperreduction (OPTIONAL)**
   Apply hyperreduction method

5. **ROM solving**
   For each time-step
   Find POD coefficients \( \mathbf{a} \) to minimize the unsteady nonlinear Residual
   \( \min_{\mathbf{a}} \| \mathbf{R}(\mathbf{Ua} + \bar{\mathbf{w}}) \|_{L^2}^2 \)

6. **Output**
   Gust response for untried gust parameters
Theoretical Approach | nonlinear ROM formulation

I. compute the approx. solution

\[ \mathbf{w}(t) \approx \mathbf{U} \mathbf{a}(t) + \overline{\mathbf{w}} \Rightarrow \mathcal{O}(N r) \]

II. evaluate the unsteady residual

\[ \hat{\mathbf{R}}(\mathbf{w}(t)) \overset{\text{def}}{=} \mathbf{R}(\mathbf{w}(t)) + \mathbf{\Omega} \frac{\partial \mathbf{w}(t)}{\partial t} \Rightarrow \mathcal{O}(N) \]

III. solve the LS problem

\[ (\mathbf{J}^T \mathbf{J} + \lambda \mathbf{I}) \Delta \mathbf{a} = -\mathbf{J}^T \hat{\mathbf{R}} \]

\[ \Rightarrow \mathcal{O}(N r) \]

The computational cost scales linearly with the dimension \( N \) of the full order model. No significant speedup can be expected when solving the minimum residual ROM and hyperreductions are necessary.
Theoretical Approach | nonlinear ROM formulation

Complexity reduction by sampling (or compute only a few entries of) the nonlinear unsteady residual vector $\mathbf{R}$

- omission of many components
- non intrusive
- approximation of the entire vector, by interpolation or by least-squares projection onto a subspace

Selecting the subset indices $\rightarrow$
- (Discrete) Empirical Interpolation Method
- Missing Point Estimation

Greedy: minimize

$$\left\| \mathbf{U}(\mathbf{U}^T \mathbf{P} \mathbf{P}^T \mathbf{U})^{-1} \mathbf{U}^T \mathbf{P} \mathbf{P}^T \right\| = \frac{1}{\sigma_{\min}(\mathbf{P}^T \mathbf{U})}$$
Results | Supercritical airfoil

- RANS equations with SA turbulence model
- $V_\infty = 223$ m/s, Mach = 0.754, $Re = 5.9 \times 10^6$
- Angle of attack iteratively adjusted for a target $C_L$ of 0.25
- 2 greedy missing point estimation levels (3k and 6k points)

**CFD settings (DLR – TAU Code)**
- Dual time stepping
- Min unsteady residual: $1e-4$
- Max inner iterations: 300
- Physical time steps: 500
- Linearly distributed time steps

**ROM settings (DLR SMARTy)**
- Truncation of the POD modes to 99.9999% of their energy content (~100 modes)
  (Additional variations in the paper)

Selected points around the airfoil
Results | Supercritical airfoil

- Training with two 1-cos gusts and evaluating influence of model reduction

Lift coefficient response for $L_g/c_{ref} = 2.26$

Lift coefficient response for $L_g/c_{ref} = 26.41$

Nondimensional Gust lengths: 2.26 & 26.41
Gust amplitudes: CS25

Shift is due to modal basis truncation (see paper for other truncation levels)

Following results will compare minimum and maximum values
Results | Supercritical airfoil

- Altering the gust amplitude for a fixed gust length and extracting the minimum and maximum lift coefficient

- Training signal based on the response seen one slide before

Linear increase due to model assumption

Nonlinear increase over gust amplitude

Proposed unsteady nonlinear reduced order model is capable of predicting linear and nonlinear dynamic responses
Results | Supercritical airfoil

- Altering the gust length for amplitudes based on CS25 and extracting the minimum and maximum lift coefficient

Training signal based on the responses seen two slides before

No dynamic flow separation possible with LFD method

Focusing on point furthest away from training signals

Proposed unsteady nonlinear reduced order model is capable of predicting variations in gust length
Results | Supercritical airfoil

- Time response behavior for gust with nondim. length of 11.31

Prediction

Analyze surface pressure distributions

Lift coefficient response for \( \frac{L}{c_{\text{ref}}} \)

Instantaneous surface pressure distributions

Nondim. Gust lengths: 11.31
Gust amplitude: CS25

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**Results** | *NASA CRM*

- Half-body configuration without VTP
- Mesh consists of roughly 7 million points
- RANS equations with SA turbulence model
- $V_\infty = 261$ m/s, $\text{Mach} = 0.86$, $H = 9142$ m
- One GMPE level with 5% of all points

**CFD settings (DLR – TAU Code)**
- Dual time stepping
- Min unsteady residual: $1e^{-6}$
- Max inner iterations: 500
- Physical time steps: 300
- Linearly distributed time steps

**ROM settings (DLR SMARTy)**
- Truncation of the POD modes to 99.9999% of their energy content (~100 modes)

Selected points around the aircraft
Results | NASA CRM

- Training with one 1-cos gust and evaluating influence of model reduction

Gust length: 213 m
Gust amplitude: CS25

Reconstruction

Lift coefficient response

Pitching moment coefficient response
Results | NASA CRM

- Predicting significantly shorter 1-cos gust

Gust length: 92 m
Gust amplitude: CS25

Analyze surface pressure distributions

Lift coefficient response

Pitching moment coefficient response
Results | NASA CRM

- Instantaneous surface pressure distributions at maximum lift coefficient

Gust length: 92 m
Gust amplitude: CS25

Proposed unsteady nonlinear ROM offers significantly better surface pressure distributions compared to LFD
Results | NASA CRM

- Comparing of section-wise integrated forces and moment for all methods

Gust length: 92 m
Gust amplitude: CS25

If only section-wise integrated forces and moments are used all methods offer very similar results.
Conclusions

- **Unsteady nonlinear ROM** method presented based on least-squares residual minimization
- **Parametric** model for gust responses
- Predictions presented for an airfoil and an aircraft at transonic flight conditions
- Results compared to the FOM with good agreement
- Superior accuracy compared to the LFD for distributed surface quantities

Outlook

- Extending the methodology towards coupled fluid-structure analysis
- Enhancing the model creation and evaluation process by including an analytically derived Jacobian matrix
- Including unsteady ROM based gust loads in an MDO process chain
The research leading to this work has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement number 636053.
Results | NASA CRM Computational Cost

- Preliminary assessment of computational cost for different approaches

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<th>Offline Cost</th>
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<td>ROM MPE 5%</td>
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<td>LFD</td>
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Some points to mention about this (more details in the paper):
- Residual currently fully evaluated in the DLR-TAU code but subset used for LS-problem (approx. 70% of the ROM cost)
- Some overhead due to data-transfer (approx. 25% of the ROM cost)
- Various other improvements possible (Cauchy convergence, adaptive time-stepping etc.)