

ALGEBRAIC FRACTIONS

Algebraic fractions contain numbers and letters.

You have already learned how to deal with numbers in 'ordinary fractions', and you know how to work with powers and letters, so you should find these straight-forward, as the same rules apply, particularly with regard to **CANCELLING**, and **FINDING** the **LOWEST COMMON DENOMINATOR**.

An important habit to develop is "**IF IN DOUBT – WRITE IT OUT!**"

Example 1

$$\text{Simplify } \frac{9am^3}{6a^2m} = \frac{9 \times a \times m \times m \times m}{6 \times a \times a \times m}$$

CANCEL, where possible giving:

$$\frac{3 \times m \times m}{2 \times a} = \frac{3m^2}{2a}$$

Example 2

$$\text{Simplify } \frac{x^2y^5}{3xy} = \frac{x \times x \times y \times y \times y \times y \times y}{3 \times x \times y}$$

CANCEL, giving:

$$\frac{x \times y \times y \times y \times y}{3} = \frac{xy^4}{3}$$

Example 3

$$\begin{aligned} & \frac{3ab}{5c^2d} \times \frac{15cd^4}{9a^2b^2} \\ &= \frac{3 \times a \times b}{5 \times c \times c \times d \times d \times d} \times \frac{15 \times c \times d \times d \times d \times d}{9 \times a \times a \times b \times b} \end{aligned}$$

CANCEL, to give $\frac{d}{abc}$

Example 4

$$\frac{4ax^2}{5p} \times \frac{12a^3}{25p^4}$$

As in fractions, turn the second fraction upside down then multiply to give

$$\frac{4ax^2}{5p} \times \frac{25p^4}{12a^3}$$

$$\frac{4 \times a \times x \times x \times x}{5 \times p} \times \frac{25 \times p \times p \times p \times p}{12 \times a \times a \times a}$$

Cancelling gives

$$\frac{5p^3x^2}{3a^2}$$

Example 5

Simplify $\frac{x^2 - y}{xy - y}$

STOP! You cannot cancel until you have factorised the expressions.

It becomes $\frac{x(x - y)}{y(x - 1)}$

As you can see the terms inside the brackets are the same. You may cancel these. **Whole brackets as well as single numbers or letters can be cancelled.**

This now becomes $\frac{x}{y}$

If you are entering at Level 1, you do not need to study anymore of this pack – however, please attempt Exercise 1.

Example 6

Simplify $\frac{x^2 - 4}{x^2 - 2x}$

Factorise first $\frac{(x - 2)(x + 2)}{x(x - 2)}$

Cancel $(x - 2)$

Answer = $\frac{x + 2}{x}$

Example 7

$$\frac{4b-2}{3b^2-15} \times \frac{5b^2-25}{8b-4}$$

becomes

$$\frac{2(2b-1)}{3(b^2-5)} \times \frac{5b(b-5)}{4(2b-1)}$$

Cancel $(2b-1)$ and (b^2-5)

Cancel 2 into 4

$$\text{Answer} = \frac{5}{6}$$

Example 8

$$\frac{2a-5}{3a+2} \div \frac{4a^2-10a}{9a^2+6}$$

Multiply by inverted second fraction

$$\frac{2a-5}{3a+2} \times \frac{9a^2+6}{4a^2-10a}$$

Factorise

$$\frac{2a-5}{3a+2} \times \frac{3a(3a+2)}{2a(2a-5)}$$

Cancel $(2a-5)$

$(3a+2)$

and a, to give the answer $\frac{3}{2}$

Exercise 1

Simplify these

1. $\frac{4ab^2}{8a^2b}$

2. $\frac{a^2b^3}{3ab^4}$

3. $\frac{3xy}{5m^2n^3} \times \frac{15mn^4}{9x^2y^2}$

4. $\frac{12ax^3}{10p} \div \frac{12a^3x}{15p^3}$

5. $\frac{a^2 - a}{ab - b}$ **Stop here for level 1**

6. $\frac{a^2 - 9}{a^2 - 3a}$

7. $\frac{4a - 2}{3a^2 - 15} \times \frac{5a^2 - 25}{8a - 4}$

8. $\frac{2m - 5}{3m + 2} \div \frac{4m^2 - 10m}{9m^2 + 6m}$

ADDITION AND SUBTRACTION OF ALGEBRAIC FRACTIONS

Before you start:

- As in ordinary fractions, the denominators **MUST** be the same, before you can **ADD** or **SUBTRACT** the **NUMERATORS**.
- Look for the LCM (the smallest number which all the denominators will “go” into).

E.g. LCM of 5p and 3pq is 15pq
LCM of c^2y and cy^2

Look for the **HIGHEST POWER** of c and y – both ².

Therefore LCM = c^2y^2

E.g. LCM of $5c^4y^4$, $10c^2y^3$, $6c^4y$
LCM of 5, 10 and 6 is 30

Highest power of c is 4
Highest power of y is 4

Therefore, LCM = $30c^4y^4$

Exercise 2

Now find the LCM of the following

1. 2ab, $4a^2b$

2. $4a^2b^2$, $12a^3b$

3. 3, 12c

4. 2cy, 4y, 6c

5. 3a, 4b

6. $3a^2bc$, $9ab^2c$, $12abc^2$

ADDING AND SUBTRACTING (the real thing!)

Example 1

$$\frac{4a}{3} - \frac{5a}{6}$$

First find the LCM of the denominator

LCM of 3 and 6 = 6

Proceed as for fractions:

$$= \frac{2(4a) - 1(5a)}{6}$$

$$= \frac{8a - 5a}{6}$$

$$= \frac{3a}{6}$$

CANCEL 3 INTO 3 AND 6 Answer = $\frac{a}{2}$

Example 2

$$\frac{2}{3x} + \frac{1}{5x^2}$$

First find the LCM of the denominator

LCM of $5 \times 3 = 15$

Highest power of x is 2

Therefore, LCM = $15x^2$

STEP BY STEP, this looks like:

1. $\frac{\quad}{15x^2}$ (LCM)

2. $\frac{2}{15x^2} + \frac{1}{15x^2}$

3. $\frac{2(5x) + 1(3)}{15x^2}$

4. $\frac{10x + 3}{15x^2}$ **This cannot be simplified any further**

Example 3

$$\frac{5x}{12} - \frac{7x}{18}$$

LCM of 12 and 18 = 36

1. $\frac{\quad}{36}$

2. $\frac{(5x)(-7x)}{36}$

3. $\frac{3(5x) + 2(-7x)}{36}$

4. $\frac{15x - 14x}{36}$

5. $\frac{x}{36}$

Example 4

$$\frac{1}{a} - \frac{5}{3a} + \frac{4}{5a}$$

LCM of 3 and 5 is 15

Highest power of a is 1

LCM is 15a

1. $\frac{\quad}{15a}$

2. $\frac{1(-5)(+4)}{15}$

3. $\frac{15(1) + 5(-5) + 3(4)}{15a}$

4. $\frac{15 - 25 + 12}{15a}$

5. $\frac{2}{15a}$

Exercise 3

1. $\frac{a}{2} + \frac{a}{3} + \frac{a}{6}$

2. $\frac{x}{4} + \frac{y}{12}$

3. $\frac{3g}{10} + \frac{h}{5}$

4. $\frac{1}{m} + \frac{1}{2n}$

5. $\frac{3}{2x} - \frac{4}{x^2}$

ANSWERS

Exercise 1

Simplify

$$1. \quad \frac{4ab^2}{8a^2b} = \frac{b}{2a} \qquad 2. \quad \frac{a^2b^3}{3ab^4} = \frac{a}{3a} \qquad 3. \quad \frac{3xy}{5m^2n^3} \times \frac{15mn^4}{9x^2y^2} = \frac{n}{mxy}$$

$$4. \quad \frac{12ax^3}{10p} \div \frac{12a^3x}{15p^3} = \frac{12ax^3}{10p} \times \frac{15p^3}{12a^3x} = \frac{3x^2p^2}{2a^2}$$

$$5. \quad \frac{a^2 - a}{ab - b} = \frac{a(a-1)}{b(a-1)} = \frac{a}{b}$$

$$6. \quad \frac{a^2 - 9}{a^2 - 3a} = \frac{(a-3)(a+3)}{a(a-3)} = \frac{a+3}{a}$$

$$7. \quad \frac{4a-2}{3a^2-15} \times \frac{5a^2-25}{8a-4} = \frac{2(2a-1)}{3(a^2-5)} \times \frac{5(a^2-5)}{4(2a-1)} = \frac{5}{6}$$

$$8. \quad \frac{2m-5}{3m+2} \div \frac{4m^2-10m}{9m^2+6m} = \frac{2m-5}{3m+2} \times \frac{3m(3m+2)}{2m(2m-5)} = \frac{3}{2}$$

Exercise 2

LCM

$$1. \quad 4a^2b \qquad 2. \quad 12a^3b^2 \qquad 3. \quad 12c$$

$$4. \quad 12cy \qquad 5. \quad 12ab \qquad 6. \quad 36a^2b^2c^2$$

Exercise 3

$$1. \quad \frac{3a+2a+a}{6} = \frac{6a}{6} = a \qquad 2. \quad \frac{3x+y}{12} \qquad \text{NB. You cannot cancel the 3 and the 12}$$

$$3. \quad \frac{9g+2h}{30} \qquad \text{You cannot cancel any further.} \qquad 4. \quad \frac{2n+m}{2mn} \qquad \text{You cannot cancel any further.}$$

$$5. \quad \frac{3x-8}{2x^2} \qquad \text{You cannot cancel any further.}$$