

EASY PROBABILITY

When I toss a coin in the air, what are the chances of it coming down heads? You will probably answer one of the following:

$$\frac{1}{2}; \quad 50/50; \quad 1 \text{ in } 2; \quad \text{evens}; \quad 0.5$$

All of these answers are correct - they are different ways of saying the same thing.

In mathematics, probability is expressed as either a vulgar fraction or as a decimal fraction, in this case either as

$$\frac{1}{2} \text{ or } 0.5$$

We know that a fair coin is equally as likely to come down heads as it is tails. There are two possible **outcomes** (heads or tails) to this **trial** (tossing a coin). The **event** we are interested in is "landing heads".

Here is a definition of probability:

$$\text{PROBABILITY} = \frac{\text{NUMBER OF WAY THE EVENTS CAN HAPPEN}}{\text{TOTAL NUMBER OF POSSIBLE OUTCOMES}}$$

So, when a coin is tossed:

$$P(\text{ head}) = \frac{\text{NUMBER OF WAYS OF GETTING A HEAD}}{\text{TOTAL NUMBER OF POSSIBLE OUTCOMES}} = \frac{1}{2}$$

CARDS AND DICE

A pack of cards is made up of 52 cards, divided into four suits - Diamonds, Hearts, Clubs and Spades. Diamonds and Hearts are red cards. Clubs and Spades are black cards. Each suit has 13 cards. These carry the numbers from 2 - 10 PLUS Jack, Queen, Kings and Ace (called the 'picture' cards). So, each suit has the following cards:-
Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King.

A **die** (or **dice** if there are more than one die) is a six-faced cube, with numbers from 1 to 6 on each face.

Look at these examples which will include dice and cards:

Example

a) A die is rolled. What is the probability of getting a four?

$$P(4) = \frac{\text{NUMBER OF WAYS OF GETTING A HEAD}}{\text{TOTAL NUMBER OF POSSIBLE OUTCOMES}} = \frac{1}{6}$$

- b) A card is drawn from a pack. What is the probability that it is a king?

$$P(\text{King}) = \frac{\text{NUMBER OF WAYS "King" CAN HAPPEN}}{\text{TOTAL NUMBER OF POSSIBLE OUTCOMES}} = \frac{4}{52} = \frac{1}{13}$$

- c) What is the probability that the card drawn is a heart?

$$P(\text{Heart}) = \frac{\text{NUMBER OF WAYS GETTING "HEART"}}{\text{TOTAL NUMBER OF POSSIBLE OUTCOMES}} = \frac{13}{52} = \frac{1}{4}$$

- d) What is the probability that the card drawn is the Ace of Spades?

$$P(\text{Ace of Spades}) = \frac{1}{52}$$

Exercise 1

1. There are twenty buttons in a box. Five are red, three are blue and twelve are green. One is drawn at random. What is the probability that the button
 - a) is blue?
 - b) is green?
 - c) is red or blue?
2. A die is rolled. Calculate the probability that it will give
 - a) a 1
 - b) an odd number
 - c) a number greater than four.
3. A letter is chosen from the word **stranger**. What is the probability that it is:-
 - a) a vowel (a e i o u are the vowels)
 - b) an r.

SAMPLE SPACE

When the situation is complicated in any way e.g. we have 2 coins or 2 dice it is sometimes helpful to make a list of possible outcomes. Such a list can be called a 'sample space'.

Example 1

If I toss two coins list the possible outcomes.

Find the probability of getting i) 2 heads and ii) one head and one tail in any order. Possible results are HH, HT, TH, TT.

Notice that HT and TH are **both** included first coin head and second tail and the other way round.

There are 4 possible results.

$$p(2 \text{ heads}) = \frac{1}{4}$$

$$p(\text{head \& tail}) = \frac{2}{4} = \frac{1}{2}$$

Example 2

If I roll 2 dice list the set of possible outcomes.

Find the probability of getting :

- i) double six ii) same number on both dice iii) total score of ten.

It is easiest here to show the results in a table, showing 1st score, 2nd score. (can you fill in the missing spaces?)

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2				
3,1					
4,1					
5,1					
6,1	6,2	6,3			

Total number of possible outcomes = 36

$$\text{Prob}(\text{double six}) = \frac{1}{36}$$

Prob (same number on both dice) = P (double six or double five or)

$$= \frac{6}{36} = \frac{1}{6}$$

Prob (Total score of 10) = P (double 5 or 6 and 4 or 4 and 6)

$$= \frac{3}{36} = \frac{1}{12}$$

THE PROBABILITY SCALE

If an event is certain to happen the probability that it will happen is 1 (one).

P (a tossed coin will come down again) = 1

P (one day 1 shall die) = 1

If an event can never happen the probability that it will happen is **0 (zero)**.

P (I will live for ever) = 0

P (I can fly without mechanized help) = 0

All probabilities must lie between 0 and 1. Probabilities can be expressed as decimal fractions or fractions.

Both of the following are acceptable as answers:

$$P(\text{a die will show '2'}) = \frac{1}{6} \text{ or } 0.16667 \text{ (correct to 5 dec. places)}$$

TOTAL PROBABILITY

If the event we are interested in happens, this is often thought of as a "**success**". If the event does not happen, this is called a "**failure**".

Total probability covering all possible outcomes of an event must be 1. Therefore,

$$P(\text{success}) + P(\text{failure}) = 1$$

What is the probability of obtaining a 5 when throwing a die? What is the probability of not getting a 5?

$$P(5) = \frac{1}{6}$$

$$P(\text{not getting a 5}) = 1 - \frac{1}{6} = \frac{5}{6}$$

This shows us a very useful method for many probability examples.

Example

What is the probability that, when a card is drawn from a pack it is **NOT** an ace?

Solution

$$P(\text{Ace}) = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{not Ace}) = 1 - \frac{1}{13} = \frac{12}{13}$$

Exercise 2

1. A box contains 100 coloured balls. 25 balls are red; 30 are blue. The rest are yellow or green. One ball is drawn at random.

What is the probability that:-

- a) it is red?
- b) it is red or blue?
- c) it is not red?
- d) it is black?

2. Five red balls are now removed from the box, and then one ball is drawn at random. What is the probability that:-
- a) it is red?
 - b) it is blue'?

HOW TO UNDERSTAND PROBABILITY

When a coin is tossed, the probability of getting a head is 0.5. This does not mean that if we tossed a coin ten times we would get five heads and five tails.

$P = \frac{1}{2}$ would tell us **what to expect** in the long run.

If we tossed a coin 100 times, it would be very unlikely that we would get exactly 50 heads, but if we repeated this experiment a large number of times, we would **expect** that the average results would be 50 heads. If we tossed a coin 100 times and got 95 heads, we would suspect that someone had given us an unfair coin (in this case, weighted to favour heads).

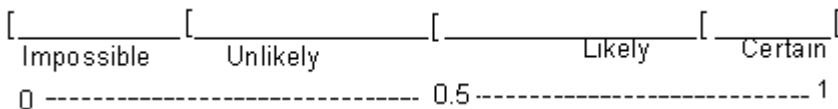
Remember that probability can never tell us what **will** happen, but it can tell us what is **likely** to happen.

Probability = $\frac{99}{100}$ means **very likely to happen**

Probability = $\frac{1}{100}$ means **very unlikely to happen**

and so on.

PROBABILITY SCALE



EMPIRICAL (EXPERIMENTAL) PROBABILITY

In the examples we have done so far, we have been able to work out the probabilities from our knowledge of the situation. We know that all coins have two faces; that packs of cards are made up from 52 cards and four of them are kings.

In many situations, this kind of information is not available, and in practice, the probabilities are worked out from information gathered from samples that have been taken or experiments that have been done.

If we are told that the probability of rain on a June day is $\frac{13}{30}$ this will have been arrived at by studying the rainfall records for June, over, let us say, the past 20 years. Obviously, information about every June day that ever was or ever shall be is not available, so the probability has been estimated from a sample.

If a manufacturer tells us that he is 99% certain ($P = 0.99$) that an electric light bulb will last for 200 hours, this figure will have been arrived at from experiments which have tested samples of light bulbs.

Think about - how you would give an estimate of the probability of a 25-year old passing the driving test at his first attempt?

INDEPENDENT EVENTS

I toss a fair coin. Four times in a row, it comes down heads. What would you call for the next toss? It might be tempting to think that if we have had four heads in a row, the next toss is more likely to be a tail! But, no, if the coin is a fair coin, the probability of getting a head on the next go, is still 0.5, just as it was on all the other goes.

This is an example of independent events, because what has happened on one trial has no effect on what happens on any other trial.

Example

I draw a card from a pack. It is the king of hearts. I put the card back and draw again. What is the probability of drawing a heart this time?

The answer is - $\frac{1}{4}$

The two events are independent. The outcome of the first trial cannot affect the outcome of the second trial. Now suppose after I drew the first card, I did not put it back into the pack. What is the probability of drawing a heart this time?

The answer is not $\frac{1}{4}$ because the events are not independent.

The answer depends on what happened on the first draw. There are two possibilities -

- a) If the first card drawn was a heart, then on the **second** draw, I have a pack of 51 cards, containing 12 hearts.

$$\text{Therefore, } P(\text{heart}) = \frac{12}{51}$$

- b) If the first card drawn was not a heart, then on the second draw, I have a pack of 51 cards containing 13 hearts.

$$\text{Therefore, } P(\text{heart}) = \frac{13}{51}$$

Exercise 3

1. A card is drawn from a pack.
 - a) What is the probability that it is an ace?
 - b) The card is replaced and another one drawn. What is the probability that this is a diamond?

2. A card is drawn from a pack. It is not replaced. A second card is drawn. What is the probability that this card is an ace if:
 - a) the first card drawn was an ace?
 - b) the first card drawn was not an ace'?
3. On throwing a die, what is the probability of turning up a
 - a) 1
 - b) a number greater than 3 c a number more than 6 d an even number.
4. In a pack of playing cards, what is the chance of drawing a
 - a) Jack?
 - b) a spade?
 - c) the Ace of Diamonds?
 - d) an 8, 9 or 10?
5. In a box containing 100 oranges. 5 are bad. What is the probability of picking out a bad one.
6. Two dice are thrown. What are the probabilities that the **total score** is:
 - a) 4
 - b) 1
 - c) 11
 - d) 12
 - e) 5
 - f) less than 5
 - g) more than 5.
7. A box contains 40 coloured pencils, 15 red, 10 blue and the rest yellow. What is the probability of choosing:
 - a) a red pencil?
 - b) a yellow pencil?
 - c) a red or a yellow pencil?
 - d) a white pencil?

ANSWERS

Exercise 1

1. 20 buttons 5 red, 3 blue, 12 green.

$$\text{a) } P(\text{blue}) = \frac{3}{20}$$

$$\text{b) } P(\text{green}) = \frac{12}{20} = \frac{3}{5}$$

$$\text{c) } P(\text{red or blue}) = \frac{5+3}{20} = \frac{8}{20} = \frac{2}{5}$$

2. A die 1 2 3 4 5 6

$$\text{a) } P(1) = \frac{1}{6}$$

$$\text{b) } P(\text{odd numbers}) = \frac{3}{6} = \frac{1}{2}$$

$$\text{c) } P(\text{numbers greater than 4}) = \frac{2}{6} = \frac{1}{3}$$

3. S T R A N G E R

$$\text{a) } P(\text{vowel}) = \frac{2}{8} = \frac{1}{4} \quad (\text{vowels are A and E})$$

$$\text{b) } P(R) = \frac{2}{8} = \frac{1}{4}$$

Exercise 2

1. 100 balls 25 red. 30 blue 100 - 55 = 45 yellow or green.

$$\text{a) } P(\text{red}) = \frac{25}{100} = \frac{1}{4}$$

$$\text{b) } P(\text{red or blue}) = \frac{55}{100} = \frac{11}{20}$$

$$\text{c) } P(\text{not red}) = \frac{(100-25)}{100} = \frac{75}{100} = \frac{75}{100} = \frac{3}{4}$$

$$\text{d) } P(\text{black}) = \frac{0}{100} = 0 = \frac{0}{8} = \frac{0}{4}$$

2. 95 balls - 20 red, 30 blue $95 - 50 = 45$ yellow or green.

a) $P(\text{red}) = \frac{20}{95} = \frac{4}{19}$

b) $P(\text{blue}) = \frac{30}{95} = \frac{6}{19}$

Exercise 3

1. a) $P(\text{ace}) = \frac{4}{52} = \frac{1}{13}$

b) $P(\text{diamond}) = \frac{13}{52} = \frac{1}{4}$

2. a) 51 cards 3 aces left

$$P(\text{ace}) = \frac{3}{51} = \frac{1}{17}$$

b) 51 cards 4 aces left

$$P(\text{ace}) = \frac{4}{51}$$

3. Die 1 2 3 4 5 6

a) $P(1) = \frac{1}{6}$

b) $P(\text{number greater than 3}) = P(4,5 \text{ or } 6) = \frac{3}{6} = \frac{1}{2}$

c) $P(\text{number more than 6}) = 0$

d) $P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$

4. a) $P(\text{Jack}) = \frac{4}{52} = \frac{1}{13}$

b) $P(\text{spade}) = \frac{13}{52} = \frac{1}{4}$

c) $P(\text{ace of diamonds}) = \frac{1}{52}$

d) $P(8, 9 \text{ or } 10) = \frac{12}{52} = \frac{3}{13}$

5. $P(\text{bad}) = \frac{5}{100} = \frac{1}{20}$

6.

	1 st die					
	1	2	3	4	5	6
2 nd die	1	2	3	4	5	6
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11
	n = 36					

a) $p(4) = \frac{3}{36} = \frac{1}{12}$

b) $P(1) = 0$

c) $P(11) = 0 = \frac{2}{36} = \frac{1}{18}$

d) $P(12) = \frac{1}{36}$

e) $P(5) = \frac{4}{36} = \frac{1}{9}$

f) $P(\text{less than 5}) = \frac{6}{36} = \frac{1}{6}$

g) $P(\text{more than 5}) = \frac{26}{36} = \frac{13}{18}$

Note: $P(\text{more than 5}) = 1 - P(\text{not more than 5})$

$$= 1 - \left(\frac{4}{36} + \frac{6}{36} \right) = 1 - \frac{10}{36} = \frac{26}{36} = \frac{13}{18}$$

7. 40 pencils 15 red, 10 blue, yellow pencils = $40 - (15 + 10) = 15$

a) $P(\text{red}) = \frac{15}{40} = \frac{3}{8}$

b) $P(\text{yellow}) = \frac{15}{40} = \frac{3}{8}$

c) $P(\text{red or yellow}) = \frac{30}{40} = \frac{3}{4}$

d) $P(\text{white}) = 0$