Learning Development

University of Northampton

# INEQUALITIES

**There are four signs which we use in dealing with inequalities.**

$>$ means greater than
$\geq$ means greater than or equal to
$<$ means less than
$\leq$ means less than or equal to

**Example**

6 > 2 reads 6 is greater (or bigger) than 2.
-3 < 0 reads minus 3 is less (or smaller) than 0.

**RULES OF INEQUALITIES**

## Note

1. $>$ greater than

E.g. $c > 5$ Reads c is **GREATER** than 5, c could be 6 or 100 or even $5\frac{1}{2}$ but could not be 4.

2. $<$ less than

E.g. $c < 3$ Reads c is **LESS** than 3, c could be 2 or -6 or even $2\frac{3}{4}$ but could not be 4.

3. $\geq$ greater than or equal to

E.g. $y \geq 7$ Reads y is **GREATER** than or **EQUAL** to 7. y could be 7 or 100 or even $7\frac{1}{2}$ but could not be 6.

4. $\leq$ less than or equal to

E.g. $y \leq 0$ Reads y is **LESS** than or **EQUAL** to zero. y could be 0 or -5 or even -0.1 but could not be 1.

**Remember the arrow always points to the smaller quantity**

5. The equation $c = 6$ has only **ONE** solution, but in an inequality such as $c < 6$, c can have **ANY** numerical values less than 6.

6. Inequalities can be solved in similar ways to equations.

**Example 1**

Consider the inequality $2c > 6$ (Read 2c is greater than 6)

1. Add an equal quantity to both sides
$2c + 4 > 6 + 4$ i.e. $2c + 4 > 10$

2. Subtract an equal quantity from both sides
$2c - 6 > 6 - 6$ i.e. $2c - 6 > 0$

3. Multiply each side by the same **POSITIVE** number
$2c(3) > 6(3)$ i.e. $6c > 18$

4. Divide each side by the same positive number
$2c + 2 > 6 + 2$ i.e. $c > 3$

After each of the operations 1 to 4 the inequality remains unaltered.

**MULTIPLYING AND DIVIDING BY A NEGATIVE NUMBER**

**Note** $3 < 7$

When both sides of the inequality are multiplied by a negative number, for example -2, the inequality becomes

$-6 < -14$

**BUT** this is **NOT TRUE** if the inequality sign is reversed

$-6 > -14$

the statement is now **TRUE**

Similarly $20 > 15$

If both sides of the inequality are divided by a negative number, for example -5, **the inequality sign needs reversing.**

$20 > 15$
+ (-5) $-4 < -3$

## Summary

**When MULTIPLYING or DIVIDING an inequality by a NEGATIVE number.**
**the inequality sign must be reversed.**

Read through the following examples.

1. Solve for c: $2c + 4 > 10$

   Treat this like a simple equation but replace the equals sign with >.

   $2c + 4 > 10$ See pack on Linear Equations

   $2c > 10 - 4$

   $2c > 6$

   $c > \frac{6}{2}$

   $c > 3$

   On a number line this would be represented as

2. Solve for c: $16 \leq 3c - 5$

   $16 + 5 \leq 3c$

   $21 \leq 3c$

   $\frac{21}{3} \leq c$

   OR

   $7 \leq c$

   $c \geq 7$

   On a number line this would be represented as

3. Solve for c: $3c + 15 > 4c - 6$

   $15 + 6 > 4c - 3c$

   $21 > c$

   On a number line this would be represented as

4. Solve for c: $16 < 4 - 2c$

   $16 - 4 < -2c$

   $\frac{12}{2} < -c$

   $6 < -c$

   **To find +c MULTIPLY BOTH SIDES of the INEQUALITY by -1. REMEMBER THIS CHANGES THE INEQUALITY SIGN.**

   $-6 > c$

   OR

   $c < -6$

   On a number line this would be represented as

**Exercise 1**

Solve the following for c

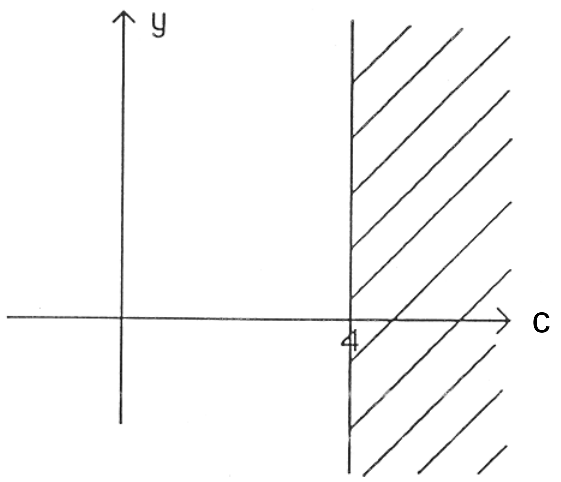
1. $3c + 4 > 16$
2. $3 + 5c > 23$
3. $14 < 10 - 2c$
4. $6 \leq 8c - 18$
5. $5 > 2 + 3c$
6. $2c - 7 > -9$
7. $-4 \leq 5 - 3c$
8. $-3 < 7 - 2c$
9. $5c - 4 \geq -14$
10. $2c - 3 \leq 1$

**GRAPHS OF INEQUALITIES**

You can indicate on a diagram the regions represented by inequalities, by shading as shown below.

**Example 1**

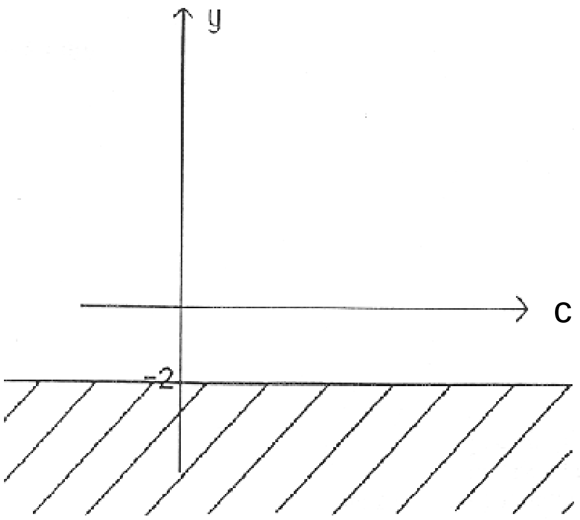
$c \geq 4$




Draw the line c = 4 and shade to the right of it as the values of c on this side are bigger than 4.

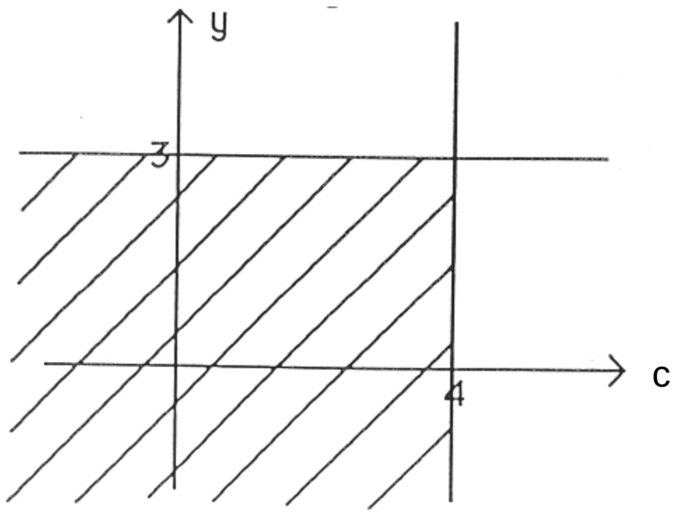
**Example 2**

$y \leq -2$



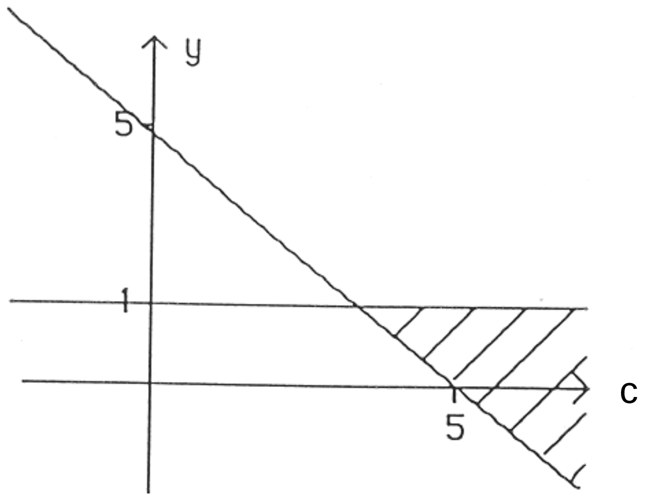

**Example 3**

Denote by shading the region where $c \leq 4$ and $y \geq 3$



**Example 4**

Indicate, by shading, the region which represents c > 0, y < 1 and c + y ≥ 5.
First we have to draw the line c + y ≥5.
See Unit on Simple Graphs.
Rearranging c + y ≥ 5 we get y ≥ -c + 5.
Draw this line as shown and shade the appropriate area where y ≥ -c + 5, c > 0 and y < 1 intersect.



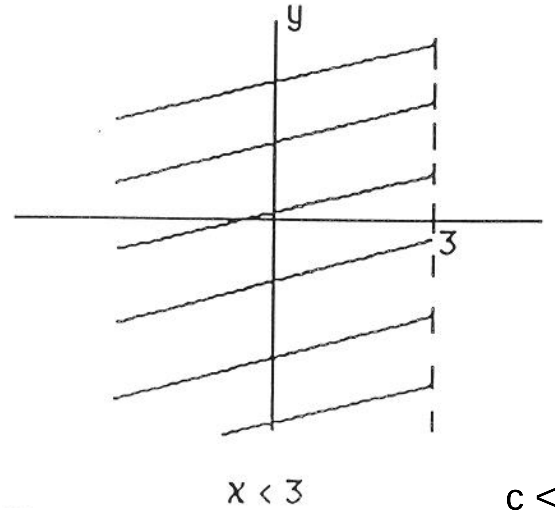

**Exercise 2**

Shade on separate diagrams

1. c ≥ 2
2. y ≤ 3
3. c ≥ -3
4. y ≥-1
5. c ≥0
6. y ≥4,c ≤ 2 indicate the required region by shading
7. c ≥-3, Y ≤2 indicate the required region by shading
8. c ≤ 3, Y ≥ 4, c + y ≥ 2 indicate the required region by shading
9. c ≥ 2, Y ≤ 5, 2c + y ≥ 6 indicate the required region by shading
10. c ≤ 0, Y≥0 , c + y ≤ 5 indicate the required region by shading

So far all the inequalities have been ≤ or ≥. There is a convention for writing < and >. Instead of a complete line a broken line is used.

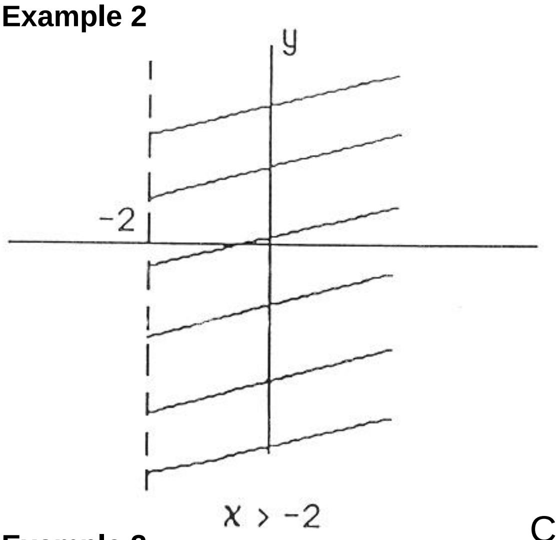
**Example 1**




c

c < 3

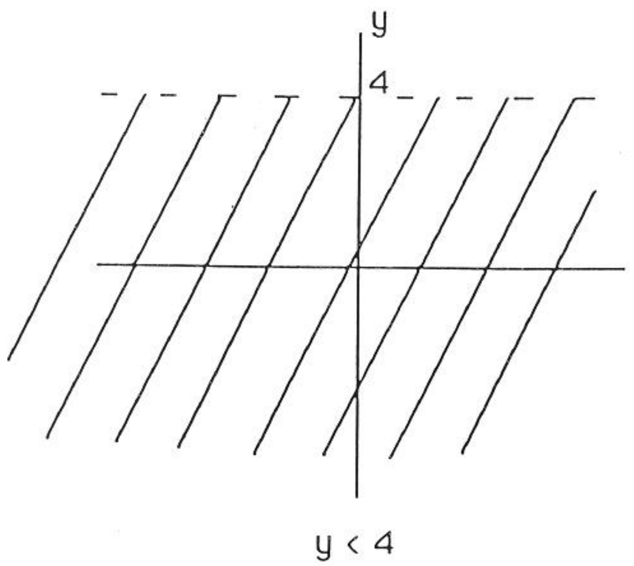
**Example 2**




c

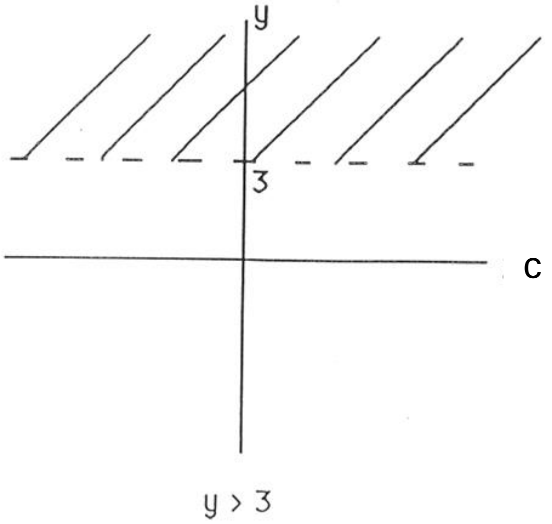
C > -2

**Example 3**

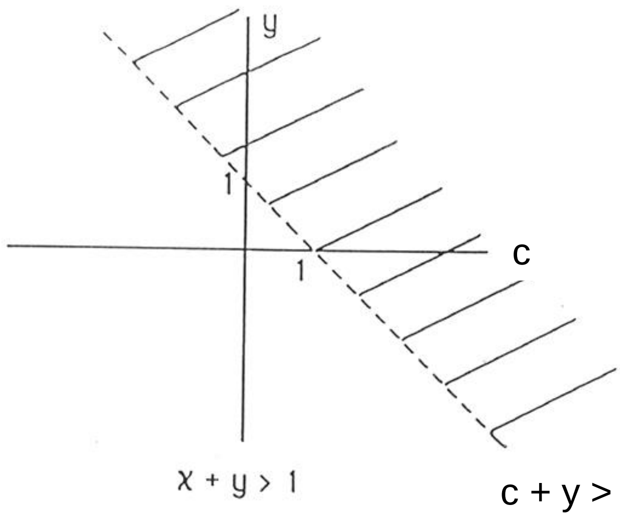



c

**Example 4**

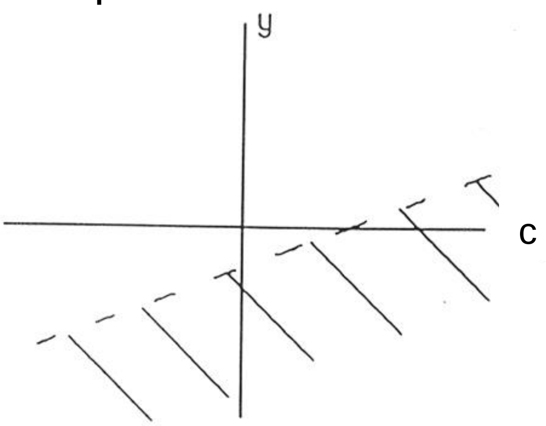



**Example 5**




c + y > 1

**Example 6**




y < ½ c – 2

2y < c - 4

**Exercise 3**

Shade on separate diagrams

1. $c < 4$
2. $y > -3$
3. $Y > 2$, $c < -2$ indicate the required region by shading
4. $c \geq 3$, $Y < 2$ indicate; the required region by shading 5. $c + y > 2$
6. $c + y > 1$, $c \geq 2$ indicate the required region by shading
7. $c > 1$, $Y \leq 2$, $c + y \geq 5$ indicate the required region by shading

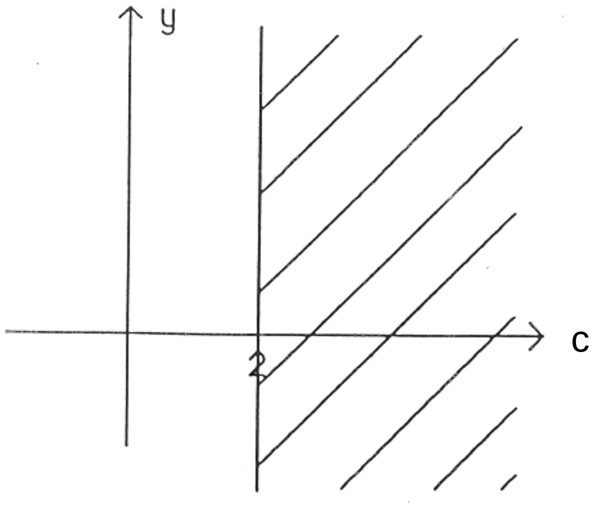
## ANSWERS

### Exercise 1

1. $c > 4$
2. $c > 4$
3. $c < -2$
4. $3 \leq c$ **OR** $c \geq 3$
5. $1 > c$ **OR** $c < 1$
6. $c > -1$
7. $c \leq 3$
8. $c < 5$
9. $c \leq -2\ 1$
10. $c \leq 2$

### Exercise 2

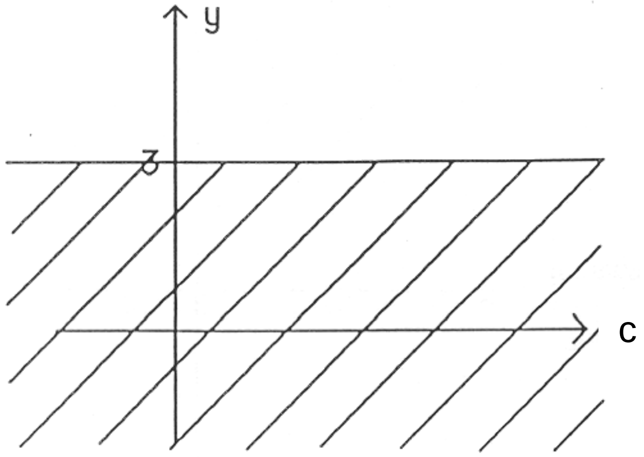
1.



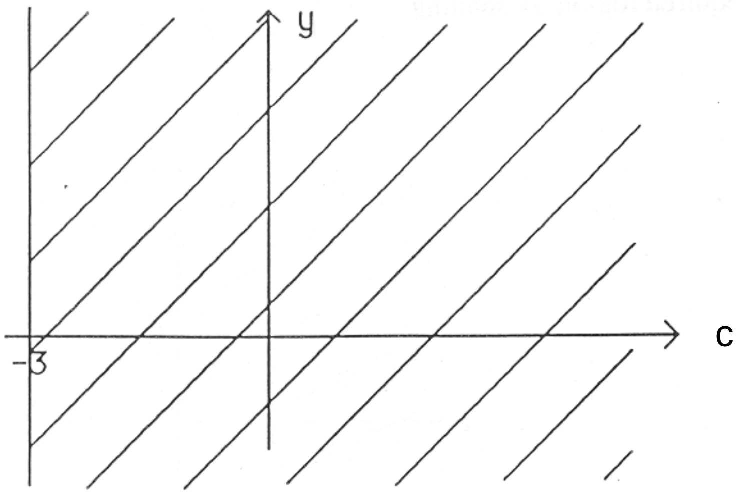
Learning Development

University of Northampton

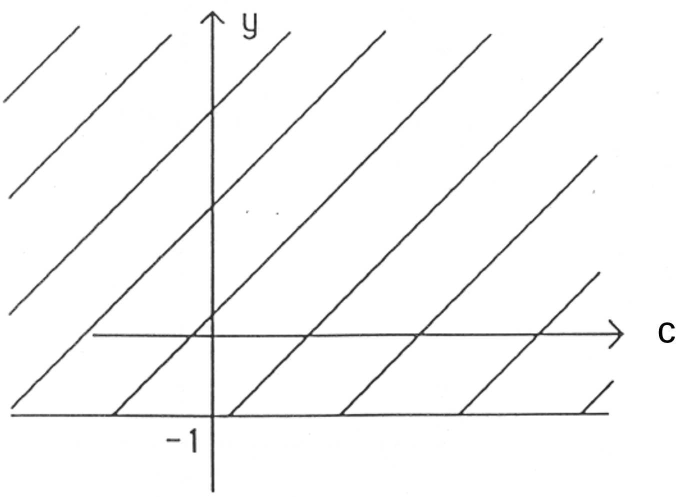
2.



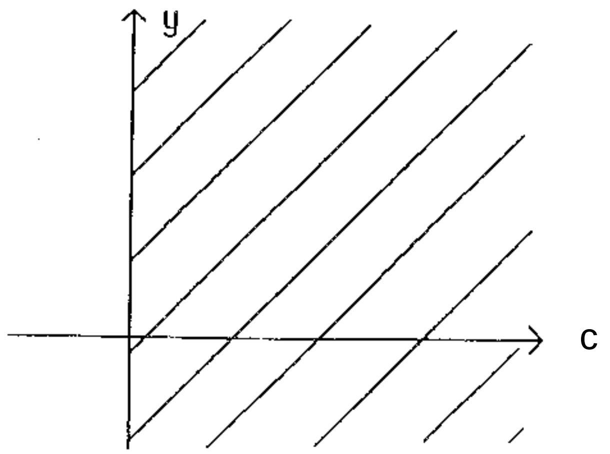

3.



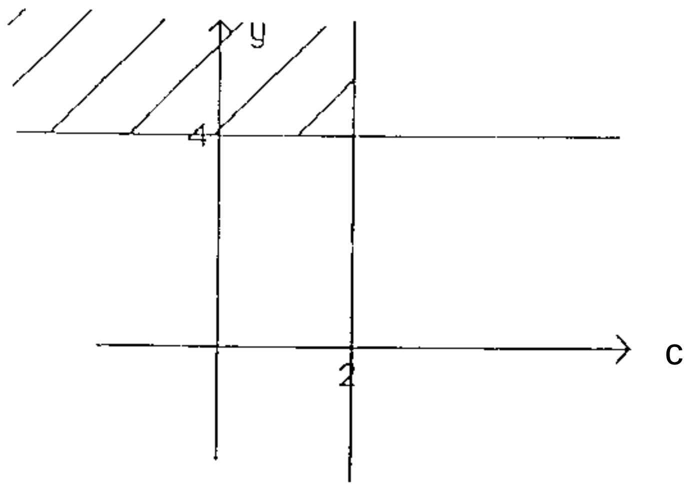

4.



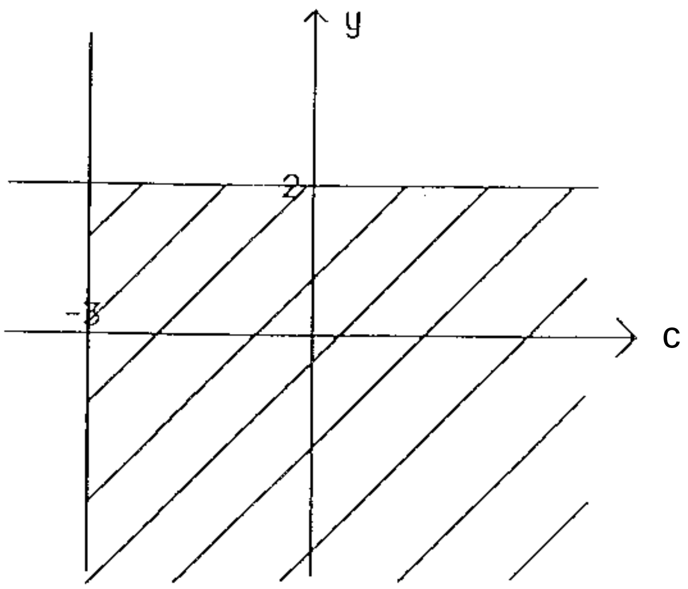
5.



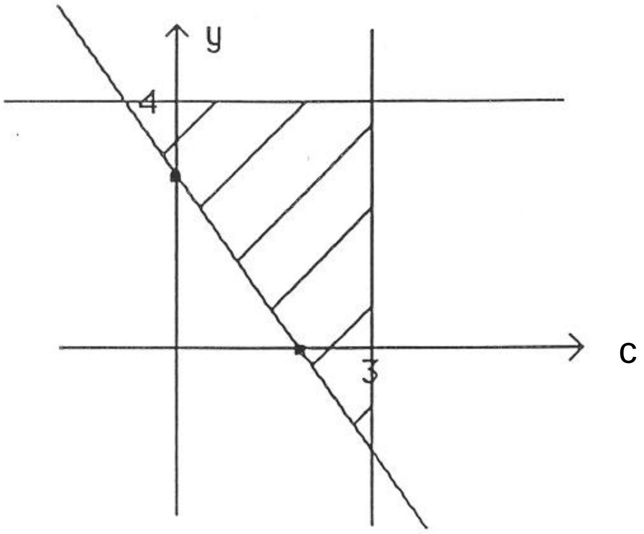

6.



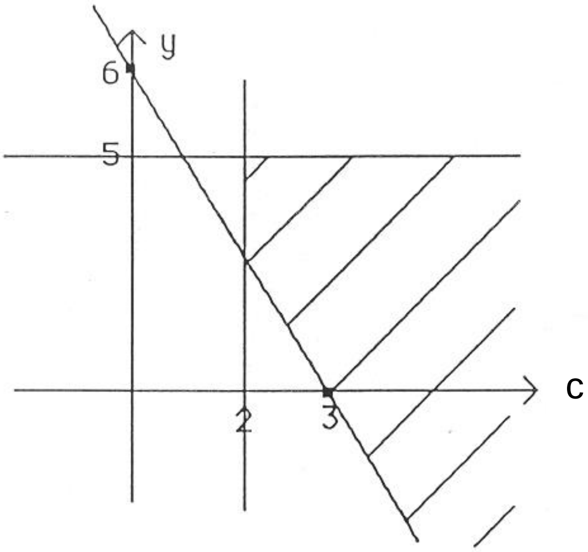

7.



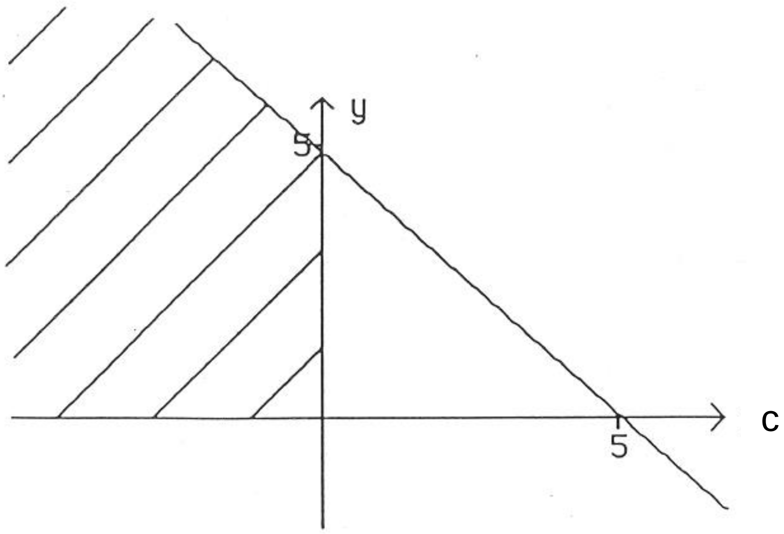
8.




9.

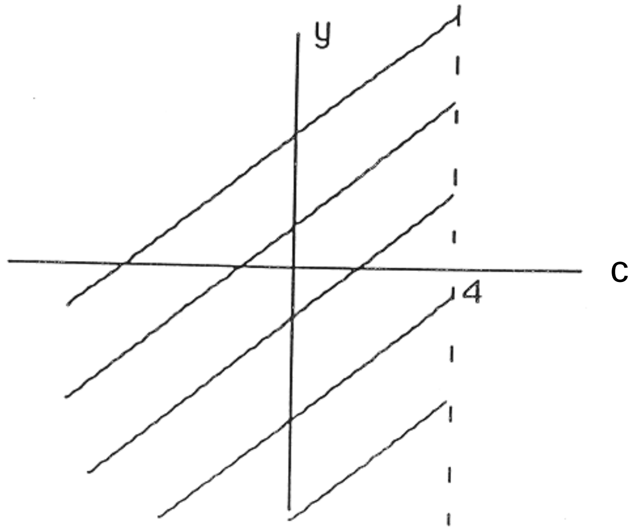



10.

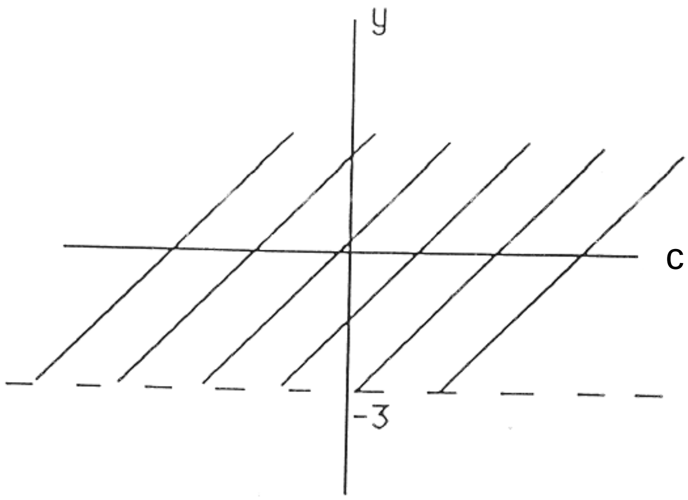


Exercise 3

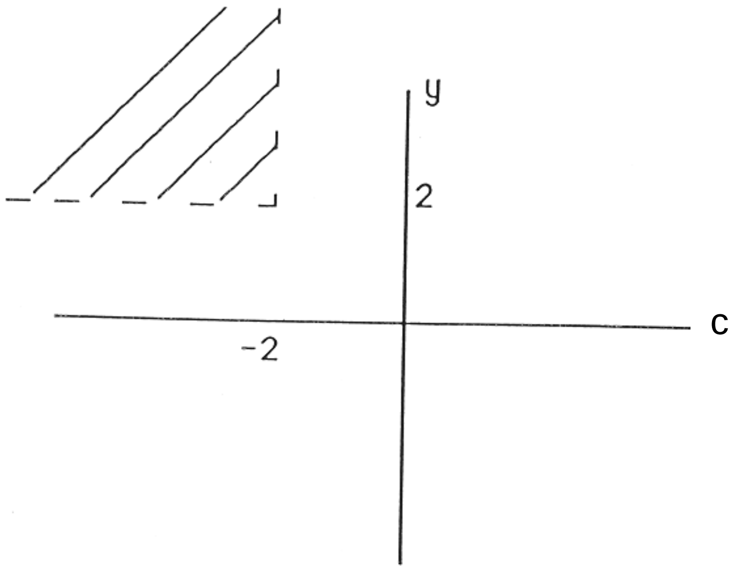
1.



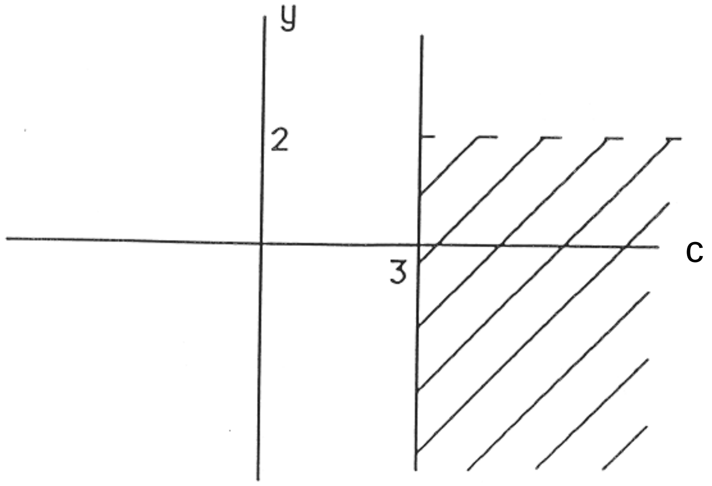

2.



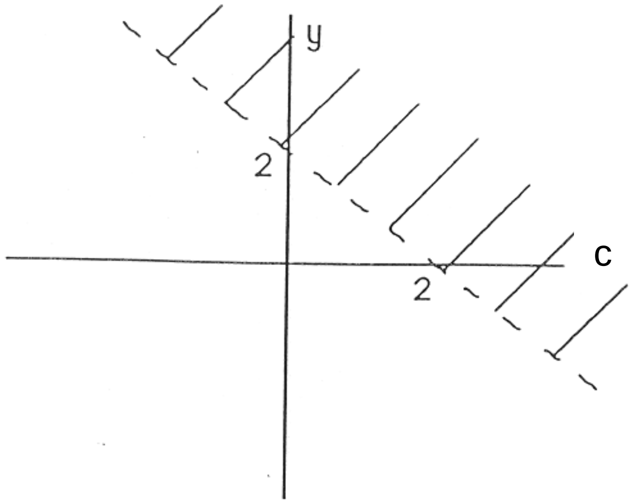

3.



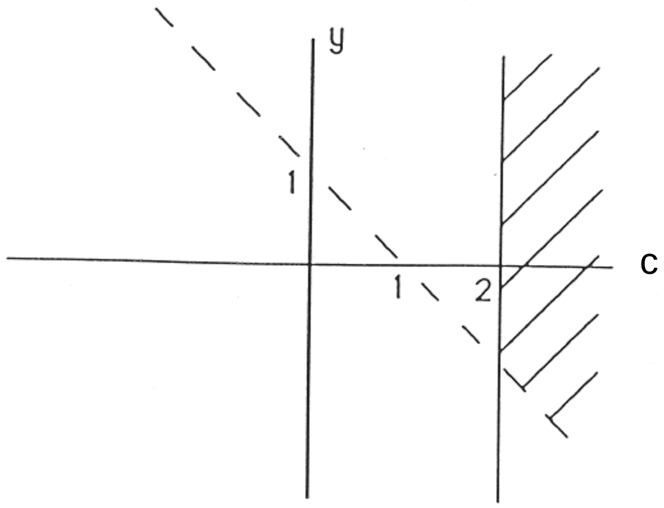
4.




5.




6.



7.