Learning Development

University of Northampton

HARDER PROBABILITY

MUTUALLY EXCLUSIVE EVENTS AND THE ADDITION LAW OF PROBABILITY

Two events are said to be mutually exclusive if the occurrence of one excludes the occurrence of the other.

Example 1

Throwing a 3 with a single die excludes the possibility of throwing a 4. Therefore, throwing a 3 and a 4 are mutually exclusive.

Example 2

It is impossible to cut both a King and a Jack with a single cut of a pack of cards. Therefore, the one excludes the other.

Example 3

Choosing an odd number excludes the choice of an even one.

Example 4

A coin cannot show both heads and tails, simultaneously, etc..

Example 5

Find the probability of throwing an even number with a single fair die. This event consists of three separate mutually exclusive events:

throwing a 6

throwing a 4

throwing a 2.

The probability of throwing an even score P is

the probability of throwing a 2 P^1
plus the probability of throwing a 4 P^2
plus the probability of throwing a 6 P^3

$P = P^1 + P^2 + P^3$

$$P = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

If P1 P2 P3... are the separate probabilities of a set of mutually exclusive events, then the

probability of one of the events happening is $P = P^1 + P^2 + P^3 + \ldots$.

Therefore, P (A or B) = P(A) + P(B)

The addition law is sometimes called the **or law**, because we require the probability that one event OR another event will happen.

Example 6

What is the probability of throwing a 5 or a 6 with a single die?

$$P(5) = \frac{1}{6} \quad P(6) = \frac{1}{6}$$

Mutually exclusive events, therefore **adding** gives

$$P(5 \text{ or } 6) = \frac{1}{6} + \frac{1}{6} + \frac{3}{6} = \frac{1}{3}$$

Example 7

A pack of cards is cut once. Find the probability that the card will be the 7 of Hearts, the King of Diamonds, or the Ace of Spades.

$$P(\text{7 Hearts}) = \frac{1}{52} \quad P(\text{King Diamonds}) = \frac{1}{52} \qquad (\text{Ace Spades}) = \frac{1}{52}$$

Mutually exclusive events, therefore **add**, to give, $\frac{3}{52}$

Example 8

In a partly used box of tissues, there are 12 purple, 15 green, 17 white, 10 yellow, and 6 blue tissues.

Assuming the box is filled with colours at random what is the probability that the last one is:

a) either blue or yellow
b) not white?

Answer

a) Total number of tissues = 60

$$P(\text{last one blue}) = \frac{6}{60} \quad P(\text{last one yellow}) = \frac{10}{60}$$

Therefore, $P(\text{last one blue or yellow}) = \frac{6}{60} + \frac{10}{60} = \frac{16}{60} = \frac{4}{15}$

b) $P(\text{last one white}) = \frac{17}{60}$

P(last one **NOT** white) $= 1 - \frac{17}{60} = \frac{43}{60}$

INDEPENDENT EVENTS: THE MULTIPLICATION LAW OF PROBABILITY

An independent event is one which has no effect on subsequent events e.g.

1. Drawing a card from a pack and tossing a coin.
2. Drawing from a pack, replacing it, and then drawing another card.
3. When a die is thrown twice, what happens in the first throw does not affect what happens on the second. The two throws are therefore independent events.
4. The probability of developing lung cancer is **not** independent of the probability of smoking.
5. The probability of having naturally brown eyes and naturally dark hair are **not** independent. Brown eyes occur more often with dark hair than fair hair.

If P1, P2, P3... are the separate probabilities of a set of independent events, then the probability that ALL the events will occur is

$P = P^1 X P^2 X P^3 \ldots$

P(A and B) = P(A) x P(B)

The multiplication law is sometimes called the **and law**, because we require the probability that one event **and** another will happen.

Example 1

What is the probability of throwing a 4 and then a 5, with 2 throws of a die?

$$P(4) = \frac{1}{6} \qquad P(5) = \frac{1}{6}$$

Independent events, therefore **multiply** to give

$$P(4 \text{ then } 5) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Example 2

What is the Probability of tossing a coin and obtaining a head and throwing a die and obtaining a 6?

$$P(h) = \frac{1}{2} \qquad P(6) = \frac{1}{6}$$

Independent events, therefore **multiply**, giving

$$P(H \text{ and } 6) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

Example 3

The probability that Billy Banana will get a "hole in one" is $\frac{1}{8}$

The probability that his partner, Ollie Orange will get a "hole in one" is $\frac{1}{12}$

What is the probability that:

a) either will get a hole in one
b) both will get one
c) neither will get one?

a) P(Billy OR Ollie) = P(Billy) + P(Ollie)

$$=\frac{1}{8}+\frac{1}{12}$$

$$=\frac{3+2}{24}=\frac{5}{24}$$

b) P(Billy **AND** Ollie) = P(Billy) x P(Ollie)

$$=\frac{1}{8}\times\frac{1}{12}=\frac{1}{96}$$

c) P(Neither) = P(Billy will not get one) x P(Ollie will not get one)

$$=\frac{7}{8}\times\frac{11}{12}=\frac{77}{96}$$

Example 4

It is Granddad's Birthday. The probability that Granddad will be given a bottle of whisky is $\frac{1}{12}$. The probability that Granddad will be given a tie is $\frac{1}{4}$. The probability that Granddad will be given a pair of socks is $\frac{2}{3}$.

What is the probability that Grandad will receive:

a) a tie or socks

b) a bottle of whisky and a tie

c) a bottle of whisky, or a tie or socks

d) a bottle of whisky, and a tie and socks?

Answer

a) P(tie **OR** socks) = P(tie) + P(socks)

$$=\frac{1}{4}+\frac{2}{3}=\frac{3+8}{12}=\frac{11}{12}$$

b) P(tie **AND** whisky) = P(tie) x P(whisky)

$$=\frac{1}{12}x\frac{1}{4}=\frac{1}{48}$$

c) P(whisky **OR** tie **OR** socks) = P(whisky) + P(tie) + P(socks)

$$=\frac{1}{12}+\frac{1}{4}+\frac{2}{3}=1$$

d) P(whisky **AND** socks **AND** tie) = P(whisky) x P(socks) x P(tie)

$$=\frac{1}{12}x\frac{2}{3}x\frac{1}{4}=\frac{1}{72}$$

TREE DIAGRAMS

Note

The multiplication law can be illustrated by a "probability tree". This method helps you see what is taking place. Study the examples below, because they show you both methods in operation for the same problem.

All possible outcomes, together with their probabilities, can be shown on a tree diagram.

Example 1

A coin is tossed 3 times. Find the probabilities of

a) 3 heads b) only 1 head.

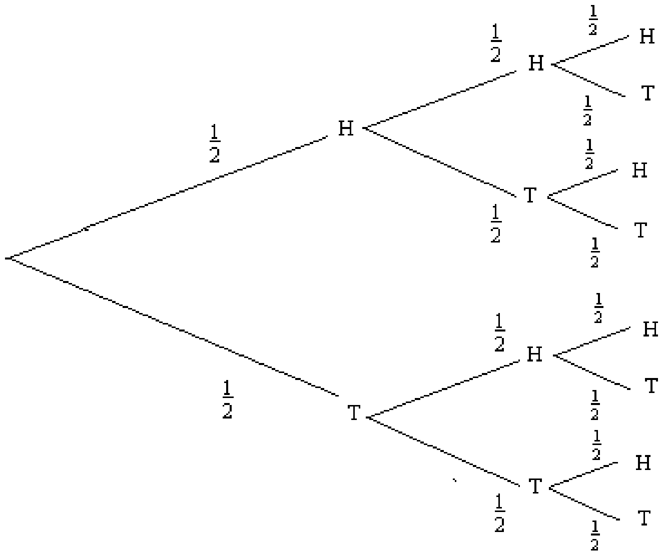
Answer

a) P(3H) $=\frac{1}{2}X\frac{1}{2}X\frac{1}{2}=\frac{1}{8}$

b) P(1H) there are three ways in which one head can be obtained

i) HTT ii) THT iii) TTH

Probability of any of these is $=\frac{1}{2}X\frac{1}{2}X\frac{1}{2}=\frac{1}{8}$



Example 2

A bag containing 10 marbles, has 7 red and 3 black marbles. A marble is drawn at random and then replaced. What is the probability that both marbles are red?

$$\text{P(first marble is red)} = \frac{7}{10}$$

The marble is returned to the bag making 10 marbles.

$$\text{P(second marble is red)} = \frac{7}{10}$$

$$\text{So P(both marbles are red)} = \frac{7}{10} \times \frac{7}{10} = \frac{49}{100}$$

$$\text{Similarly, P(both marbles black)} = \frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$$

$$\text{P (one red marble then one black marble)} = \frac{7}{10} \times \frac{3}{10} = \frac{21}{100}$$

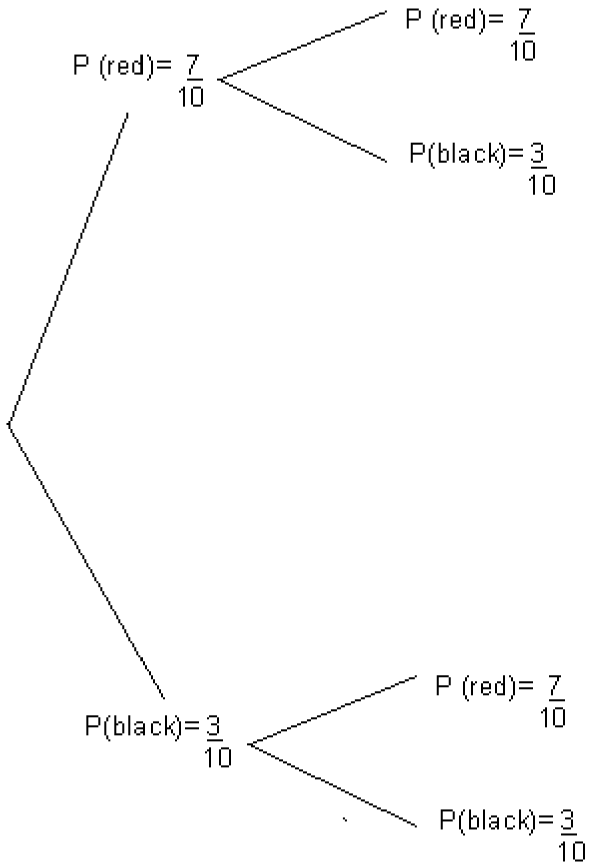
But if one black then one red is equally acceptable, that is the black appears first instead of the red then

$$\text{P (one black marble then one red)} = \frac{3}{10} \times \frac{7}{10} = \frac{21}{100}$$

So, provided the order of drawing the marbles is not important,

$$\text{P (one red, one black, any order)} = \frac{21}{100} + \frac{21}{100} = \frac{42}{100}$$

This can easily illustrated by a tree diagram.



P (red, red) $\frac{7}{10} \times \frac{7}{10} = \frac{49}{100}$

P (red, black) $\frac{7}{10} \times \frac{3}{10} = \frac{21}{100}$

P (black, red) $\frac{3}{10} \times \frac{7}{10} = \frac{21}{100}$

P (black, black) $\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$

An alternative method

Since the sum of all the probabilities is 1

P(one red one black) = 1- P (all red) – P (all black)

$$= 1 - \frac{49}{100} - \frac{9}{100}$$

$$= 1 - \frac{58}{100}$$

P (one red, one black) $= \frac{42}{100}$

Exercise

1. P, Q and R are three mutually exclusive events, with

$P(P) = \frac{1}{6}$ $P(Q) = \frac{1}{3}$ $P(R) = \frac{1}{2}$

Calculate:
a) the probability of either P or R occurring
b) the probability of either P or Q occurring.

2. What is the probability of throwing 6 heads in 6 tosses of a coin?

3. A bag contains ten marbles, six are blue and four are yellow. One marble is drawn at random. Its colour is noted and then it is replaced. This is repeated once more. Find the probability that the following will occur:

a) two blue marbles are drawn one after the other
b) their colours are different.

4. In a fishing competition, the probability of catching a trout is $= \frac{1}{3}$

the probability of catching a barbel is $= \frac{1}{5}$

the probability of catching a chub is $= \frac{1}{4}$

What is the probability of catching on one day:

a) a trout and a chub?
b) a chub or a trout?
c) a trout, a barbel and a chub?

5. A bag contains twenty sweets, eight toffees and twelve chocolates. A sweet is drawn at random and replaced. A second and third sweet are also chosen, and replaced. What is the probability that the following sweets are chosen?

a) three toffees
b) one toffee, following by two chocolates.

6. The probability that Anne and Jack will pass their driving tests are

$\mathbf{\frac{2}{3}}$ **and** $\mathbf{\frac{1}{4}}$ **respectively.**

Find the probability that:

a) Anne and Jack pass
b) Anne or Jack pass
c) they both fail
d) only Anne passes.

7. Find the probability of drawing 3 Kings in succession from a pack of cards, if the cards are **not** replaced after each draw.

8. Find the probability of drawing either a Diamond or a Heart from a pack of cards at the first draw.

9. Find the probability of drawing a King or a Queen or a Jack from a pack of cards at the first attempt.

10. It is only possible for three beauty queens to win a major prize. The probabilities are as follows:

Miss A $\frac{1}{3}$, Miss B $\frac{3}{8}$ and Miss C X

Which girl is most likely to win? What is the probability that it is Miss A or Miss B that wins the prize?

ANSWERS

1. a) P(P or R) = P(P) + P(R)

$$=\frac{1}{6}+\frac{1}{2}=\frac{1}{6}+\frac{3}{6}=\frac{4}{6}=\frac{2}{}$$

b) P(P or Q) = P(P) + P(Q)

$$=\frac{1}{6}+\frac{1}{3}=\frac{1}{6}+\frac{2}{6}=\frac{3}{6}=\frac{1}{2}$$

2. P(6 heads) = P(H) x P(H) x P(H) x P(H) x P(H) x P(H)

$$=\frac{1}{2}X\frac{1}{2}X\frac{1}{2}X\frac{1}{2}X\frac{1}{2}X\frac{1}{2}$$

3. a) P(blue followed by blue) = P (blue) x P (blue)

$$\frac{6}{10}\times\frac{6}{10}\text{ Cancel!! }\frac{9}{25}$$

b) P(blue followed by yellow) = P(blue) x P(yellow)

$$\frac{6}{10}\times\frac{4}{10}=\frac{6}{25}$$

P(yellow followed by blue) = P(yellow) x P(blue)

$$\frac{4}{10}\times\frac{6}{10}=\frac{6}{25}$$

P(blue followed by yellow or yellow followed by blue)=

P(blue) P(yellow) + P(yellow) P(blue)

$$\frac{6}{25}+\frac{6}{25}=\frac{12}{25}$$

4. a) P(trout and chub)= P(trout x P(chub)

$$\frac{1}{2}\times\frac{1}{4}=\frac{1}{12}$$

b) P(chub or trout) = P(chub) + P(trout)

$$=\frac{1}{4}+\frac{1}{3}=\frac{3+4}{12}=\frac{7}{12}$$

c) P(trout, barbel and chub) = P(trout x P(barbel) x P(chub) .

$$=\frac{1}{3}X\frac{1}{5}X\frac{1}{4}=\frac{1}{60}$$

5. a) P(three toffees) = P(toffee) x P(toffee) x P(toffee)

$$=\frac{8}{20}X\frac{8}{20}X\frac{8}{20}=\frac{8}{125}$$

b) P(toffee followed by 2 chocolates) = P(toffee) x P(chocolate) x P(chocolate)

$$=\frac{8}{20}X\frac{12}{20}X\frac{12}{20}=\frac{18}{125}$$

6. a) P(Anne and Jack) = P(Anne) x P(Jack)

$$=\frac{2}{3}\times\frac{1}{4}=\frac{1}{6}$$

b) P(Anne or Jack) = P(Anne) + P(Jack)

$$\frac{2}{3}+\frac{1}{3}=\frac{8+3}{12}=\frac{11}{12}$$

c) P(Anne fail + Jack fail) = P(Anne fail) x P(Jack fail)

$$=\frac{1}{3}\times\frac{3}{4}=\frac{1}{4}$$

7. P(3 Kings when cards not replaced) = P(King) x P(King) x P(King)

$$=\frac{4}{52}\times\frac{3}{52}\times\frac{2}{50}=\frac{1}{5525}$$

8. P(Heart or Diamond) = P(Heart) + P(Diamond)

$$=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$

9. P(King or Queen or Jack) = P(King)+P(Queen) + P(Jack)

$$=\frac{4}{52}+\frac{4}{52}+\frac{4}{52}=\frac{3}{13}$$

10. P(A) $=\frac{3}{8}$ P(B) $=\frac{1}{3}$ P(C)=x

P(A) + P(B) + P(C)=1

$=\frac{3}{8}+\frac{1}{3}+\frac{9+8}{24}=\frac{17}{24}$

Therefore, P(C) $=1-\frac{17}{24}=\frac{7}{24}$

Miss A is more likely to win

P(Miss A or Miss B) = P(Miss A) + P(Miss B)

$=\frac{3}{8}+\frac{1}{3}=\frac{17}{24}$