Learning Development

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BASIC ALGEBRA

In algebra, letters are used as well as numbers.

MAKING ALGEBRAIC EXPRESSIONS

This is really like making up sentences and is quite straight forward once you have learned the rules. If you are asked to make up an algebraic expression you may choose which letter(s) you wish.
Look at these examples where you are asked to write down the sentences in algebraic forms.

Q means question
A means answer

Q1. **Five times a number**

A1. Let the number be d
Five times d = 5d
5 x d can be written as
5d or 5.d
The more common way is 5d
REMEMBER THIS!

Q2. **Three more than a number**

A2. Let the number be a
Three more than a = a + 3
or = 3 + a
(whichever way round, the answer is the same)

Q3. **Seven less than a number**

A3. Let the number be g
Seven less than g = g – 7

Q4. **The sum of two numbers**

A4. Let the numbers be j and k
The sum of j and k = j + k

Q5. **A number multiplied by itself**

A5. Let the number be c
c multiplied by itself = c x c
= c^2

Q6. **Half the number**

A6. Let the number be s
Half of s = $\frac{s}{2}$

Q7. **The product of two numbers**
(product means multiply)

A7. Let the two numbers be *y* and z
The product of y and z = *y*z or z*y*
(also *y* x z or *y*.z)

Q8. **One number divided by three times another number**

A8. Let one number be m and the other number n
One number divided by three times the other

$$= \frac{m}{3n}$$

The χ used in algebraic is 'curly', so that it cannot be confused with the multiplication sign. Practise writing the 'curly' χ.

Exercise 1

Write down the following as algebraic expressions

1. Four times a number.
2. A quarter of a number.
3. Eight less than a number.
4. Six more than a number.
5. The sum of three numbers.
6. Three times the product of two numbers.
7. Six times a number, plus five times a second number.
8. Four times a number, minus another number.

SUBSTITUTION

When a letter is replaced by a number in an expression this is called substitution.
In the following 9 examples a = 2, b = 3, c = 4.

1. 5a means 5 x a
   thus 5 x 2 =10

2. b + c gives 3+ 4 = 7

3. c – a gives 4 – 2 = 2

4. 5b + 12a means
   5 x b plus 12 x a
   = 5 x 3 plus 12 x 2
   = 15 + 24
   = 39

5. ab means a x b
   = 2 x 3 = 6

6. abc means a x b x c
   = 2 x 3 x 4
   = 24

7. $\frac{bc}{a}$ means $\frac{3 \times 4}{2} = \frac{12}{2} = 6$

8. 6 – c means 6 – 4 = 2

9. 3ac means 3 x a x c
   = 3 x 2 x 4
   = 24

Exercise 2

If a = 1
b = 2
c = 3
d = 4
e = 5

Find the values of :

1.	3 + b	2.	2a
3.	c + d	4.	e – c
5.	2c + 3d	6.	4a – 2b
7.	bc	8.	de + d
9.	abcd	10.	$\frac{de}{b}$

What is different about a^3 and 3a?

If you are in any doubt about an expression
WRITE IT OUT IN FULL

So, a^3 is a x a x a and 3a is 3 x a

These, as you know, will give different answers if the value of 'a' is known.
If a = 2, then

$a^3 = 2^3 = 2 \times 2 \times 2 = 8$

but,

$3a = 3 \times a = 3 \times 2 = 6$

More examples
If a = 2, b = 3 and c = 5, find the values of the following:

1. $a^4 = a \times a \times a \times a$
   $= 2 \times 2 \times 2 \times 2 = 16$

2. $b^2 = b \times b$
   $= 3 \times 3 = 9$

3. $b^3 = b \times b \times b$
   $= 3 \times 3 \times 3 = 27$

4. $ac^2 = a \times c \times c$
   $= 2 \times 5 \times 5 = 50$

5. $3c = 3 \times c$
   $= 3 \times 5 = 15$

6. $4a = 4 \times a$
   $= 4 \times 2 = 8$

7. $2b^2 = 2 \times b \times b$
   $= 2 \times 3 \times 3 = 18$

8. $\frac{5a^2}{c} = \frac{5 \times a \times a}{c}$

   $= \frac{5 \times 2 \times 2}{5}$ Cancel 5's

   $= 4$

9. $2c^2 + 2b^3$
   means 2 x c x c plus 2 x b x b x b
   $= 2 \times 5 \times 5 + 2 \times 3 \times 3 \times 3$
   $= 50 + 54$
   $= 104$

10. $\frac{3a^2}{b} = \frac{3 \times a \times a}{b}$

$= \frac{3 \times 2 \times 2}{3}$ Cancel 3's

$= 4$

Exercise 3

If $p = 2$
$q = 3$
$r = 4$
$s = 5$

Find the values of:

1.	q^2	2.	r^3
3.	$2s^2$	4.	qp^2
5.	$3p^2 + r^3$	6.	$2s^2 + p^3$
7.	$2q^2 + 3p^2$	8.	rs^2
9.	$\frac{2q^2}{r}$	10.	$\frac{s^2}{p}$

SUBSTITUTION WITH POSITIVE AND NEGATIVE NUMBERS

If $a = 1$
$b = -2$
$c = 3$
$d = -4$
$e = 5$

Work out the following:

1. $a + b = 1 + (-2) = -1$

2. $a – b = 1 – (-2) = 1 + 2 = 3$

3. $b^2 + 2e$ $= (b \times b) + 2 \times e$
$= (-2 \times –2) + 2 \times 5$
$= 4 + 10 = 14$

4. ed^2 $= e \times d \times d$
$= 5 \times -4 \times -4$
$= 5 \times +16$
$= 80$

5. $(ed)^2 = (e \times d)^2$
$= (5 \times –4)^2$
$= (-20)^2$
$= 400$

Exercise 4

$c = -1$, $y = 2$, $z = 3$

1. $2c + 3y$
2. cyz
3. c^2y
4. $c + y + z^2$
5. $(2c + y)^2$

ADDITION AND SUBTRACTION OF ALGEBRAIC EXPRESSIONS

You can only add or substract algebraic terms **if they have the same letter(s)**
i.e. b's can only be added to b's
k's can only be added to k's
fg's can only be added to fg's
g^2's can only be added to g^2's

Example 1

$3a + 2a = 5a$
Think of it as adding 3 apples to 2 apples.
Your answer would be 5 apples – in other words **only the numbers are added.**

Example 2

$6a – 2a = 4a$

Example 3

$8a – 6a + 7a = 2a + 7a = 9a$

Example 4

$2cy + 6yc = 8cy$

Remember that yc is the same as cy

Example 5

$3ab – ab = 2ab$
(ab is really 1ab)

Example 6

$12cy + 5cy – 6cy \quad = 17cy – 6cy$
$= 11cy$

Example 7

$a + b = a + b$

Example 8

3a + 2b – a + 3b

FIRST – collect the ‘a’ terms together keeping the same sign in front of each term.
+3a – a + 2b + 3b
= +2a + 5b Answer

Example 9

$a^2 + 2a^2 + 3a$
$= 3a^2 + 3a$ Answer

NB you cannot add a^2 to a.
NB the number in front of the letter is called **CO-EFFICIENT**.
If there is no number in front of the letter, it must be assumed to be 1.
ab^2 means $1ab^2 = 1 \times a \times b \times b$
If there is no **SIGN** in front of the letter this is, as you know, assumed to be positive.

Exercise 5

1. 4a + 10a
2. 11a – 6a
3. 6cy + 2cy
4. 6a – 2a +3a
5. 11b +2b – 7b
6. 3a +2b –a
7. 8b – 6a – 2b – 7a
8. $3b^2 + 2b + 5b^2$
9. $6s^2t + 3s^2t$
10. $2b^2 + 3b + 6b^2 + 4b$

MULTIPLICATION AND DIVISION OF ALGEGRAIC FRACTIONS

The same rules apply as those in the unit of DIRECTED NUMBERS.
Read through the following examples to clarify the rules.

c times y = cy

5c times 3y = 5 x 3 x c x y

= 15cy

Multiply the numbers together and then the algebraic terms

Remember the rules on **INDICES**

Example 1

A times a = a x a = a^2

Example 2

4b x 3b x 5b= 4 x 3 x 5 x b x b x b

= $60b^3$

Example 3

2y x 5z = 2 x 5 x y x z

= 10yz

Example 4

a x (-b) = -ab

Example 5

(-2a) x (5b) = -10ab

Example 6

(2a) x (-5b) = -10ab

Example 7

(-2a) x (-5b) = +10b

Example 8

$3a^2$ x 2a = 3 x a x a x 2 x a

= $6a^3$

Example 9

$\frac{4a}{2b} = \frac{2a}{b}$

Example 10

$$\frac{3c}{4y} = \frac{3c}{4y} \quad \text{(no change!)}$$

Example 11

$$\frac{4a^3}{2a} = \frac{4 \times a \times a \times a}{2 \times a} \qquad \text{Cancel 2 and a}$$

$$= 2a^2$$

Example 12

$$\frac{12a^3bc^2}{4a^2c} = \frac{12 \times a \times a \times a \times b \times c \times c}{4 \times a \times a \times a \times c} \qquad \text{Cancel 4, a and c}$$

$$= 3abc$$

Exercise 6

1. $2a \times 3a$
2. $2a \times 3b$
3. $4 \times 6a$
4. $(-2a) \times (6a)$
5. $(2s) \times (-6t)$
6. $(-2s) \times (-6t)$
7. $4a^2 \times 2a^2$
8. $3b^2 \times 2a^2$
9. $(-a) \times (-b)$
10. $\frac{12a^2}{3a}$
11. $\frac{4a^2b}{2ab^2}$
12. $\frac{8a^2b^2c^2}{2abc}$
13. $\frac{(-a)}{b}$
14. $\frac{(-6c)}{(-2cy)}$
15. $\frac{9a^2bc}{27a^2bc}$

BRACKETS

Brackets are used in mathematics as a type of shorthand. When removing the brackets, everything inside the bracket is multiplied **BY THE EXPRESSION OUTSIDE THE BRACKET**.

Example 1

2(a + b) becomes 2a + 2b

Example 2

3(f + g) becomes 3f +3g

Example 3

A(j + k) becomes aj +ak

Example 4

2(a - b) becomes +2a – 2b

Example 5

4(3a – 2b) = 12a – 8b

Example 6

3a(5b – 6c) = 15ab – 18ac

Example 7

$2c(3+2c) = 6c + 4c^2$

-2c(3 + c) means that +3 and +c must both be multiplied by –2c.
Write it like this

(-2c) x (3) + (-2c) x (c)
$= -6c - 2c^2$

RULE
When a bracket has a minus sign in front of it, the signs inside the bracket are changed when the bracket is removed.
Look at the following examples:

1. -2(3 + 6a) = -6 – 12a
2. -3(4 – 3b) = -12 – 9b
3. -(a – b) = -1(a – b) = -a +b
4. -(a + b) = -1(a +b) = -a – b

In 3 and 4 there was only a minus sign in front of the bracket, but really this is a short way of saying that –1 is in front of the bracket. This is a **VERY IMPORTANT POINT TO REMEMBER.**

Exercise 7

1.	2(c +3)	2.	4(a + b)
3.	6(a – b)	4.	5(c – 3)
5.	3(4c +2y)	6.	-(m + n)
7.	-2(3c +5)	8.	-3(4 – 6c)
9.	-(2p +3q)	10.	4b(3a – b)

REMOVING BRACKETS AND SIMPLIFYING

In this type of question you have to multiply out the brackets **FIRST** and then collect all the '**LIKE'** terms together.

Example 1

2(c + 6) +3(c + 5)
= 2c +12 + 3c +15
= 5c + 27

Example 2

2(c + 3) + (c – 2)
= 2c +6 + c – 2
= 2c +c + 6 – 2
= 3c 4

Example 3

$c(3c + 4) - 2(c^2 - c)$
$= \quad 3c^2 + 4c - 2c^2 + 2c$
$= \quad c^2 + 6c$

Example 4

$3(a - b) - (2a - b) + 4(a - 2b)$
$= \quad 3a - 3b - 2a + b + 4a - 8b$
$= \quad 3a - 2a + 4a - 3b + b - 8b$
$= \quad a + 4a - 2b - 8b$
$= \quad 5a - 10b$

Exercise 8

1. $2(c + 2) + 3(c + 4)$

2. $3(c - 6) - 2(c - 4)$

3. $c(2c + 1) - 4(c^2 + 1)$

4. $4(a - b) - 2(a + b) + 6(a + b)$

5. $4c(c + 6) - 2(c^2 - 3) + 5(c^2 + c + 2)$

ANSWERS

Exercise 1

MAKING EXPRESSIONS

Here a, b and c have been chosen for the numbers.

1.	4a	2.	$\frac{1a}{4}$ or $\frac{a}{4}$	3.	a – 8	4.	a + 6
5.	a + b + c	6.	3ab	7.	6a + 5b	8.	4a – b

Exercise 2

SUBSTITUTION

1.	5	2.	2	3.	7
4.	2	5.	18	6.	0
7.	6	8.	24	9.	24
10.	10				

Exercise 3

MORE SUBSTITUTION

1.	9	2.	64	3.	50
4.	12	5.	76	6.	58
7.	30	8.	100	9.	4.5
10.	12.5				

Exercise 4

POSITIVE AND NEGATIVE SUBSTITUTION

1.	4	2.	-6	3.	2
4.	10	5.	0		

Exercise 5

ADDITION AND SUBSTRACTION OFALGEBRAIC TERMS

1.	14a	2.	5a	3.	8cy
4.	7a	5.	6b	6.	2a +2b
7.	6b –13a	8.	$8b^2$ +2b	9.	$9s^2t$
10.	$8b^2$ +7b				

Exercise 6

MULTIPLICATION AND DIVISION

1.	$6a^2$	2.	6ab	3.	24a
4.	$-12a^2$	5.	-12st	6.	+12st
7.	$8a^4$	8.	$6a^2b^2$	9.	ab
10.	4a	11.	$\frac{2a}{b}$	12.	4abc

13. $\frac{-a}{b}$ or $\frac{a}{-b}$ or $-\frac{a}{b}$ NB minus sign can go on top or bottom

14. $\frac{3}{y}$

15. $\frac{1}{3}$

Exercise 7

BRACKETS

1.	2c + 6	2.	4a + 4b
3.	6a – 6b	4.	5c – 15
5.	12c + 6y	6.	- m – n
7.	- 6c – 10	8.	-12 + 18c
9.	-2p – 3q	10.	$12ab - 4b^2$

Exercise 8

MORE BRACKETS

1. 2c + 4 + 3c + 12 = 5c + 16

2. 3c – 18 – 2c + 8 = c - 10

3. $2c^2 + c - 4c^2 - 4 = -2c^2 + c - 4$

4. 4a – 4b – 2a – 2b + 6a + 6b = 8a

5. $4c^2 + 24c - 2c^2 + 6 + 5c^2 + 5c + 10 = 7c^2 + 29c + 16$