INDICES/ INDEX

The index is the power of a number,

2^3 is said 'two to the power three' **OR** 'two cubed' and
means 2 x 2 x 2 (Ans = 8).

3^2 is said 'three to the power two2 **OR** ' three squared' and
means 3 x 3 (Ans = 9).

Now look at these examples with indices (plural of index!)

1. 10^4 is said 'ten to the power four'
and means 10 x 10 x 10 x 10 = 10000
2. 5^3 is said 'five cubed'
and means 5 x 5 x 5 = 125
3. a^4 is said 'a to the power 4'
and means a x a x a x a
4. g^6 is said 'g to the power 6'
and means g x g x g x g x g x g
5. z^7 is said 'z x z x z x z x z x z x z

Remember! $x^1 = x$

The following examples illustrate the rules which apply to indices. You are advised to learn them.

MULTIPLYING NUMBERS OR LETTERS WITH POWERS

Example 1

2^3 x 2^2 = 2^1 x 2^1 x 2^1 x 2^1 x 2^1
= 2^5 = 32

A **QUICK** way of doing this is to add the powers when you are multiplying numbers with powers.

Example 2

a^6 x a^3 = a^{6+3}
= a^9

Example 3

b^4 x b= b^{4+1}
= b^5

NOTE b^1 is the same as b

Example 4

$3k^2 \times 4k \quad = 3k^2 \times 4k^1$

Written out fully, this is

$3 \times k^1 \times k^1 \times 4 \times k^1$

Multiply the numbers (3 x 4 = 12)
Add the powers 1 + 1 + 1

Answer $= 12 \times k^3$
$= 12k^3$

DIVISION OF NUMBERS OR LETTERS WITH POWERS

Example 1

$$\frac{2^3}{2^2} = \frac{2^1 \times 2^1 \times 2^1}{2^1 \times 2^1}$$

Now cancel, which gives 2^1 which is the same as 2

A **QUICK** way to divide letters or numbers with powers is to subtract the powers

$2^{3-2} = 2^1$ or 2

Example 2

$$\frac{a^6}{a^3} = \frac{a^1 \times a^1 \times a^1 \times a^1 \times a^1 \times a^1}{a^1 \times a^1 \times a^1}$$

$= a^{6-3}$
$= a^3$

Example 3

$$\frac{b^4}{b} = \frac{b^1 \times b^1 \times b^1 \times b^1}{b^1}$$

$= b^{4-1}$
$= b^3$

REMEMBER b^1 is the same as b

Example 4

$$\frac{12s^2}{4s} = \frac{12 \times s^1 \times s^1}{4 \times s^1}$$

Divide the numbers $\frac{12}{4} = 3$

Subtract the powers $s^{2-1} = s^1$

So, the answer is

$3s^1$ which is the same as 3s

Anything to the power zero is 1

Remember this – it is important!

2^0	= 1	6^0	= 1
147^0	= 1	a^0	= 1
b^0	= 1	2.3^0	= 1

BUT $6a^0$ = 6 **WHY**?
Well, $6a^0$ written out fully is
$6 \times a = 6 \times 1$
$= 6$
Try to remember this example!

BRACKETS

Example 1

$(a^2)^3$ which means $a^2 \times a^2 \times a^2$
$a^{2+2+2} = a^6$
A **quick** way of doing this is to **multiply the power inside the bracket by the power outside the bracket**.

$a^{2 \times 3} = a^6$
You get the same answer, but the latter is quicker!

Example 2

$(b^4)^5 = b^{4 \times 5} = b^{20}$

Example 3

$(2b^3)^2 = (2^1 \times b^3)^2$
$= 2^2 \times b^{2 \times 3} = 4b^6$

Example 4

$(4^2b^1)^2 = (4^2 \times b^1)^2$
$= 4^{2 \times 2} \times b^{1 \times 2}$
$= 4^4 \times b^2$
$= 256b^2$

Example 5

$2(a^3)^4$ **NOTE – only a^3 is inside the bracket**
The 2 is not affected by the power outside the bracket!
$= 2 \times a^{3 \times 4}$
$= 2a^{12}$

NEGATIVE INDICES

Example 1

9^{-1} means $\frac{1}{9^1}$ or $\frac{1}{9}$

9 can be written as $\frac{9}{1}$

9^{-1} is written as $\frac{1}{9}$ and is known as the RECIPROCAL of nine.

When you see a negative power think 'one over'

Example 2

$10^{-1} = \frac{1}{10^1} = \frac{1}{10}$

$10^{2} = \frac{1}{10^2} = \frac{1}{100}$

Example 3

$a^{-1} = \frac{1}{a^1}$ and $a^{-4} = \frac{1}{a^4}$

BUT

Example 4

$4d^{-1} = \frac{4}{1} \times \frac{1}{d} = \frac{4}{d}$

$4d^{-3} = \frac{4}{1} \times \frac{1}{d^3} = \frac{4}{d^3}$

In this example the minus sign **ONLY APPLIES** to the letter d not to the number 4!

FRACTIONAL INDICES

Example 1

$16^{\frac{1}{2}} = \sqrt[2]{16^1} = \sqrt[2]{16} = 4$

$4 \times 4 = 16$

This means the 'square root' of 16, i.e. which number multiplied by itself gives 16 – answer is 4. When you see fractional indices think 'Root sign'.

Example 2

$$8\frac{1}{3} = \sqrt[3]{8^1} = \sqrt[3]{8}\ 8 = 2$$

2 x 2 x 2 = 8

This means the '**cube root**' of 8, i.e. which number multiplied by itself three times gives 8 – the answer is 2.

Example 3

$$32\frac{1}{5} = \sqrt[5]{32^1} = \sqrt[5]{32} = 2$$

2 x 2 x 2 x 2 x 2 = 32

This means the '**fifth root'** of 32, i.e. which number multiplied by itself five times gives 32 - answer is 2.

Example 4

$$81\frac{1}{4} = \sqrt[4]{81}^1 = \sqrt[4]{81} = 3$$

3 x 3 x 3 x 3 = 81

This means the '**fourth root**' of 81, i.e. which number when multiplied by itself four times gives 81 – answer is 3.

Example 5

$$a\frac{1}{3} = \sqrt[3]{a^1} = \sqrt[3]{a}$$

Example 6

$$d\frac{1}{2} = \sqrt[2]{d^1} = \sqrt[2]{d}$$

If you are entering for level 1 the following examples can be left out.

Example 7

$$27\frac{2}{3}$$

This should be done without a calculator.

Firstly, work out the cube root of 27 $= \sqrt[3]{27} = 3$

Then square this i.e. $3^2 = 9$

Example 8

$$16^{\frac{3}{4}} = \sqrt{16^3} = 2^3 = 8$$

Example 9

$$100^{\frac{3}{2}} = \sqrt{100^3} = 10^3 = 1000$$

FRACTIONAL AND NEGATIVE INDICES

Example 1

$$16^{-\frac{1}{2}} = \frac{1}{16^{\frac{1}{2}}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$$

Example 2

$$16^{-\frac{3}{4}} = \frac{1}{16^{+\frac{3}{4}}} = \frac{1}{\sqrt[4]{16^3}} = \frac{1}{2^3} = \frac{1}{8}$$

Example 3

$$100^{-\frac{3}{2}} = \frac{1}{\sqrt[2]{100^3}} = \frac{1}{10^3} = \frac{1}{1000}$$

Example 4

$$8^{\frac{1}{2}} \text{ x } 8^{\frac{3}{2}} = 8^{\frac{1}{2}+\frac{3}{2}} = 8^{\frac{4}{2}}$$
$$= 8^2 = 64$$

Example 5

$$8^{-\frac{1}{2}} \text{ x } 8^{\frac{1}{2}} = 8^{-\frac{1}{2}+\frac{1}{2}} = 8^0$$
$= 8^0 = 1$ (Anything to power zero is 1)

Example 6

$$\frac{3^3 \text{x} 3^2}{3^8} = \frac{3^5}{3^8} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

These are very important examples which you should know.

Exercise 1

1. $a^4 \times a^3$
2. $b^5 \times b$
3. $3s \times 4s^5$
4. 3^2
5. 3^{-3}
6. 8^2
7. 8^{-2}
8. 2^5
9. 2^{-5}
10. $16^{\frac{1}{2}}$
11. $9^{\frac{1}{2}}$
12. $32^{\frac{1}{5}}$
13. $27^{\frac{1}{3}}$
14. $81^{\frac{1}{4}}$
15. $81^{-\frac{1}{4}}$
16. $8^{\frac{2}{3}}$
17. $8^{-\frac{2}{3}}$
18. $100^{-\frac{3}{2}}$
19. $125^{\frac{2}{3}}$
20. a^0
21. $8y^0$
22. $4^{\frac{1}{2}} \times 4^{\frac{3}{2}}$
23. $9^{-\frac{1}{2}} \times 9^{\frac{1}{2}}$
24. $9^{-\frac{3}{2}} \times 9^{\frac{1}{2}}$
25. $10^0 \times 10^2 \times 10$

STANDARD FORM

Very large and very small numbers must sometimes be expressed in **STANDARD FORM.**

$A \times 10^n$

Where $1 < A < 10$ and n is an integer.
Translated (!) this means that A must be a number between 1 and 10 and n is a positive or negative number.

Here are some examples to clarify this.

Example 1

$87000 = 8.7 \times 10^4$
The display on the right of the equals sign is 87000 written in standard form.

Example 2

$0.000026 = 2.6 \times 10^{-5}$
If we talk about the decimal point moving to give us a number between 1 and 10, you will see that if the point moves to the LEFT, the power is POSITIVE.
If the point moves to the RIGHT, the power is NEGATIVE.

The number of PLACES the point moves, gives us the number in the power.

Example 3

$146.2 = 1.462 \times 10^{2}$
Point 'moves' to left – sign is +
Point 'moves' 2 places – power is 2.
Look at these examples and watch for these relationships.

Example 4

$26 = 2.6 \times 10$ which is 2.6×10^{1}

Example 5

$0.9 = 9.0 \times 10^{-1}$

Example 6

$265 = 2.65 \times 10^{2}$

Example 7

$0.0095 = 9.5 \times 10^{-3}$

Example 8

$5 \times 10^{-5} \times 3 \times 10^{2}$
Take this step by step as shown below:

First multiply the numbers **without powers**
$5 \times 3 = 15$

Secondly, multiply the number **with powers**
$10^{-5} \times 10^{2} = 10^{-5+2} = 10^{-3}$

Answer = 15×10^{-3}

NOW, this must be converted to standard form as shown:
$1.5 \times 10^{1} \times 10^{-3} = 1.5 \times 10^{+1-3}$
$= 1.5 \times 10^{-2}$ which is the answer.

Exercise 2

Write these in **STANDARD FORM**

1. 6500
2. 0.0082
3. 132.3
4. 0.5
5. 43
6. 2660000
7. 0.35
8. $0.7 \times 10^{5} \times 3 \times 10^{4}$
9. $6 \times 10^{3} \times 2 \times 10^{-2}$
10. $9 \times 10^{-1} \times 3 \times 10^{-1}$

ANSWERS

Exercise 1

1.	a^7	2.	b^6
3.	$12s^6$	4.	27
5.	$\frac{1}{27}$	6.	64
7.	$\frac{1}{64}$	8.	32
9.	$\frac{1}{32}$	10.	4
11.	3	12.	2
13.	3	14.	3
15.	$\frac{1}{3}$	16.	4
17.	$\frac{1}{4}$	18.	$\frac{1}{1000}$
19.	25	20.	1
21.	8	22.	16
23.	1	24.	$\frac{1}{9}$
25.	1000		

Exercise 2

1.	6.5×10^3	2.	8.2×10^{-3}
3.	1.323×10^2	4.	5×10^{-1}
5.	4.3×10	6.	2.66×10^6
7.	3.5×10^{-1}	8.	2.1×10^9
9.	1.2×10^2	10.	2.7×10^{-1}