Uncertainty Quantification of Aeroelastic Systems with Structural or Aerodynamic Nonlinearities

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In this work, various aeroelastic approaches are used in the uncertainty quantification of a generic UAV wing. The different methods that are employed investigate varying levels of model fidelity, representing methods that could be used for low-order, first-case studies, up to much higher fidelity methodologies. Results consider geometrically-exact structural and aerodynamic nonlinearities, and investigates the validity of using low-order simulations to predict deterministic and uncertainty bounds versus those of higher-order approaches. It is shown how correlated loads envelopes comparing strip theory aerodynamics show good agreement with higher order panel methods, even for very flexible wings, but it is also seen how differences in aerodynamic modelling (which would be equivalent in a structurally linear analysis) can effect the results, particularly torque. It is also shown how aerodynamic fidelity can potentially affect the uncertainty bounds of the computed aerodynamic loads, suggesting that low-order potential flow solvers significantly underestimate the uncertainty bounds compared to higher-order RANS approaches.

I. Introduction

From current trends in aerospace design, next-generation commercial aircraft and unmanned aerial vehicles (UAVs) are likely to exhibit considerable nonlinear behaviour, either due to structural or aerodynamic contributions. For example, aircraft featuring slender structures for increased aerodynamic performance may exhibit structural nonlinearities due to large deformations, and structure to flight-mechanics couplings. Similarly, due to large induced angles of attack on such slender structures, nonlinear aerodynamic phenomena due to flow separation may occur. Compressible effects and transonic shock are additional sources of nonlinearities which may arise if considering current and future high-speed aerospace applications. One important aspect of aircraft design is understanding how the structural loads are affected by atmospheric turbulence; as aerospace designs feature more nonlinear behaviour, the traditionally linear approaches to investigate gust disturbances on an aircraft must be extended into the nonlinear regime.

In aeroelastic modelling (and computational modelling in general), a high-fidelity model is almost always capable of predicting more accurate results than a lower-fidelity approach, at the cost of higher computational and user/engineer resources. On top of this, probabilistic analyses with uncertain system parameters can add an additional level of complexity to the analysis, and make higher-fidelity tools even more computationally expensive. Because of this, low-fidelity approaches are often still valuable tools, and can provide insight to the problem.

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In this work, the impact of both nonlinearities and modelling fidelity are considered on the aeroelastic response of an aircraft wing, with uncertain structural properties, subject to gust excitations. The following questions are posed:

1. How important are geometric nonlinearities on the predicted uncertainty bounds of a gust loads analysis?
2. How important is the aerodynamic modelling fidelity on the predicted uncertainty bounds of a gust loads analysis?

To answer these questions, various aeroelastic modelling approaches are used, with increasing degrees of complexity. To address the first question, nonlinear structural models coupled with either strip theory or panel methods are used to carry out uncertainty quantification on flexible variants of a wing structure in order to accentuate the nonlinear effects. Uncertain structural properties are introduced, and the probabilistic bounds compared for the two methods, including comparisons to linearised results. This study should determine whether linear approaches to an aeroelastic gust can predict the same trends seen in the full nonlinear simulations, or whether uncertainty quantification (UQ) of the nonlinear equations is necessary for accurate prediction.

To address the second question, linear structural models are coupled with aerodynamics models of increasing fidelity from unsteady strip theory, through to panel methods up to full turbulent Navier-Stokes solvers. Uncertainties in structural properties are introduced, and the bounds from probabilistic analyses are compared for the different fidelity approaches to ascertain whether results from the lower-fidelity approaches are useful, or whether they lack too much detail compared to the higher fidelity approaches to predict uncertainty propagation accurately.

In this paper, the theory behind the three aeroelastic methodologies is introduced in §II followed by the uncertainty quantification approaches. Then the generic UAV wing test case used in this work is described in §III along with definitions of the probabilistic density functions that are used. Finally, results are presented in §IV and conclusions drawn.

II. Methodology

In this section, the aeroelastic approaches used in this report are outlined, followed by the methods used to perform the uncertainty quantification.

A. Aeroelastic Models

Three different approaches to aeroelastic modelling are used here in order to understand the effects of modelling fidelity and nonlinearities on the uncertainty quantification process. These methods are summarised here, with discussion of the kinds of nonlinearities that may arise in certain aerospace structures.

In the classic case of aeroelastic trim analysis (in ZAERO for example) it is assumed that both the aerodynamics as well as the structure is linear. An equilibrium is sought for the forces (inertial and aerodynamic forces), and the deflections are computed as a result of these forces. Consider a schematic representation of a beam with aerodynamic panels (Figure 1a). The structure may be deformed considerably. However, the panels do not change orientation throughout this procedure, thus the effectiveness of the panels to provide lift remains unchanged. No iterations are required as the aerodynamic panel orientation does not change and hence the trim solution will remain the same. From the structural perspective the displacements due to the aerodynamic forces are computed in one step, assuming that the displacements remain small compared to the length of the beam. As these displacements remain small the assumption that the aerodynamic forces remain unchanged after the deformation is valid. However, when the displacement becomes larger due to, for example, more flexible structures, this assumption is no longer valid. Correct modelling of this effect is essential in aeroelastic analysis of flexible structures, and the following two methods are presented as approaches which are capable of capturing large aerodynamic panel rotations.

Further to the correct rotation of the panels due to large displacements, differences in the way that an angle of attack is included into the formulation can have a large effect on the results. Figure 2 illustrates two ways that an aerofoil could be given an effective angle of attack. Figure 2a shows the approach commonly used in linear tools such as ZAERO and Nastran where the aerodynamic panels are given a downwash term to change the angle of attack - essentially rotating the aerodynamic panel independently of the structure.
Conversely, Figure 2b shows the inflow vector itself being rotated. Because in potential flow theory the lift vector is always normal to the inflow (i.e. no drag is inherently modelled in the flow direction), the two approaches exert slightly different forces on the structure. Most notably, a small in-plane component of lift (due to leading-edge suction) arises if the inflow is rotated, which while small, particularly for small-angles, can become more influential in a nonlinear system. For example, consider if the aerofoil sections in Figure 2 were deflected vertically – a small in-plane component of lift would generate a torque at the root of the wing which would not appear if the aerodynamic panel had been rotated instead of the inflow vector. Ultimately, the correct method is debatable since neither approach predicts drag correctly, but the distinction is important as it hints at the importance of drag modelling on aeroelastic loads where it is traditionally ignored for simplicity and convenience.

A table of structural fidelities versus aerodynamic fidelities is summarised in Table 1, denoting a letter to particular combinations and indicating which of the three approaches considered in this work can model it. The methods are introduced in detail next. The two studies proposed in this work will investigate collections of these combinations in order to understand the effect of increasing fidelity on the analysis.

1. Nonlinear Beam Model with Unsteady Strip-theory Aerodynamics

In this approach, the structure is represented as a geometrically-exact nonlinear beam, using an intrinsic formulation based on the work of Hodges (representing method C in Table 1). This nonlinear structural model is coupled with an unsteady strip theory aerodynamics model, which uses a rational function approximation to Theodorsen’s function in an indicial response approach to determine the forces and moments on the
structure. The strips are able to move through arbitrarily large rotations, giving the correct orientation of the wing aerodynamic surfaces as illustrated in Figure 1b. Furthermore, angles of attack are included in the formulation by rotating the inflow vector, as shown in Figure 2b. A span-wise lift distribution calculated from a vortex-lattice solver on an undeformed geometry is used to correct the strip theory for 3D effects and improve the predictions. A more in-depth description of the approach outlined here can be found in Cook et al. This approach represents a very low-order, low-state approach to solving a nonlinear aeroelastic problem.

In addition to the full nonlinear system, the equations are linearised to understand at which point the nonlinearities begin to show significant changes in the results. The linear simulations can be carried out about an undeformed geometry, representative of a standard solution in Nastran (method A in Table 1), or about a nonlinear trim geometry (method B in Table 1) which is used to determine whether the differences that are seen are fundamentally due to nonlinearities or just a linear solution carried out about an incorrect state. It should be noted that methods B and C share the same nonlinear trim deflection at the very beginning of the simulation.

2. Linear Beam Model (about a nonlinear trim geometry) with Panel Method Aerodynamics

A methodology has been proposed in Aalbers et al. to increase the fidelity of a static aeroelastic analysis for flexible structures by taking into account the change in orientation of the aerodynamic panels as the structure deforms. This represents method D in Table 1. Nonlinear structural solvers gradually introduce the load onto the structure to update the direction of the forces such that its direction remains perpendicular to the structure (note that the aerodynamic panels generate forces normal to their surfaces, and exert these onto the structure). The newly found structural shape may be considerably deflected, and so the orientation of the aerodynamic panels must rotate accordingly (Figure 1b). The new orientation of the panels requires a new trim calculation as the panels have changed. This exchange of forces and deformations has to be iterated until a converged state has been reached.

The methodology is implemented by coupling Nastran, for the structural deformations, and ZAERO, for the aerodynamic forces. The aerodynamic forces from ZAERO (based on a doublet-lattice method (DLM) without corrections) are input for the nonlinear static analysis (SOL106) in Nastran. As such, the effective angle of attack is included into the formulation by applying downwash on the aerodynamics panels (as in Figure 2a) rather than by rotating the inflow vector as in methods A–C. Nonlinearities are taken into account such as follower forces using the LGDISP,1 parameter. In the nonlinear analysis, gravity forces are taken into account as well. Once a solution has been computed, the deformed state becomes the input for the flow solver by updating the aerodynamic model. The aerodynamic panels are redefined such that the (rigid) aerodynamic model corresponds with the structural deflection computed in the nonlinear analysis. Unless stated otherwise, this threshold is set to be 1 millimetre in the vertical (z) direction.

Once the trim state has been obtained, a pre-stressed modal analysis is performed. These mode shapes are used to compute the gust response. So the trim state is computed in a nonlinear way, and the gust response in a linear way, but based on the pre-stressed trimmed geometry. The assumption that a linear gust analysis is sufficient is motivated by the research of Cook et al. This approach represents a very low-order, low-state approach to solving a nonlinear aeroelastic problem.

3. Linear Beam Model with RANS-based Aerodynamics

In order to represent higher-order aerodynamics, the flow solver that has been used for this method is the unstructured FINE™/Open solver from Numeca. This represents method E in Table 1. It is a multi-block

<table>
<thead>
<tr>
<th>Structural Models</th>
<th>Linear (about undeformed)</th>
<th>Linear (about nonlinear trim geometry)</th>
<th>Nonlinear</th>
</tr>
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<tr>
<td>Aerodynamic Models</td>
<td>Strip Theory</td>
<td>Panel Method</td>
<td>RANS-based</td>
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<tr>
<td>A</td>
<td></td>
<td></td>
<td>E</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>D</td>
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<tr>
<td>C</td>
<td>-</td>
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</tr>
</tbody>
</table>

Table 1: Table of possible fidelity of simulation that can be run in this work.
finite volume density based solver that can deal with structured and unstructured hexahedral grids with hanging nodes. A complete description of the solver can be found in Patel.\textsuperscript{9}

Numeca has developed a Fourier decomposition ROM for gust in its unstructured chain FINE\textsuperscript{TM}/Open. This ROM is based on the Nonlinear Harmonic method already existing for turbomachinery applications.\textsuperscript{10} Knowing the importance of fluid/structure interactions to ensure structural integrity in the aerospace industry, this method has been extended and successfully applied to external aerodynamics by Debrabandere.\textsuperscript{11} It has been further improved to include unsteady fluid loads in the structural deformation.\textsuperscript{12,13}

The structural ROM that has been retained for the present study is the Oofleie\textsuperscript{®} Computational Structure Mechanics solver. It is a general purpose finite element software for structural and heat transfer analysis. It computes a modal representation of the structure that is used with the aerodynamics ROM to produce an aeroelastic ROM.

B. Studies

Two studies will be conducted in this work to independently assess the effects of aerodynamic and structural fidelity on the predicted uncertainty predictions. These are described next.

1. Geometric Nonlinearity Study (Study 1)

Analysis methods A, B and C will be run and compared to understand how structural nonlinearities affect the uncertainties. These results will be also compared to method D in order to introduce the idea of higher-fidelity 3D aerodynamics via the use of panel methods (DLM) into the geometric nonlinear study. Simulations will be carried out on a linear system, linearised about the undeformed and trim geometries, as well as the full nonlinear simulations about the trim geometry.

2. Aerodynamic Fidelity Study (Study 2)

Methods A, D and E in Table 1 will be run and compared in order to focus on the effect of changing aerodynamic fidelity on the uncertainties. Simulations will be carried out about the undeformed geometry in this case (apart from method D which is about a trim geometry).

C. Uncertainty Quantification

Two methods are used for the uncertainty quantification, depending on the aeroelastic approach. Non-intrusive methods based on polynomial chaos expansions (PCE) are used in methods A, B, C and D, and a probabilistic collocation method is used for method E. for completeness, these methods are outlined here.

1. Probabilistic Collocation Method

The uncertainty propagation method used for method E is the non-intrusive probabilistic collocation method.\textsuperscript{14} It is based on the expansion of the solution into Lagrange interpolating polynomials. The basis points of the polynomial expansion (the collocation points) are chosen as the Gauss quadrature points by means of the Golub-Welsh algorithm for general Probability Density Function shapes.\textsuperscript{15} A system of uncoupled simulations can be derived, which has the advantage that this UQ propagation technique can be wrapped around the flow solver in a non-intrusive way.

Based on the output of the performed simulations, statistical moments of any output quantity are automatically calculated, by taking the weights from the Gauss quadrature. The mean, variance, skewness and kurtosis are calculated following Eq. 1 and Eq. 2. This information is calculated for a selected number of scalar output quantities as follows:

\begin{equation}
\mu_1 = \sum_{k=1}^{N_u} w_k \phi_k(\vec{x}, t) \tag{1}
\end{equation}
and the second (variance, \( n = 2 \)), third (skewness, \( n = 3 \)) and fourth moment (kurtosis, \( n = 4 \)) are calculated as follows:

\[
\mu_n = \sum_{k=1}^{N_p} w_k (\varphi_k(\vec{x}, t) - \mu_1)^n
\]  

(2)

Industrial design challenges are usually characterized by a multitude of simultaneous uncertainties, that cannot be considered one-by-one, whether they are correlated or not. Given that quadrature is used to determine the collocation points, the problem of multiple simultaneous uncertainties boils down to a multi-dimensional quadrature problem. The standard approach is a tensor product, which, however, suffers from the “curse of dimensionality”. This means the computational cost increases exponentially with the number of uncertainties considered. In the current work a sparse grid quadrature, based on Smolyak’s quadrature method is used. Table 2 summarizes the number of quadrature points contained in linear growth sparse grid for levels of 1 and 2 in comparison with a full-tensor grid with the same number of 1D quadrature points.

<table>
<thead>
<tr>
<th>Level</th>
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</tr>
<tr>
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<tr>
<td>5</td>
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<td>11</td>
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<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>10</td>
<td>59049</td>
<td>21</td>
</tr>
</tbody>
</table>

Table 2: Number of quadrature points \( N_p \) contained in the full tensor grid and the sparse grid.

The significant reduction in number of runs makes the simultaneous treatment of many uncertainties in complex 3D CFD problems accessible.

2. Polynomial Chaos

The uncertainty propagation methodology used in methods A-D is the polynomial chaos expansion (PCE) as implemented with an in-house code for methods A-C, and in the Dakota software package for method D. For the normal distributions used as input distributions in the current report, the methodology basically fits the output distribution with Hermite polynomials. To accurately estimate the mean and variance of a distribution third order Hermite polynomials are sufficient, which means that three analysis evaluations are required. As the model which propagates the uncertainties is nonlinear, the output probability distribution need not be symmetric. To accurately capture higher order moments of the probability distribution higher degree polynomials are required. As a straightforward tensor quadrature would result in an excessive number of evaluations, the sparse grid methodology of Dakota is used for method D, which implements the sparse tensor product quadrature (see Table 2). Alternatively, a normally distributed Latin-hypercube sampling approach is used with methods A-C.

3. Uncertainty Quantification of 2-parameter correlated loads

Two-parameter correlated loads are of interest during aircraft design to identify critical loading events. The 2D correlated loads are plotted in so-called potato plots where the time histories of two interesting quantities (IQ), such as wing bending or torsion, are plotted against each other for a series of load cases. In the current report, the load cases will be a series of “one minus cosine” (1MC) gusts of different lengths (defined later in IIIB). The potato plot is the convex hull of the time traces. Hence the defining points of the potato are 2D loads at specific time instants. When the structural properties of the wing change, the defining points may originate from other time instants, and even the number of points may change. In order to have a rigorous
definition of the uncertainty quantification of a potato plot, it is proposed to parameterise the shape of the potato and perform the UQ analysis on the parameters describing it. Consequently, the results from the UQ analysis can be graphically shown by a potato representing the mean loads or 99-percentile values of the loads. The latter potato will show the maximum extent of the 2D correlated loads satisfied by 99% of the cases. In this work, two different approaches to the UQ of correlated loads plots are carried out.

In the first approach (used by methods A-D) the basic idea of the parameterisation is to create a ‘spider’ plot from the center of the potato, rays emanating from the center. The distance from the center to the intersection of the ray with the potato plot is the parameterisation of the potato plot in the ray’s direction. With a sufficient number of rays, the potato plot is well represented. In practice, the potato can be highly skewed, so the parameterisation is not performed on the potato directly, but on a scaled and rotated potato which is more akin to a circle. The scaling and rotation parameters are obtained from a principal component analysis of the convex hull data. The steps of the process are shown in Figure 3.

The parameterisation must be well defined over a series of potato plots (which will arise from the UQ analysis). Therefore the center and the scaling and rotation parameters must be fixed. These parameters are obtained from a reference potato plot, typically the potato plot computed from the nominal values of the uncertainty parameters. In a UQ analysis it may happen that the different potato plots for different parameter settings are so different from the reference potato that the center of the reference potato is not contained inside the current potato plot. In such a case the scaling of the current potato will result in a curve which does not contain the origin, and the parameterisation will break down. There is no obvious solution, since in theory the different potato plots may be completely disjoint from each other. To circumvent this
problem, the potato plots will be based on incremental loads (with respect to the trim state), rather than
the absolute loads. In terms of the evaluation of critical loads this is an unsatisfying solution, since the
trim loads are just as important as the dynamic loads. Therefore, in the presentation of the results, also
the uncertainty in the trim loads will be shown. Clearly, the uncertainties in the trim loads and the gust
loads cannot be summed, but given the fact that the parameterisation will break down, this is the next best
solution.

In the second approach (used in method E) the error bounds are known for all loads at a given time
step. Therefore, when the correlated loads are scattered on a graph, the mean values are used for the convex
hull computation to identify the mean correlated load plot. In addition to the mean values, the uncertainty
bounds based on the standard deviation of the loads at any given time give an additional set of points around
which another convex hull can be formed. This second envelope is the 99% bound of the loads.

III. Test Case Definition

In this section, the test case that will be studied is outlined. First, the aircraft model that is analysed
will be introduced, followed by definitions of which variables are to be considered uncertain, and with what
PDFs.

Figure 4: A rendering of the UAV wing used in this paper.

A. Aircraft Model

The specific test case that is being studied in this work is a representative high aspect ratio (HAR) UAV
wing, designed for the AEROGUST project. The wing is based roughly on the size and modal response
of a Global Hawk, has an aspect-ratio of 25 and a uniform chord of 2m. It is unswept, untapered, with
no dihedral or twist. The aerelastic model is fixed at the root with a cantilever boundary condition. An
illustration of this wing is shown in Fig. 4. The individual wing mass is 425kg and the full aircraft mass is
7,000kg. The structure of the wing is defined as a simple rectangular section with constant outer dimensions
(a width of 1m and height of 0.2m) and a linearly varying thickness (from 6mm at the root to 1.5mm at
the tip). The centre of this rectangular section lies 40% chord behind the leading edge. The wing section
is made of aluminium with Young’s modulus of 69GPa, and a shear modulus of 27GPa. The first, second
and third bending modes occur at 1.79Hz, 9.84Hz, and 26.35Hz, respectively. The first torsion mode occurs
at 15.26Hz, and the first fore-aft/in-plane bending mode occurs at 25.23Hz. The flight case for this aircraft
is an altitude of 55,000ft at Mach 0.55, although the compressibility effects are ignored in the aerodynamics
models used for methods A-C.

Despite the baseline wing having quite a high aspect ratio, it is still fairly stiff, and therefore under normal
flight conditions does not exhibit large deformations. To encourage larger deformations and emphasise the
effects of geometric nonlinearity on the behaviour of the wing, flexible variants of the wing are created by
multiplying the stiffness properties of the structure by some factor. Figure 5 shows the trim geometry of the
wing as seen from the front, comparing the intrinsic beam approach of methods A-C with both vortex-lattice

method (VLM) and modified strip theory aerodynamics, to a linear approach using Nastran (linear structure and doublet-lattice aerodynamics without compressibility effects). It can be seen how the baseline and 50% stiffness wing trim shape agrees very well with the results of Nastran, but at 25% of the baseline stiffness and below, tip shortening effects can be seen in the nonlinear code that are not present in the linear analysis. Additionally, it can be seen that there is little difference, even at high deformations, between modified strip theory and VLM, even though the strip theory is given a lift distribution based on the undeformed geometry. Below 15% of the baseline stiffness, the wing is unable to generate sufficient lift to be able to be trimmed. The structural properties and air density defined here form the mean values used in the UQ analysis, with the standard deviations defined as in [III].

B. Gust Excitation

In this work, three 1MC gusts are considered, with lengths taken from the FAA/EASA regulation description.[21][22] Gust gradients of 9.144m and 106.68m are chosen to represent the shortest and longest advised gust lengths in the regulations, with a 45.72m gust additionally defined as the midpoint of these extremes. The gust intensities used are also taken from the regulations for the flight case altitude of the test case defined earlier.

C. Uncertain Variables

In order to carry out an uncertainty quantification, appropriate parameters must be nominated as having uncertain properties, and furthermore the probability distribution of these parameters must be defined. In this work, Young’s modulus, \( E \), and shear modulus, \( G \), are considered to be independent uncertain variables. The uncertainty distributions will be treated differently for the two studies in this work.
1. Study 1 Uncertainties

For the geometric nonlinearity study, the baseline, 50%, 25% and 15% cases are considered, with various uncertainty configurations. The global uncertainty in Young’s modulus and shear modulus are considered as one case (two independent uncertainties). In addition, two more cases are introduced where the wing is split into zones, and the Young’s modulus and shear modulus can change separately in these regions. In the first of these two extra cases there are two zones; the first zone is from the root to 60% span and the second is from 60% span to the tip (four independent uncertainties). In the second case, there are three zones; root to 40% span; 40% span to 80% span; and 80% span to tip (six independent uncertainties). In all three cases, both Young’s modulus and shear modulus uncertainty distributions are assumed to be normal distributions, with the mean value as defined in §IIIA. The standard deviations are chosen such that the $3\sigma$ bounds are plus or minus 30% of the mean value for $E$ and $G$.

2. Study 2 Uncertainties

In the aerodynamic fidelity study, Young’s modulus and shear modulus are varied globally along the whole wing for the baseline case only (two independent uncertainties). The uncertain variables have normal distributions, with the same mean and standard deviations as those defined in study 1. In addition to the structural stiffness uncertainties, method E includes an uncertainty in structural damping, defined as a beta distribution (where $\alpha = 1$, $\beta = 4$, and the most likely value is 0). However, it can be seen that the effect of structural damping on the peak loads is small, and hence not included in the analysis of the other methods A and D. Only the 106.68m gust gradient is used for this study.

IV. Results

Here, results of the two studies outlined in the previous sections are presented. First, the results considering geometric nonlinearities are presented, followed by the results of the aerodynamic nonlinearities.

A. Study 1 - Aeroelasticity with Nonlinear Structures

The output PDF $1g$ trim loads of interest (bending moment and torque in this case) for the baseline wing for methods B/C and D (B and C have the same trim condition) are plotted in Fig. 6 in order to illustrate differences in the two methods. For both static and dynamic results, methods B/C uses 25 sample points from a LHS, using first order PCE terms. For method D, results are obtained using a sparse, level 2 quadrature.
The PDFs for root bending moment in Fig. 6b show a similar skewed distribution, but with slightly different mean and standard deviations; method B shows a larger uncertainty in the loads. However, the differences between the means of these these distributions shows a difference of only around 1%, which is a very good agreement. The standard deviations of these loads is also very small, showing that even though the stiffness properties are changing by a large amount, the output loads are largely invariant to this. This can be explained by the fact that, while the stiffness of the wing is changing, and in turn the displacements, the displacement to structural load relationship is still largely linear for this case. Hence, when calculating the internal loads, the two factors almost cancel out (and would perfectly cancel out in a truly linear analysis).

A similar result was shown in previous work on a HARW operating in the linear regime.

In contrast the root torque, plotted in Fig. 6a, shows an much bigger difference in the mean and standard deviation of the output PDFs. The reason for these differences is rooted in differences in the way the lift vector is rotated between the two methods (see Figure 2), which in turn affects the in-plane components of lift and goes on to affect the torque loads as the wing deforms (the vertical moment arm increases).

![Figure 7: 2D Correlated Loads Distributions with 99% Uncertainty Bounds for the Baseline Wing](image)

The 2D correlated loads are then plotted in Fig. 7 for incremental loads, plotting the mean values from the UQ analysis, surrounded by the 99% uncertainty bound. It can be seen in this plot how the linear equations with strip theory, (linearised about the undeformed geometry - method A) agrees qualitatively in shape with the linear equations with panel methods (linearised about the trim geometry - method D) in terms of the mean values and the 99% error bounds also. Noticeably, the loads from the panel method approach are higher. However, once the trim geometry is included in the strip-theory based analysis (method B), the correlated load envelopes shift as the torque loads decrease significantly, with some reduction in bending moment. Furthermore the uncertainty bounds increase, particularly around the top of the plot where a ‘double-peak’ uncertainty appears. The cause of this is once again based in the differences in the rotation of the aerodynamics lift vector, hence why when linearised about the undeformed geometry, the match with the panel methods is better (the vertical moment arm is zero so there is no in-plane contribution to the torque). The full nonlinear strip-theory method results are also included in Fig. 7 it can be seen how they agree in shape reasonably closely to the linear results about trim condition, indicating that providing the linearisation
is performed about the appropriate condition, the prediction of the uncertainties can be estimated well, and significantly faster, using linear methods.

![Figure 8: Trim Root Load Uncertainty PDFs for the 15% baseline stiffness case](image)

The different shapes of the 2D correlated plots complicate the comparison between the different methods. A simple metric to compare the methods is the increase in area of the mean correlated loads envelope to the 99% bound. Table 3 summarises the areas of the mean correlated loads envelopes, with the percentage increase introduced by the 99% bound. The increase in area of the mean between methods A-C for the baseline wing case is quite comparable, despite the significant increase in 99% bound area when going from method A to methods B and C. The mean area for method D is almost 50% larger than method A, say, but the 99% bound represents a smaller percentage increase compared to method A. Clearly, the inclusion of in-plane contributions to the aerodynamics in methods B and C increase the uncertainty by coupling torsion and bending effects, compounding the uncertainty and increasing the bounds.

<table>
<thead>
<tr>
<th>Method</th>
<th>Baseline Wing Mean Area ($N^2m^2$)</th>
<th>99% Bound (%)</th>
<th>15% Baseline Wing Mean Area ($N^2m^2$)</th>
<th>99% Bound (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$1.034 \times 10^8$</td>
<td>25.61</td>
<td>$1.031 \times 10^8$</td>
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<tr>
<td>B</td>
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<td>62.47</td>
<td>$1.468 \times 10^8$</td>
<td>71.29</td>
</tr>
<tr>
<td>C</td>
<td>$1.111 \times 10^8$</td>
<td>68.37</td>
<td>$1.525 \times 10^8$</td>
<td>82.06</td>
</tr>
<tr>
<td>D</td>
<td>$1.491 \times 10^8$</td>
<td>16.66</td>
<td>$0.919 \times 10^8$</td>
<td>55.17</td>
</tr>
</tbody>
</table>

Table 3: Table of possible fidelity of simulation that can be run in this work

The same process is repeated for more flexible variants of the aircraft wing. In Fig. 8, the PDFs of the 1g trim loads are compared for the 15% baseline wing in a similar way to the baseline wing earlier. Here, the PDFs for root bending moment show a close agreement, similar to the baseline case. Here, the difference between the mean values is still small (around 2%), but the standard deviations of the loads are now much larger than for the baseline case. The torque loads also exhibit a much more significant difference in terms of mean and standard deviation. Because this flexible wing is bending much more than the baseline case was, the vertical moment arm is much longer, meaning the in-plane components of lift have even more influence on the torque.

The 2D correlated loads for the different methods on the 15% baseline wing are potted in Fig. 9. Here, the linear results about an undeformed condition with strip theory predicts a larger envelope than the linear about a trim condition with panel methods. This is because the nonlinear trim geometry rotates the aerodynamic
panels correctly, reducing the influence of the vertical gust on the structure, whereas the aerodynamic panels for the linearised about undeformed geometry case remain flat. Furthermore, the uncertainty bound is higher once linearised about the trim geometry as compared to the linear about undeformed. However, in the strip theory approach, once linearised about the trim condition, the in-plane component of lift greatly affects the torque, as seen by the drastic change in shape of the 2D correlated load. The wing at 15% stiffness deforms so much more than the baseline case, which exacerbates this effect for the more flexible wing when compared to the baseline wing. The uncertainty bounds are also larger. Comparisons of the full nonlinear results with results from the linear simulations (linearised about trim geometry) with strip theory shows that the mean and uncertainty bounds are still comparable, even for this highly flexible case. This implies that qualitative UQ analysis can be carried out efficiently on an aircraft wing exhibiting large deformations by using a linear approximation (provided it is linearised about the appropriate condition), and these results are still representative of the nonlinear results.

The areas of the correlated loads envelopes in Figure 9 are also summarised in Table 3 alongside the results from the baseline wing. Here is can be seen that the mean area and 99% bounds are quite consistent for method A when comparing the baseline wing to the 15% stiffness case. However, for all other cases, the uncertainty bounds are much bigger when looking at the more flexible wing.

Correlated loads envelopes are included in the Appendix at the end of this paper for intermediate flexibilities (50% and 25% [Figs. 13a and 13b respectively]) to illustrate how the increased flexibility of the aircraft begins to affect the root wing loads, particularly the torque.

Additional plots are included to illustrate the differences in uncertainty bound when including more uncertain variables in the analysis. Figure 10 shows the mean and 99% uncertainty bounds for the correlated envelopes of the baseline wing. All four methods are shown separately, with three uncertainty bounds associated to the three uncertainty configurations modelled. It can be seen how the uncertainty bounds vary very little between the different studies, but the case with uniform $E$ and $G$ uncertainty (i.e., one wing zone) seem to give the most conservative bounds of all the studies, with some small exceptions. This gives
Figure 10: Comparisons of the different UQ Studies on the Correlated Loads Distributions with 99% Uncertainty Bounds for the Baseline Wing

an indication that a global variable changes may give a good first approximation to uncertainty of a wing without requiring the added complexity of including numerous zones of independent uncertainty.

A similar plot is shown in Figure 11 for the 15% baseline stiffness case. Similarly to the case for the baseline wing, the three uncertainty bounds show a close degree of similarity, for methods A and D, where the uniform uncertainty case forms a conservative envelope around all the results. However, for methods B and C, it can be seen that the cases where two and three zones on the wing were associated with uncertain stiffness properties exhibit larger envelopes that the uniform uncertainty case. It is not clear whether this difference is due somehow to the in-plane component of lift combined with large deformations, or perhaps due to the differences in sampling methods used in the UQ analysis. Further work will be required to understand the true source of this difference.

B. Study 2 - Aeroelasticity with Nonlinear Aerodynamics

Finally, results with various levels of fidelity of aerodynamics are presented. The sample number and order of PCE for methods A-D are the same as in the previous study. For method E, 8 samples are used to perform the UQ analyses in this section, using a sparse level 1 analysis. This number of samples are used (as opposed to the 7 outlined in Table 2) to account for the beta distribution applied to the damping.

Figure 12 shows correlated loads envelopes of incremental bending (rolling) moment against incremental torque (pitching moment), comparing the lower-order, linear, potential flow aerodynamics from methods A and D to the full RANS based solution in method E. In this plot there is a reasonably close agreement
between the two potential flow-based solvers A and D, where the mean correlated envelopes are close in shape, though the panel method of method D shows a larger peak bending moment and torque. In both methods, the uncertainty in aerodynamic loads due to changes in stiffness properties is very small. Clearly for the potential flow methods, the aerodynamics response is greatly dominated by the gust velocity itself, and the effect of the structure on the aerodynamics is small. Conversely, the uncertainty bounds on the RANS-based aerodynamics approach of method E exhibits a much larger uncertainty bound. It is not clear whether this is due to differences in parameterisation of the envelope, differences in the uncertainty quantification methods, or something fundamentally different in the RANS solver which increases the sensitivity of the aerodynamic forces to structural changes. Further work here is required to understand why the uncertainty bounds are much larger for method E, but it could point to a need for more sophisticated aerodynamics approaches earlier in the design procedure if considering uncertainty quantification.

V. Conclusions

Various studies have been carried out on a slender, generic UAV wing using different aerodynamic and structural models of different fidelities in order to understand what impact these different methods have on an uncertainty analysis.

First, analyses comparing linear and nonlinear method with strip theory (method A-C) or panel methods (method D) were performed, and the results compared. The uncertain distributions of the 1g loads showed
considerable differences in torque between the two methods considered both in terms of mean values, and standard deviations. Bending moments, however, agreed more closely. It was seen how the 2D correlated envelopes agreed very well qualitatively between the methods providing they were linearised about the undeformed condition. Once the trim geometry was included in some approaches (methods B or C), the torque loads changed - an effect which was exacerbated as the wing became more flexible. This differences seen in static and dynamic torque loads could be explained by differences in aerodynamic modelling approach. All of those methods were then compared to the fully nonlinear approach (method C), where the results showed a very close agreement with the linearised results about the trim condition, even for an exceptionally flexible case. This implies that, providing the linearisation is carried out about the appropriate state, a linear model can predict reliable loads and uncertainty bounds at a fraction of the cost of a nonlinear simulation. Various numbers of uncertain variables were also considered, treating the wing as having one, two or three zones, each with independent uncertainties. It was seen how defining the stiffness properties as global uncertainties on the wing consistently gave the more conservative uncertainty bound for the baseline wing. The same was partially true for the 15% stiffness wing, though for the fully nonlinear simulations the cases where there was a larger number of independent variables resulted in a marginally larger uncertainty bound envelope. This indicates that a first case assumption of global uncertain variables is a fast and efficient method to obtain a first guess at the uncertain bounds for a wing. To answer question 1 posed in the introduction: Geometric nonlinearities have a large impact on both deterministic and probabilistic simulations, but it was also seen how linear solutions gave a reasonably close approximation at a fraction of the computational cost (provided the simulation was linearised about an appropriate initial condition, i.e., the trim geometry rather than an undeformed one).

Analyses were then carried out to compare linear structural models with varying fidelity of aerodynamic solver. Potential flow based solvers (methods A and D) were compared to a full RANS solution (method E) to determine the effects on the aerodynamic loads. It was seen how the two potential flow solutions produced similar mean envelopes, and predicted small 99% uncertainty bounds. In contrast, the uncertainty
bounds from the RANS solver were significantly larger. Although the source of this uncertainty could be due to differences in parameterisation of the envelopes or uncertainty quantification approaches, it is possible that these differences are due to the fidelity of the aerodynamics solver. To answer question 2 posed in the introduction: The aerodynamic fidelity could well be significant on the uncertainty bounds of an aircraft. This work suggests that potential flow solvers could be much less conservative than higher fidelity methods, but further work would be required to say more definitively that this is the case.

VI. Appendix

Additional correlated loads envelopes for intermediate stiffnesses of 50% and 25% baseline stiffness are included in this Appendix section. Figures 13a and 13b show the 50% and 25% baseline stiffness cases, respectively, illustrating how the effect of in-plane aerodynamics begins to affect the torque loads on nonlinear cases.

![Figure 13: 2D Correlated Loads Distributions with 99% Uncertainty Bounds](image)

(a) 50% Baseline Wing
(b) 25% Baseline Wing

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