

HARDER GRAPHS

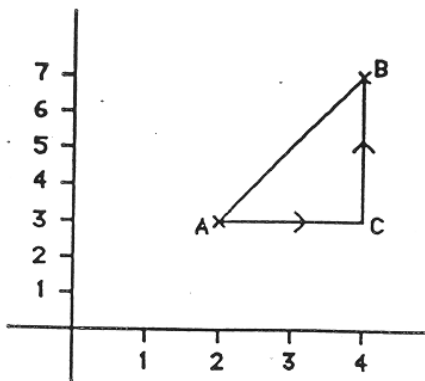
GRADIENT

The gradient of a straight line is a measure of its slope.

$$\text{Gradient} = \frac{\text{change in vertical height}}{\text{change in horizontal distance}}$$

Example 1

Find the gradient of the line joining A (2, 3) to B (4, 7)



Imagine travelling from A to B by going across the page to C then up (rather than directly)

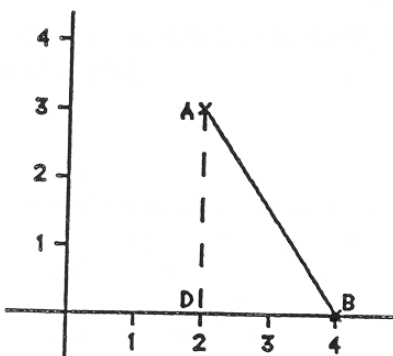
$$\text{Then gradient} = \frac{\text{dist BC}}{\text{dist AC}} = \frac{7-3}{4-2} = \frac{4}{2} = \frac{2}{1}$$

therefore gradient = 2

i.e. We go 2 units **up** for every 1 across. The line AB goes up from left to right.

Example 2

Find the gradient of the line joining A (2,3) and B (4,0)



Make a triangle so that the journey AB can be made up of a movement down and then across.

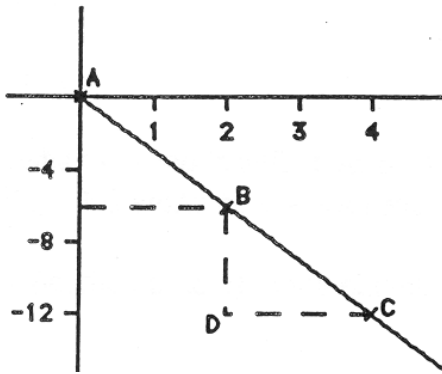
$$\text{Gradient} = \frac{\text{dist AD}}{\text{dist DB}} = \frac{-3}{2} = \frac{3}{2}$$

When the line goes **down** from left to right the gradient is **negative**.

Example 3

Find the gradient of the straight line with equation $y = -3x$

A set of 3 possible pairs of points is A(0,0), B(2, -6), C(4, -12) and the line can be sketched.



$$\begin{aligned} \text{Gradient} &= \frac{BD}{DC} \\ &= \frac{6}{2} \\ &= -3 \end{aligned}$$

Exercise 1

By sketching the lines and calculating the gradients complete the following table.

Equation of line	Gradient
1. $y = x$	
2. $y = 4x + 1$	
3. $y = -2x + 3$	
4. $y = \frac{1}{2}x + 4$	
5. $y = -\frac{1}{4}x$	

FINDING GRADIENTS BY INSPECTION

By studying your results can you see a connection between the equation of the straight line and the values found for the gradient?

Can you **predict** the gradient of the lines whose equations are:

a) $y = 5x + 1$

b) $y = 3x + 4$

We see that when the line is written as an equation beginning 'y =' the coefficient of x is the same as the gradient.

The constant (i.e. the number) in the equation can also be given a special meaning.

Look back to your graphs of exercise 1 and you will see that this number is always the same as the 'y intercept' i.e. the place where the line crosses the y axis.

We can summarise by saying if a line has an equation of the form:

$$y = mx + c$$

then $m = \text{gradient}$

and $c = \text{y intercept}$

This enables us to predict the behaviour of a line without actually drawing it.

Example 1

$$y = 2x + 3$$

In this case we know from this equation, by comparing it to the standard, that

- a) the gradient of the line m is +2 (the coefficient of the x term) the positive sign means the line slopes /.

NB. We do **not** put $2x$, just 2.

- b) the intercept is +3, so the line cuts the y axis at the point (0, 3).

Example 2

$$Y = -\frac{1}{2}x + 1$$

By comparing the equation to the standard we know

- a) the gradient of the line $m = -\frac{1}{2}$ the minus sign means the line slopes \ (downward).
- b) the intercept $c = +1$, so the line cuts the y axis at the point (0, 1).

Sometimes the equation has to be rearranged to obtain it in the standard form, so we can find m and c .

Example 3

$x + y = 3$ can be rearranged to give:

$$y = -x + 3$$

thus $m = -1$, the minus sign means the line slopes \, and $c = 3$.

Example 4

$4x - 2y = 5$ can be rearranged to give:

$$4x - 5 = 2y$$

$$2y = 4x - 5$$

or

$$y = \frac{4x}{2} - \frac{5}{2}$$

$$y = 2x - 2\frac{1}{2}$$

thus $m = 2$, the positive sign means the line slopes /, and $c = -2\frac{1}{2}$

Exercise 2

State the gradient and the intercept of the following without drawing the line:

1. $y = 5x + 1$

2. $y = -3x + 5$

3. $y = 2x - 3$

4. $y = -3x + 2$

5. $y = \frac{1}{2}x + 3$

6. $x + y = 4$

7. $2y = x + 6$ (Be careful!)

8. $3y - 2x = 5$

9. $5x - 2y = 3$

10. $-3x - 4y = 6$

DRAWING STRAIGHT LINE GRAPHS

You should already know how to plot a straight line graph from a given equation by

- a) using 3 values *of* x to make a table of values.
- b) plotting the coordinates *of* the 3 points from the table.
- c) drawing a straight line through the points.

The equations for the graph are often given in the standard form.

Example 1

$$y = 2x - 3$$

However, the equations to be plotted may not be written in the standard form and so may have to be rearranged or transposed before the table of values can be made.

Example 2

$$3x + 2y = 5 \quad \text{must be rearranged to give}$$

$$2y = -3x + 5$$

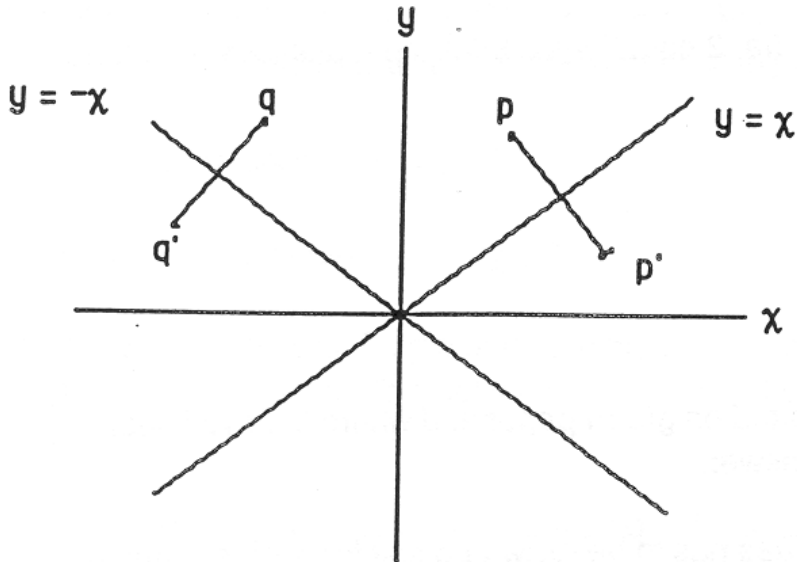
$$y = \frac{3x}{2} + \frac{5}{2} \quad \text{which is now in standard form}$$

The table of values is then prepared and the graph plotted as previously, the only difference being, that the numbers involved may be in the form of fractions or decimals needing a little more care.

Exercise 3

Plot the graph of $2y - 3x = 4$ for $x = -4$ to $+4$.

Plotting the straight line graphs $y = x$ and $y = -x$



Using suitable values for x , the two lines $y = x$ and $y = -x$ have been drawn. Note also that it is possible to reflect points in these straight lines. In the example above

p has been reflected in the line $y = x$ to give the point p' , and q has been reflected in the line $y = -x$ to give the point q'

SIMULTANEOUS EQUATIONS SOLVED GRAPHICALLY

Simultaneous equations (i.e. 2 equations containing 2 unknowns) can be solved graphically.

Example 1

$$y = x + 5$$

$$y = -2x + 3$$

The straight lines are plotted on graph paper and where the two lines intercept (cross) is the answer.

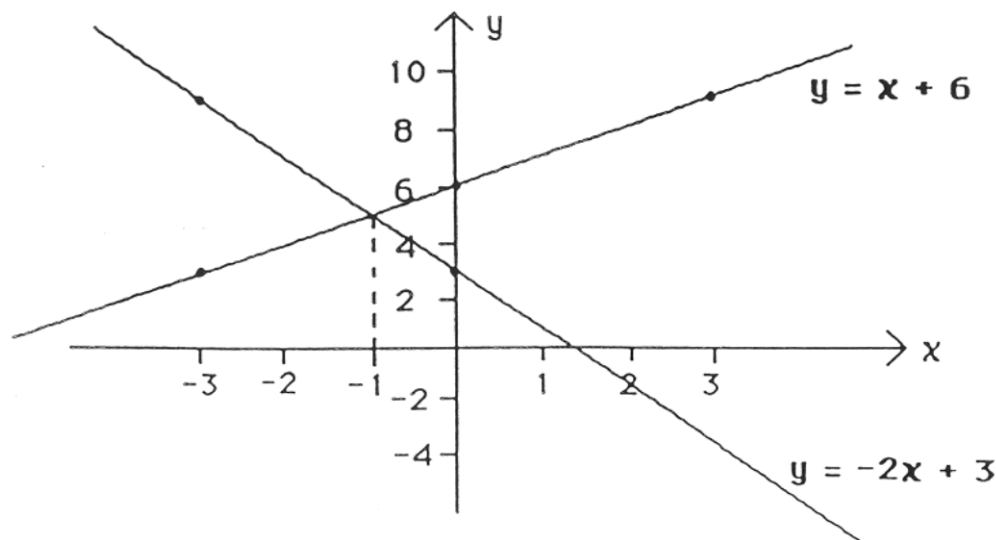
Remember a table of values has to be drawn up first for **both** equations before a suitable scale can be decided upon before plotting the lines.

$$y = x + 6$$

x	-3	0	3
y	-3	0	3
+6	+6	+6	+6
y	+3	+6	+9

$$y = -2x + 3$$

x	-3	0	3
$-2x$	6	0	-6
+3	+3	+3	+3
y	9	3	-3



Lines cross at (-1, 5)

NB

If the equations are given in the form $y - x = 6$ and $y + 2x = 3$ they have to be rearranged first so they are in the form $y = mx + c$.

i.e. $y - x - 6$ becomes $y = x + 6$

$y + 2x = 3$ becomes $y = -2x + 3$.

Then proceed as before, i.e. make up a table of values, decide upon a scale, plot the straight lines and find out where they cross from the graph.

Exercise 4

Solve the following pairs of simultaneous equations graphically.

1. $x + y = 4$ $y - x = 2$ x is between -3 to +3

2. $x + y = 5$ $x - y = 1$ x is between -3 to +3

3. $y = 3x + 2$ $x + y = 6$ x is between -3 to +3.

GRAPHS FROM EXPERIMENTAL DATA

Sometimes we are given two sets of numbers and a connection has to be found between them.

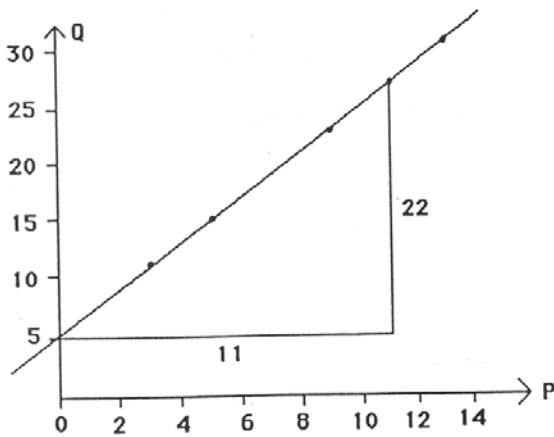
e.g. P and Q are connected by a relationship

P 3 5 9 11 13

Q 11 15 23 27 31

Plot the values of P along the horizontal axis using the scale of 1 cm = 1 unit and Q along the vertical axis using the scale of 2 cm = 5 units. Find a relationship between P and Q.

NB There are no negative values therefore you do not need the negative x and y axes.



Join the points up. They form a straight line.

The general equation of a straight line is $y = mx + c$

therefore $Q = mP + c$

c is the intercept, therefore $c = 5$

Exercise 5

1. S and T are two quantities connected by a relationship

S 1 2 4 5 7

T 7 9 13 15 19

Plot this relationship, S along the horizontal axis and T along the vertical axis. Find the gradient of the resulting line, the intercept on the vertical axis and from this, suggest the relationship between S and T.

2. Plot the following values of P along the x axis and the corresponding values of Q along the y axis. Is there a relationship between P and Q, and if so what is it?

P 10 8 7 6 4

Q 6 4 3 2 0

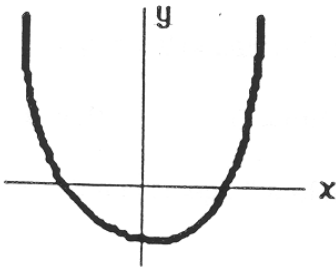
GRAPHS OF QUADRATIC FUNCTIONS

A quadratic function contains an ' χ^2 ' term and no higher power.

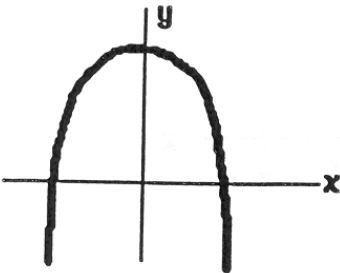
e.g. $\chi^2 + 2\chi + 6$

$$3\chi^2 - \chi + 1$$

When plotted, quadratic functions always give a 'smooth curve' known as a parabola. If the χ^2 term is positive the curve usually looks like:



If the χ^2 term is negative the curve usually looks like:



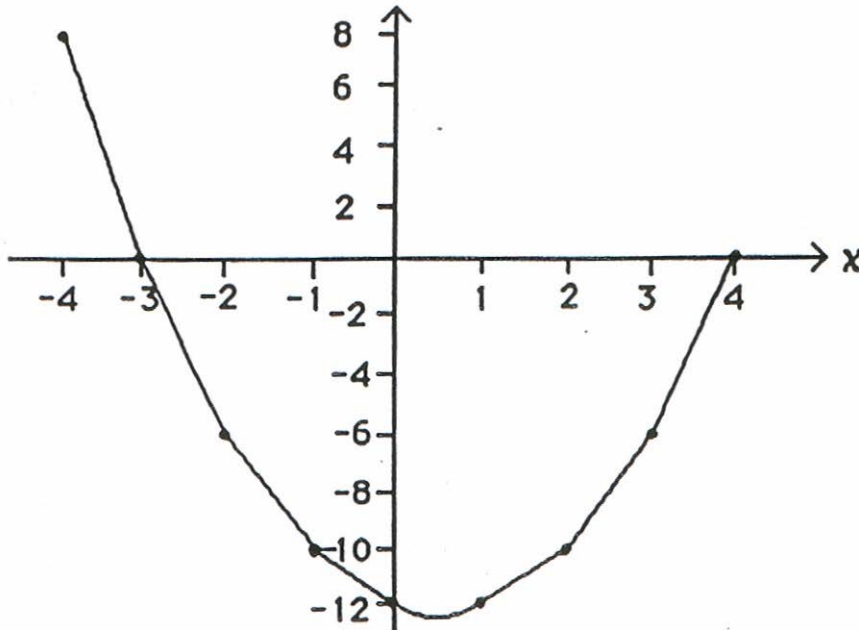
As with straight line graphs, the first thing you have to do is to make up a table of values, but this time more than just three points are needed as shown.

Now let us see exactly how we plot a given quadratic equation.

Plot the graph of $y = \chi^2 - \chi - 12$ from $\chi = -4$ to $+4$.

χ	-4	-3	-2	-1	0	1	2	3	4
χ^2	16	9	4	1	0	1	4	9	16
$-\chi$	4	3	2	1	0	-1	-2	-3	-4
-12	-12	-12	-12	-12	-12	-12	-12	-12	-12
y	8	0	-6	-10	-12	-12	-10	-6	0

Plot $(-4,8)$, $(-3, 0)$, $(-2, -6)$ etc using a suitable scale.



Do not join the points with a straight line, 'curve' it round. It is usually better to work out another set of coordinates at an intermediate value, say

$$\text{when } \chi = \frac{1}{2}.$$

Join the points up with a 'smooth' curve.

Once we have drawn the graph we can use it to solve various equations.

Example 1

Solve $0 = \chi^2 - \chi - 12$, this is the same as $\chi^2 - \chi - 12 = 0$.

We have just drawn the graph of the equation $y = \chi^2 - \chi - 12$. Comparing the above we can see that the 'y' has been replaced by '0'. Therefore, to solve the equation we have to find the values of χ where the 'y' has been replaced by the values of χ where $y = 0$. y is zero all along the χ axis and so we have to find where the graph cuts the χ axis. Looking at the graph we can see this is where $\chi = -3$ and $\chi = 4$, i.e. the solutions of the equation:

$$\chi^2 - \chi - 12 = 0 \text{ are } \chi = -3 \text{ and } \chi = 4.$$

Example 2

Solve $2x^2 - 2x = 26$

$2x^2 - 2x = 26$ can be rearranged to give $2x^2 - 2x - 26 = 0$.

We can now divide throughout by 2 to get

$x^2 - x - 13 = 0$, but we drew $x^2 - x - 12$.

Therefore we have to **add ONE** to both sides

$x^2 - x - 13 + 1 = 1$

We now solve this in a similar manner to Example 2 above by drawing the line $y = 1$ and seeing where it cuts the graph.

Exercise 6

- Plot the graph of $y = x^2 - 4$ from $x = -3$ to $+3$.

Solve a) $0 = x^2 - 4$

b) $0 = x^2 - 9$

- Plot the graph of $y = x^2 + x - 6$ from $x = -4$ to $+4$.

Solve a) $0 = x^2 + x - 6$

b) $0 = x^2 + x - 5$

- Draw the graph of $y = 2x^2 - 3x + 1$ taking values of x between -3 and $+3$.

Solve $0 = 2x^2 - 3x + 1$.

- Draw the graph of $y = 9 - x^2$ taking values of x between -4 and $+4$.

Solve $0 = 9 - x^2$.

LEVEL H ONLY

INTERSECTING GRAPHS

Sometimes we are asked to plot a curve and a straight line on the same axes and find out where they intersect.

e.g. plot $y = \chi^2 - \chi - 12$ and $y = \chi + 3$ on the same axes $\chi = -5$ to $+5$. Find the values of which satisfy the equation.

$$\chi^2 - \chi - 12 = \chi + 3$$

First we must make out a table of values as before

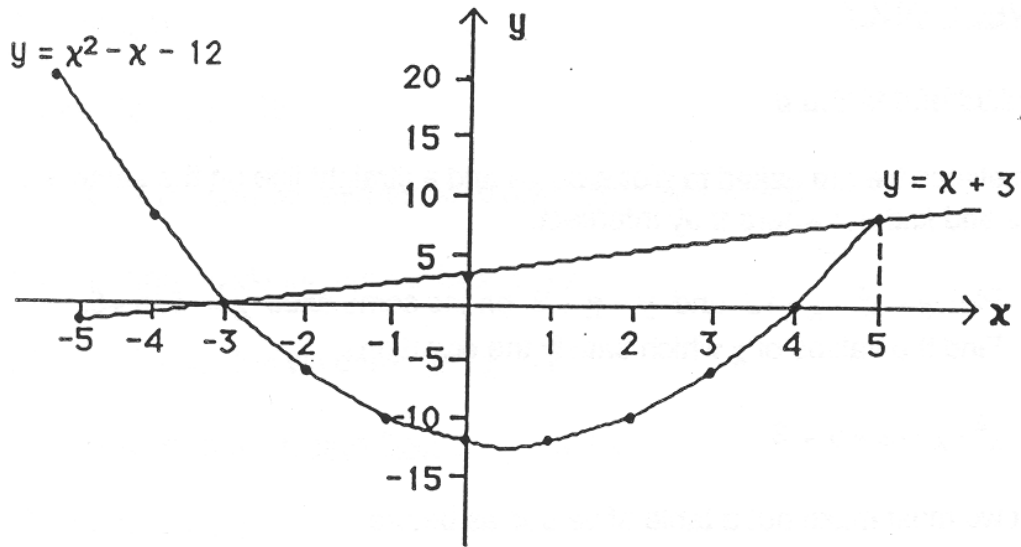
$$y = \chi^2 - \chi - 12$$

χ	-5	-4	-3	-2	-1	0	1	2	3	4	5
χ^2	25	16	9	4	1	0	1	4	9	16	25
$-\chi$	5	4	3	2	1	0	-1	-2	-3	-4	-5
-12	-12	-12	-12	-12	-12	-12	-12	-12	-12	-12	-12
y	18	8	0	-6	-10	-12	-12	-10	-6	0	8

$$y = \chi + 3$$

χ	-5	0	5
χ	-5	0	5
$+3$	+3	+3	+3
y	-2	3	8

Plot these on the same piece of graph paper. As the χ scale goes from -5 to $+5$ turn the piece of graph paper on its side.



To find the values of x which satisfy the equation $x^2 - x - 12 = x + 3$ we have to see what the values of x are where the graphs cross one another:-

$x = -3$ and $x = 5$.

Exercise 7

1. Draw the graphs of $y = \chi^2 + 4\chi - 2$ and $y = 2\chi + 1$ on the same axes, taking the values of x between -4 and +3. Solve the equation

$$\chi^2 + 4 - 2 = 2\chi + 1$$

2. Plot the graph of $y = 2\chi^2 + \chi - 6$ for values of χ from $\chi = -3$ to +2.
Use 2 cm for 1 unit on the χ -axis and 2 cm for 2 units on the y -axis.

On the same scales and axes, draw the graph of $y = -2\chi$, plotting points for $\chi = -3, -1$ and 2.

Use the graphs to solve the equations:-

i) $2\chi^2 + \chi - 2 = 0$

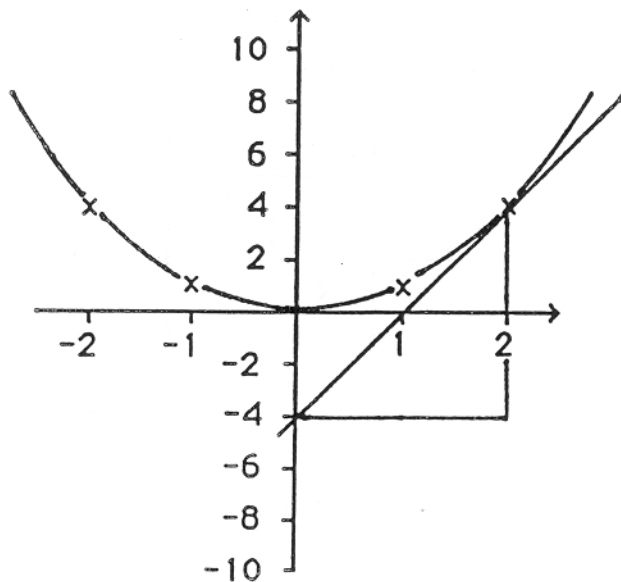
ii) $2\chi^2 + 3\chi - 6 = 0$

LEVEL H ONLY

NB. On the exam paper you may be asked to work out the gradient of a curve at a certain point.

Example

Draw the curve $y = x^2$. Find the gradient when $x = 2$.



First we draw the graph using a table of values.

Then at the point where $x = 2$ draw a tangent. (A line just touching the curve at the point $x = 2$.)

To find the gradient draw a right-angled triangle as shown

$$\text{Grad} = \frac{\text{vert. distance}}{\text{horiz. distance}} = \frac{4 - (-4)}{2} \quad \text{from diagram}$$

$$\text{Grad} = 4$$

χ	-4	0	4
$\frac{3}{2}\chi$	-6	0	6
+2	+2	+2	+2
y	-4	2	8

Exercise 4

1. $\chi = 1, y = 3$
2. $\chi = 3, y = 2$
3. $\chi = 1, y = 5$

Exercise 5

1. $T = 2S + 5$
2. $Q = P - 4$

Exercise 6

1. $y = \chi^2 - 4$

χ	-3	-2	-1	0	1	2	3
χ^2	9	4	1	0	1	4	9
-4	-4	-4	-4	-4	-4	-4	-4
y	5	0	-3	-4	-3	0	5

- i) $\chi = +2$ or -2
Draw line $y = 5$
- ii) $\chi = +3$ or -3

2. $y = \chi^2 + \chi - 6$

χ	-4	-3	-2	-1	0	1	2	3	4
χ^2	16	9	4	1	0	1	4	9	16
$+\chi$	-4	-3	-2	-1	0	1	2	3	4
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
y	6	0	-4	-6	-6	-4	0	6	14

i) $\chi = -3$ or $+2$

ii) $\chi = 1.8$ or -2.8

3. $y = 2\chi^2 - 3\chi + 1$

χ	-3	-2	-1	0	1	2	3
$2\chi^2$	18	8	2	0	2	8	18
-3χ	9	6	3	0	-3	-6	-9
$+1$	+1	+1	+1	+1	+1	+1	+1
y	28	15	6	1	0	+3	10

$\chi = \frac{1}{2}$ or 1

4. $y = 9 - \chi^2$

χ	-4	-3	-2	-1	0	1	2	3	4
9	9	9	9	9	9	9	9	9	9
$-\chi^2$	-16	-9	-4	-1	0	-1	-4	-9	-16
y	-7	0	5	8	9	8	5	0	-7

$\chi = -3$ or $+3$

Exercise 7

To check your answers for exercise 7, please see the Learning Development Maths tutor.