ROTATIONS AND REFLECTIONS USING MATRICES

Earlier in your course you looked at a variety of ways in which a shape could be moved around on squared paper. We studied:

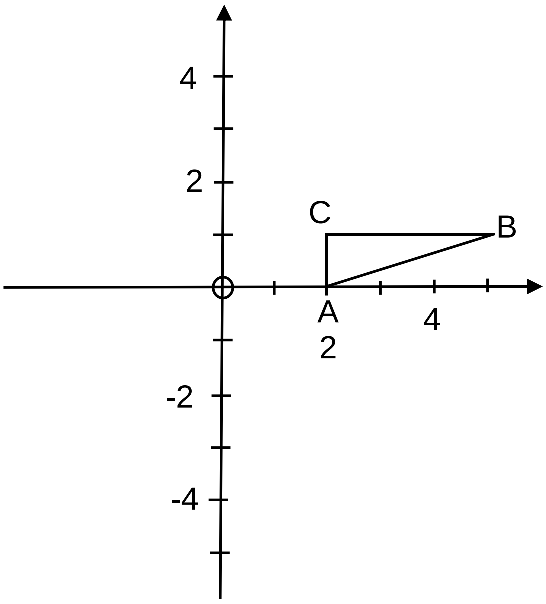
translation **reflection** **rotation**

In each of these the size of the original shape remained fixed. We also looked at the process of enlargement in which the size of the shape did change becoming either larger or (rather strangely) smaller.

These processes are all types of **transformation.** The original shape is often called the **object**. The final shape after transformation can be called the image. Work through the following example for revision.

Example 1

A triangular tile is shown below resting on a sheet of graph paper:

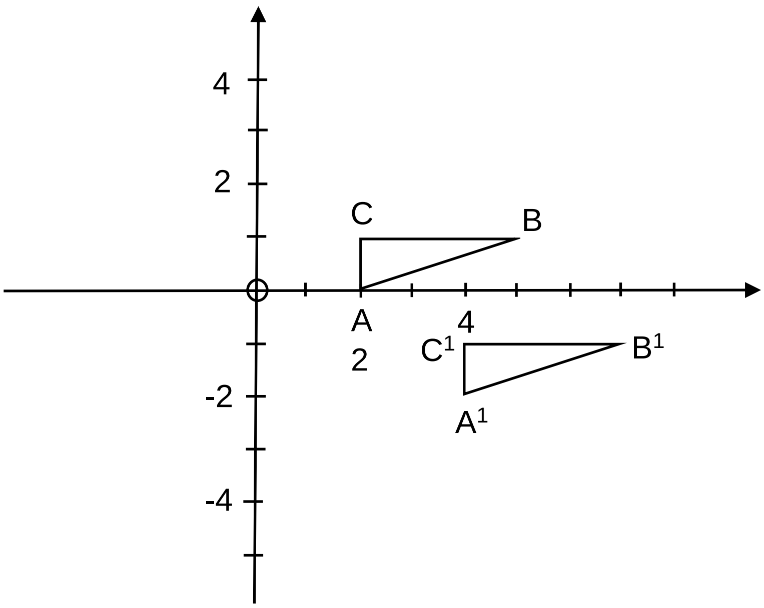


Find the image of ABC under each of the following transformations:

1. translation $\begin{matrix}2\\-2\end{matrix}$
2. rotation of 90^0 about the origin
3. reflection in the y axis
4. enlargement, centre the origin, scale factor 2.

Solution

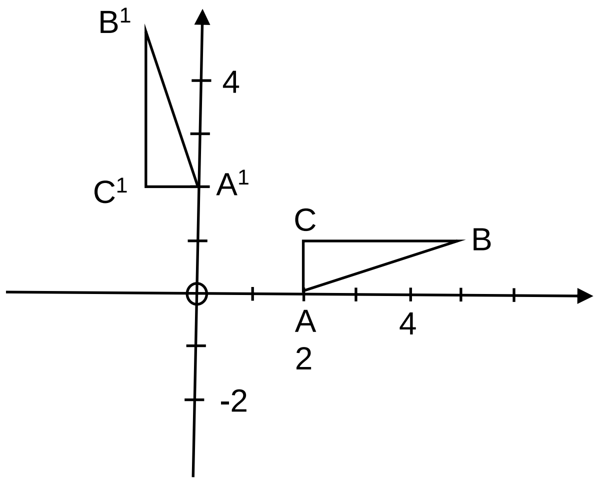
1. translation 2 means "move the triangle 2 units along parallel to the x axis and -2 parallel to the y axis". Diagram as below Ø



The tile appears to **slide** over the paper.
N.B. The sides of the image are parallel to the corresponding sides of the object and in the same direction.

2. Rotation of 90^0 about the origin.
Positive rotations are **anti clockwise** **Negative** rotations are **clockwise**

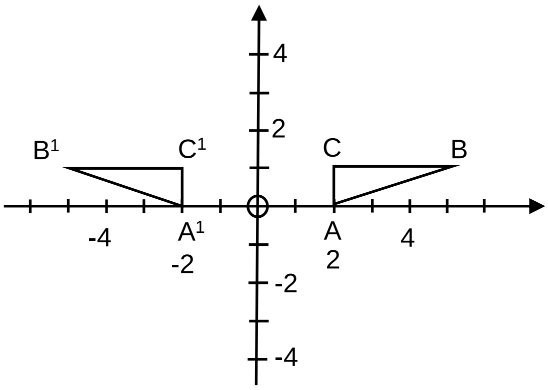
We imagine putting a pin through the origin and rotating the diagram through a right angle.



Notice that the sides of the image are pointing in a **different** direction from those of the object. The angle of rotation can be measured by looking at how any point has moved.

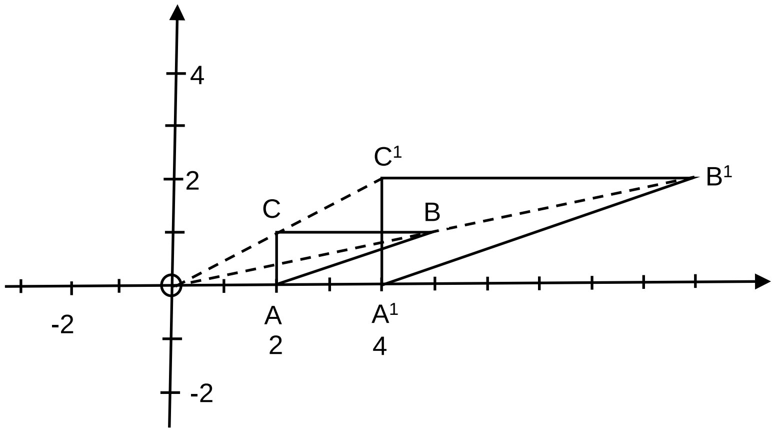
$$A\hat{O}A' = B\hat{O}B' = C\hat{O}C' = 90^0$$

3. Reflection in the y axis. This time we imagine folding the paper along the y axis. The tile will "flip over".



This time we see that we obtain a mirror image of the tile. The original shape has turned over. The circular pattern on the object is reversed in the image, just as left and right are reversed when we look in a mirror. We could say there has been a change in "handedness".

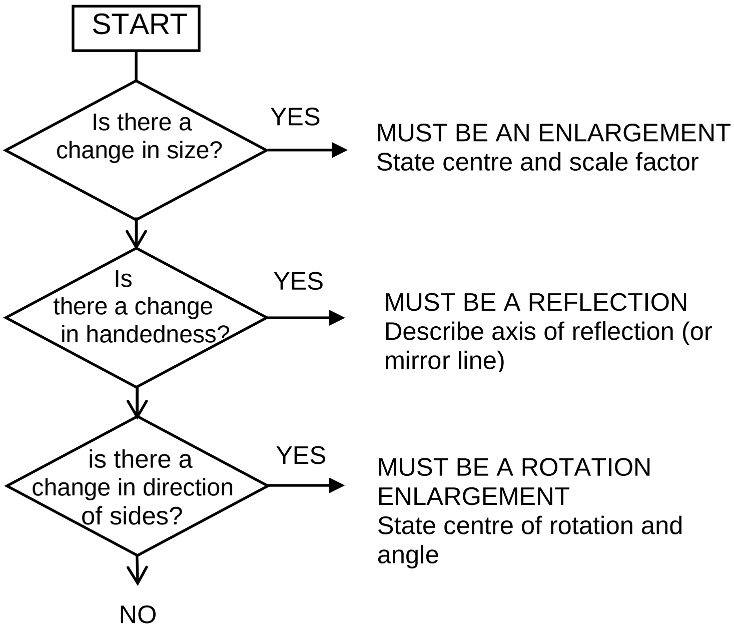
4. Enlargement, centre(). Scale factor 2. In this case we join each point to the centre and stretch by a factor of 2.



The most obvious feature is the change of **size.** Each side in the image is twice the length of the original.

You may also find problems where you are given the starting and final positions and you must describe which transformation has taken place!

When studying the object and image it may help you to use the following flow chart.

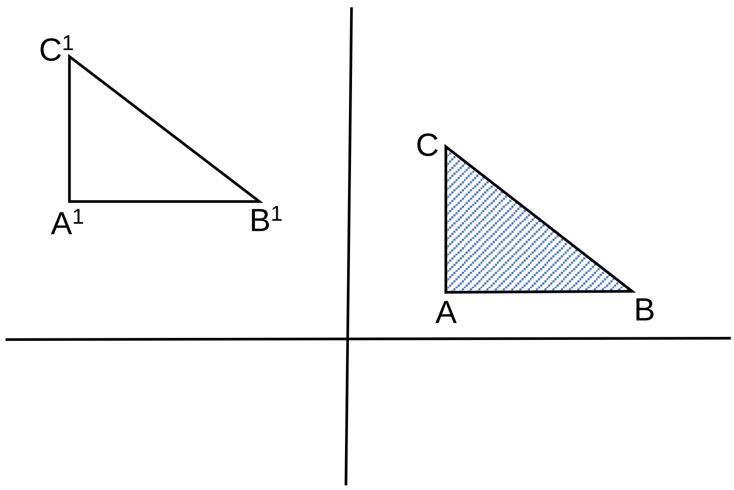


MUST BE A TRANSLATION

State distance moved parallel to x and y axes as a column vector.
Now work through example 2. (Cover the right-hand side of the page and decide for yourself before looking at the solution).

Example 2

In each triangle the shaded triangle ABC has been transformed into image 'A' 'B' 'C'. Describe <u>fully</u> the transformation used in each case.

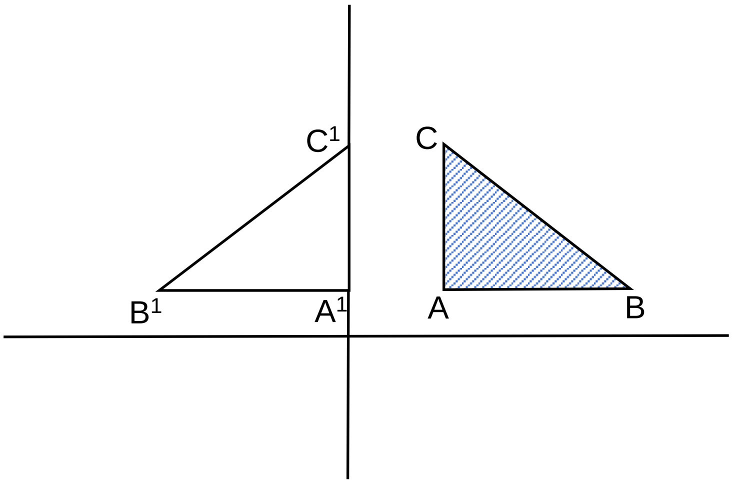


Change of size? No
Change of handedness? No
Change of direction? No

This is a TRANSLATION

$$\begin{pmatrix} -6 \\ 1 \end{pmatrix}$$

2.

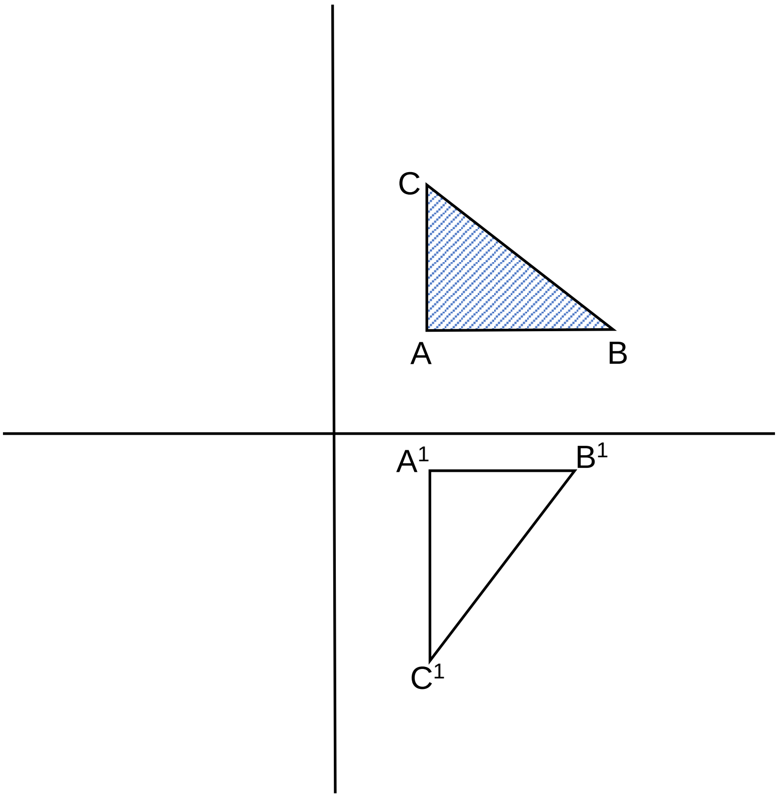


Change of size? No
Change of handedness? Yes

This is a REFLECTION

Mirror line is $x = 1$

3.

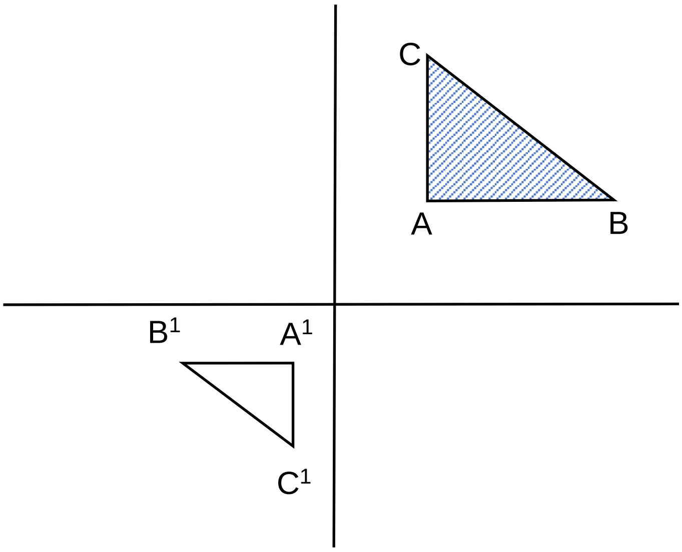


Change of size? No
Change of handedness? No
Change of direction? Yes

This is a ROTATION

Is centre at origin? Yes
Angle of rotation
$A0A^1 = -90^0$

4.



Change of size? Yes

This is ENLARGEMENT

Image is upside down and smaller.

Scale factor is $-\frac{1}{2}$

Where is centre?
Is it at origin?

AA^1, BB^1 and CC^1 all pass through 0

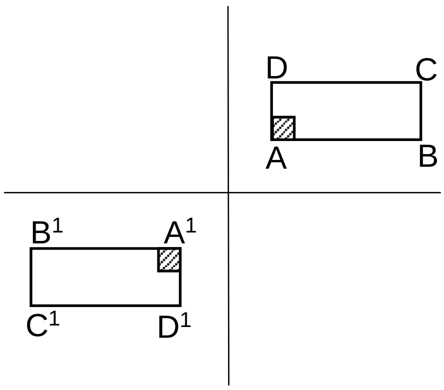
∴ Enlargement centre 0.
Scale factor $-\frac{1}{2}$

Now try the following exercise.

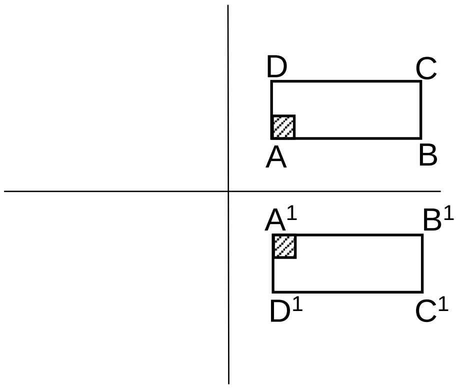
Exercise 1

1. P is point (3,2). Q is (3,4). R is (6.2). Using graph paper, draw DPQR. Find the image of DPQR under the following transformation.
   a) translation $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$
   b) reflection in line y = 1.
   c) rotation of 180^0 about (0,0).
   d) enlargement centre 0 scale factor $\frac{1}{2}$
   e) reflection in line y = $-x$.

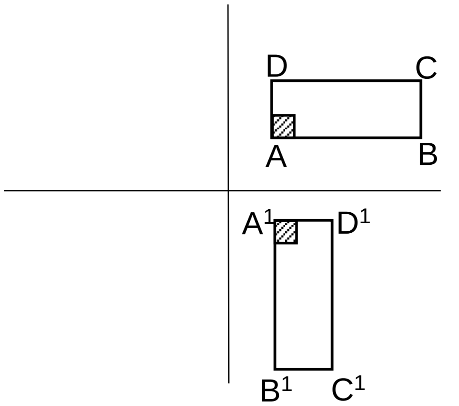
2. For the diagrams below describe completely the transformation which will take rectangle ABCD to $A^1 B^1 C^1 D^1$.
   a)



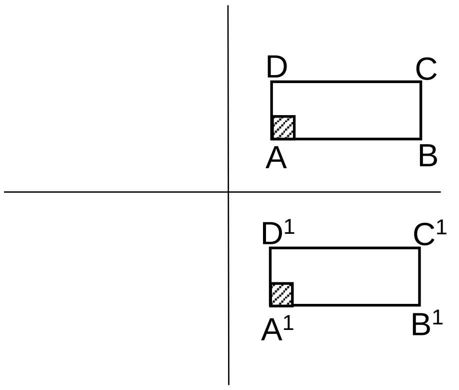
b)



c)



d)



Some harder examples

ROTATION AND ENLARGEMENT USING CENTRES OTHER THAN THE ORIGIN

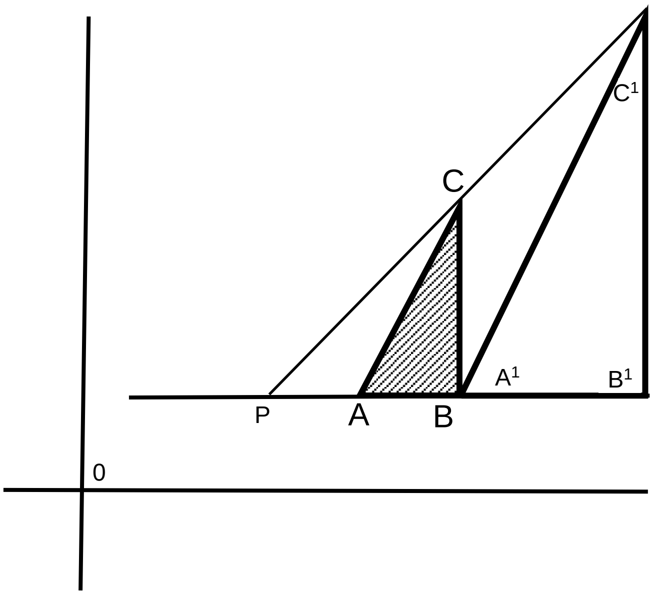
Usually the origin is used as centre but occasionally some other point may be used.

Example 1

Find the image of the triangle in the diagrams below

1. after enlargement scale factor 2 centre P(2.1)
2. after rotation of -90^0 about P (2.1)

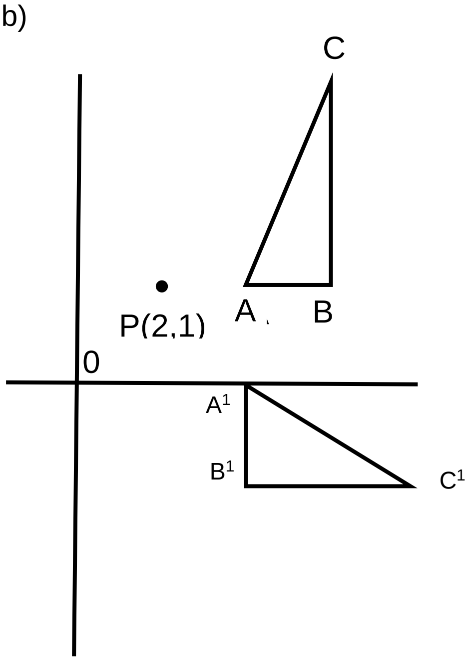
a)



Join P to A and then extend to twice length.
Similarly join P to B and extend:
Join P to C and extend.

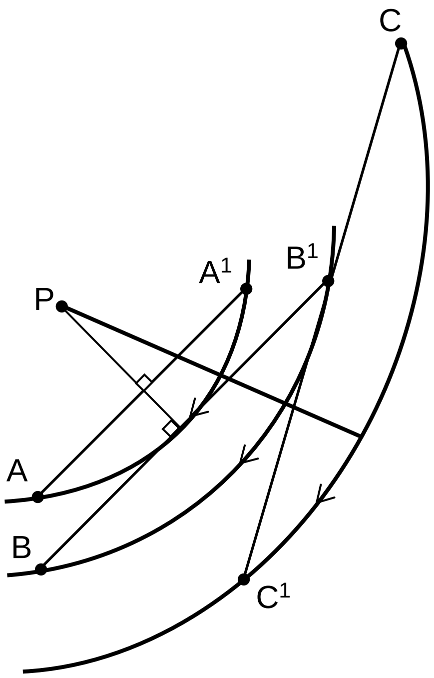
Notice that AA', BB' and CC' all pass through P.

b)



With a pin at (2,1) imagine rotating the triangle clockwise.
A will move on a **circular** path to A^1, B will move to B^1. C will move to C^1.

c)

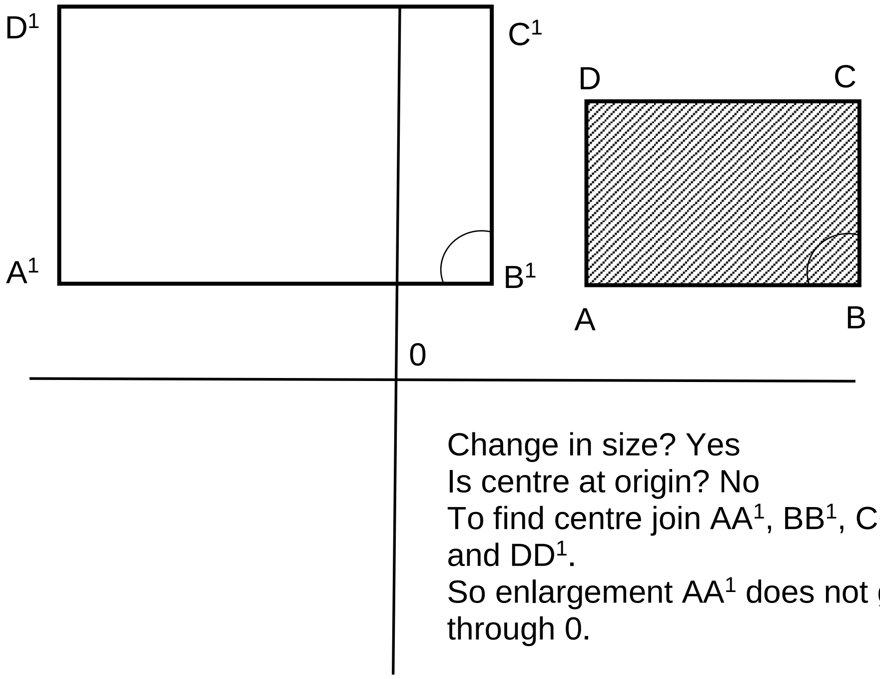


Notice that A and A’ are at equal distances from P. So, P lies on the perpendicular bisector of AA’. Also, P lies on perpendicular bisector of BB’ and CC’.

Example 2

In the diagrams below the shaded rectangle ABCD has been transformed into image A’B’C’D’. Describe fully the transformation in each case.

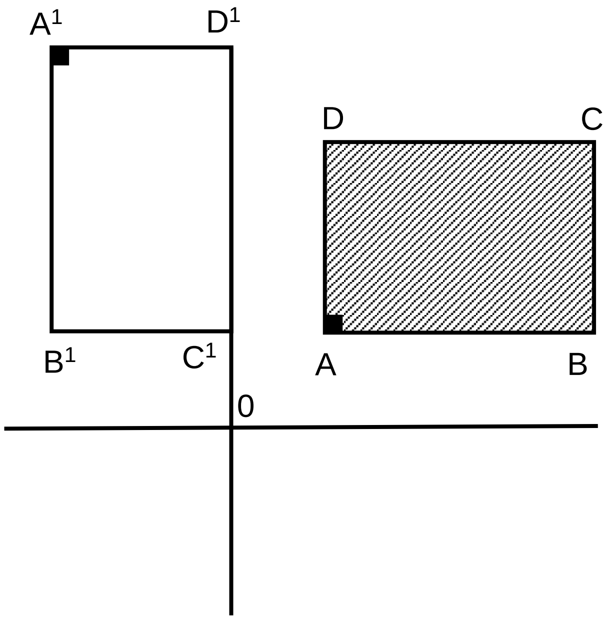
a)



Change in size? Yes
Is centre at origin? No
To find centre join AA^1, BB^1, CC^1 and DD^1.
So enlargement AA^1 does not go through 0.

Find where lines intersect. Try this.
We have enlargement scale factor 1 – 5 centre (13.1).

b)



Change of size?
Change of handedness? No
Change of direction? Yes

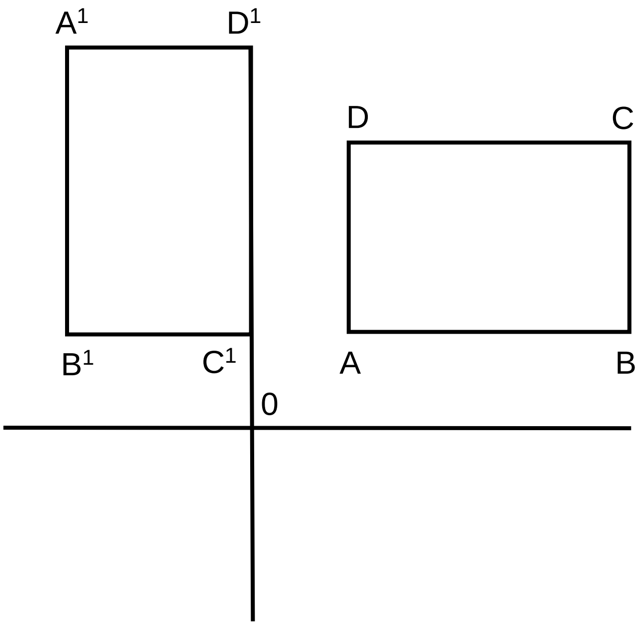
So we have ROTATION.

Is centre at 0?
No, because 0A is not equal to $0A^1$.

We need to find a point at equal distance from A and A^1, B and B^1 etc.

Join A to A' and bisect at right angles. Join B to B' and bisect at right angles. Point where bisectors intersect will give centre rotation. This is (2.5). Compare with example 3(b). (Try this on graph paper).

c)



So, we have rotation centre (2,5).

To find angle look at APA^1
This is 90^0 clockwise.

Therefore, we have rotation of 90^0 centre (2,5).

Learning Development

University of Northampton

MATRICES AND TRANSFORMATIONS

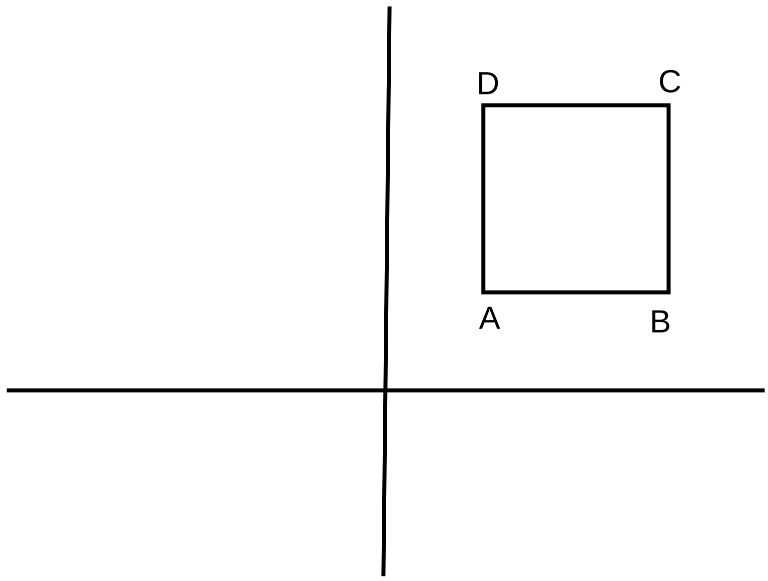
Now that we have reminded ourselves about transformations we will look at the way in which matrices provide a neat way of describing them. You have already done this for translations. When we described the translation of the previous section as

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad \begin{pmatrix} -6 \\ 1 \end{pmatrix}$$

We were using matrix shorthand. Each translation is described by a (2x1) matrix. In many problems we can set down our work neatly by also describing the positions of each point by a (2x1) matrix.

Example 1 (This is example 4 from the pack on Matrices)

Find the image of the shape ABCD where A is (2,4), B is (6,4), C (6,2), D(2,2) under the translation $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$



We see that we need to increase each x coordinate by 4 and each y co-ordinate by 5. Matrices can only be added when they have the same order so it will be convenient to use a new shorthand for the positions of the points.

We will represent position of A (2,4) by a column matrix.

$\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ (This is sometimes called a position vector).

Notice that we are just giving the same information about the position as before but using a new shorthand.

In the same way, position of B by $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$

position of C by $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$

position of D by $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$

Then calculations can then be represented as the **addition** of **two column matrices**.

A becomes $\begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$ i.e. A' is (6,9)

B becomes $\begin{pmatrix} 6 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 10 \\ 9 \end{pmatrix}$ B' is (10, 9)

C becomes $\begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 10 \\ 7 \end{pmatrix}$ C' is (10, 7)

D becomes $\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$ D' is (6, 7)

Compare this with the method used in the Matrices pack.

NB When using matrix methods we will always represent the position of a point by a 2 x 1 column matrix.

Exercise 2 - Translations

DPQR has vertices P (1,1) Q (2,1) R (2,3). Using column vector rotation find the image of DPQR under the translations:

a) $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

b) $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$

Check with a sketch.

MATRICES AND TRANSFORMATIONS II

A matrix method worked for describing translations but we now want to see whether we can use matrices to describe other types of transformation.

Example 1

Investigate the effect of **multiplying** the position vectors of A, B, C, D by the matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Where A is (2,4) B is (6,4)
C is (6,2) and D is (2,2) as before

Solution

Position vector of A is $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

We wish to calculate

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{x} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

We have a (2 x 2 matrix followed by a (2x1) matrix.

This is possible and the result is another 2 x 1 matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{x} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} (1\text{x}2) + (0\text{x}4) \\ (0\text{x}2) + (-1\text{x}4) \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

In the same way for B we find

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{x} \begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$$

For C

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{x} \begin{bmatrix} 6 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

And for D

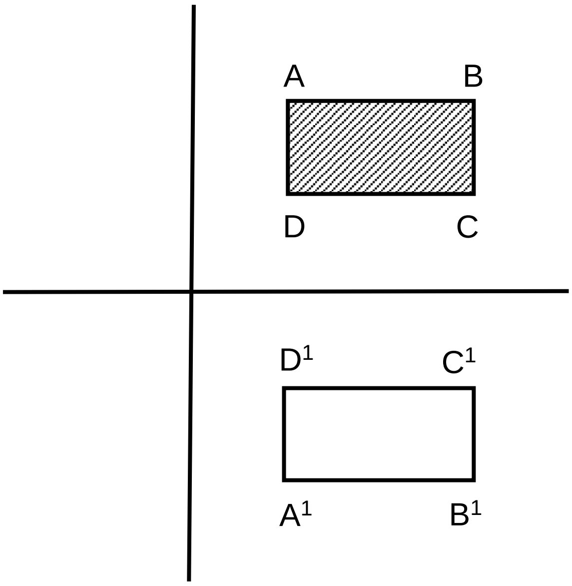
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{x} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

We find result for A is $\begin{bmatrix} 2 \\ -4 \end{bmatrix}$

corresponding to the point (2, -4). Call it A' multiplying by the matrix changes (2, 4) into (2, -4).

Similarly B gives the result (6, -4) B',
C gives the result (6, -2) C',
D gives the result (2, -2) D'.

Let us draw a sketch.



We see that this corresponds to reflecting the original rectangle ABCD in the x axis!

Multiplying by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ has had the same effect as a **reflection in the x axis.**

Exercise 3

Investigate the effect of multiplying the position vectors of A,B,C,D by each of the following matrices. In each case describe the corresponding transformation in words.

a) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

e) $\begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix}$

REPRESENTING STANDARD TRANSFORMATIONS BY MATRICES

We have seen in Exercise 3 that certain matrices formed from 0, +1, -1 can be associated with standard transformations.

We need to be able to work out the transformation given any matrix and vice versa.

We saw in the exercise that it is possible to do this by multiplying a set of given co-ordinates by the matrix and drawing a sketch. If no shape is actually given you may choose one yourself and the unit square is usually easiest, i.e. (0.0), (0.1), (1.0) and (1.1).

Example 1

What transformation is associated with $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Solution

Find the effect of $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ on the “unit square”.

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \underset{\text{origin}}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} = \underset{\text{origin}}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} \qquad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \underset{P}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} = \underset{P}{\begin{bmatrix} 0 \\ -1 \end{bmatrix}}$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \underset{R}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} = \underset{R^1}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} \qquad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \underset{Q}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}} = \underset{Q^1}{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

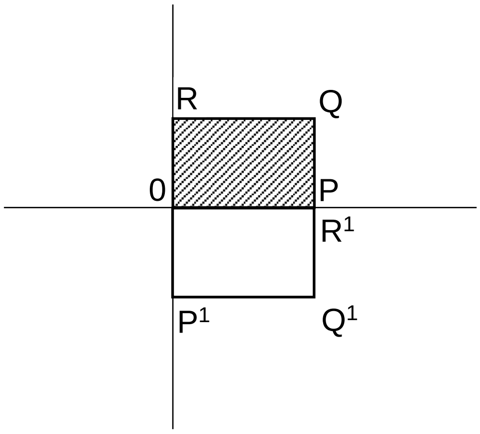
i.e.

$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ becomes $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ becomes $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ becomes $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

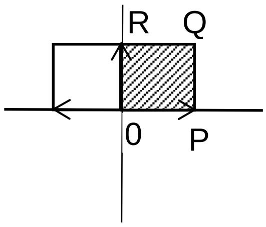
$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ becomes $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$



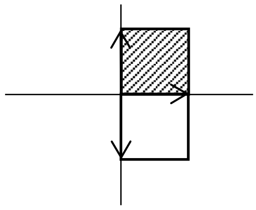
This is Rotation of -90^0 about origin.

Please see the following page for a summary of results for other transformations.

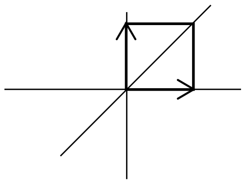
$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ Reflection in y axis



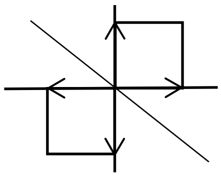
$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ Reflection in x axis



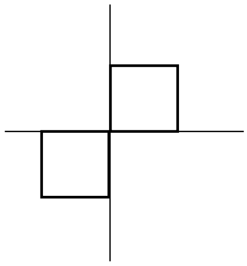
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Reflection in y = x



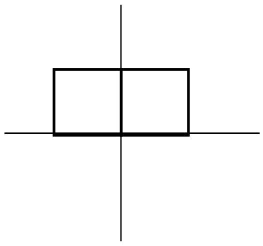
$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ Reflection in y = -x



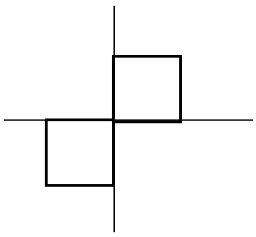
$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ Rotation of -90^0 about origin
i.e. 90^0 clockwise



$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ Rotation of +90^0 about origin
i.e. 90^0 anticlockwise



$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ Rotation of 180^0 about origin



$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Identity: No change

NB: When you have had some practice you may like to notice a **shortcut**.

The first column of the transformation matrix always tells us what will happen to $\begin{bmatrix}1\\0\end{bmatrix}$ position vector i.e. OP in the diagrams.

The second column tells us what will happen to $\begin{bmatrix}0\\1\end{bmatrix}$ i.e. OQ in the diagram.

Thus $\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$ turns (1,0) into (0.1) [first column] and (0,1) into (-1, 0) [second column].

This is particularly useful when working backwards from the transformation to the matrix.

Example 2

Which matrix is associated with a rotation of 180^0.

Solution

When rotated through 180^0 the point (1, 0) becomes (-1, 0)

So first column is $\begin{bmatrix}-1\\0\end{bmatrix}$ (0, 1) becomes (0, -1) so second column is

$\begin{bmatrix}0\\-1\end{bmatrix}$ Matrix is $\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$

COMBINATIONS OF TRANSFORMATIONS

We have seen in earlier work that a combination of two transformations may often be replaced by a single transformation.

Example 1

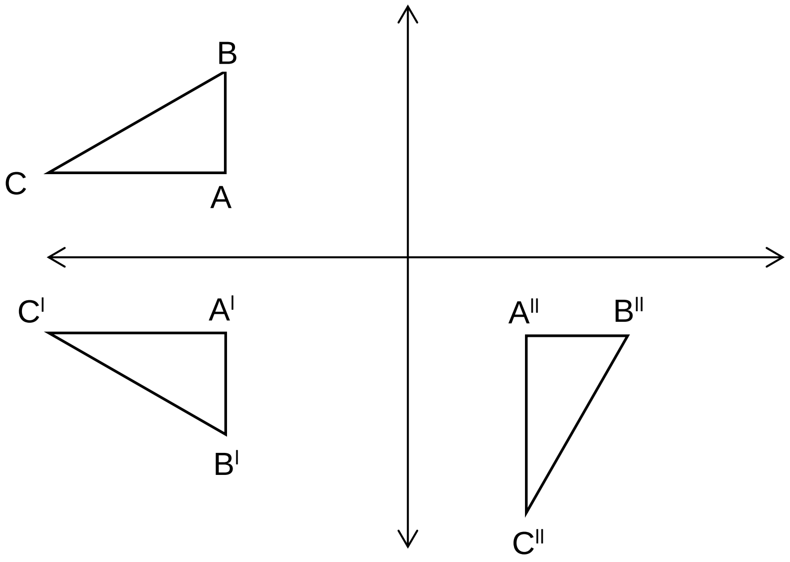
Triangle ABC has co-ordinates A (-5.2), B (-5. 4) and C (-9.2).

a) Find the image of ∆ ABC after reflection in the x axis. Call the image ∆ A'B'C'.

b) Find the image of ∆ A'B'C' after a rotation of +90^0 about the origin. Call this ∆ A"B"C".

c) What single transformation maps ∆ ABC on to ∆ A"B"C".

Solution



The single transformation taking ∆ ABC to ∆ A"B"C" is reflection in the line Y = -x. We can check this out using matrices. Let us look at what happens to point A (-5, 2). Reflection in the y axis is represented by

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$; rotation of + 90^o is $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

To find A' the image after reflection we calculate $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -5 \\ 2 \end{bmatrix}$

i.e. multiply position vector of A by reflection matrix

$= \begin{bmatrix} -5 \\ -2 \end{bmatrix}$

Then find A" (image of A' under rotation) we calculate $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

i.e. multiply position vector of A by rotation matrix

$= \begin{bmatrix} 2 \\ -5 \end{bmatrix}$

So we have worked out

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -5 \\ 2 \end{bmatrix} \right\} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$

↑ ↑

2nd transformation 1st transformation

In everyday arithmetic when multiplying three numbers for example 7, 5 and 4 we find 7 x (5 x 4) is the same as (7 x 5) x 4 i.e. we may insert brackets in either position.

(But notice that 7 – (5 – 4) is **not** the same as (7 – 5) – 4!!!)

We may wonder whether we could carry out the matrix multiplication in order below i.e. first multiply together the two 2 x 2 matrices.

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right. \qquad \left. \begin{bmatrix} -5 \\ 2 \end{bmatrix} \right\}$$

2nd transformation 1st transformation

Let us try it

$$\begin{bmatrix} 0 & +1 \\ +1 & 0 \end{bmatrix} \begin{bmatrix} -5 \\ 2 \end{bmatrix}$$

But $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is just the matrix for reflection in y = x!

We see that multiplying the matrices in the order 2nd transformation x 1st transformation gives the matrix for the equivalent transformation multiplied by 1st.

If we try $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ x $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

We get $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

This is a different result and represents reflection in y = -x.

So, to combine matrices to find an equivalent transformation we use the pattern

$$\begin{bmatrix} \text{2nd transformation} \end{bmatrix} \times \begin{bmatrix} \text{1st Transformation} \end{bmatrix}$$

Exercise 4

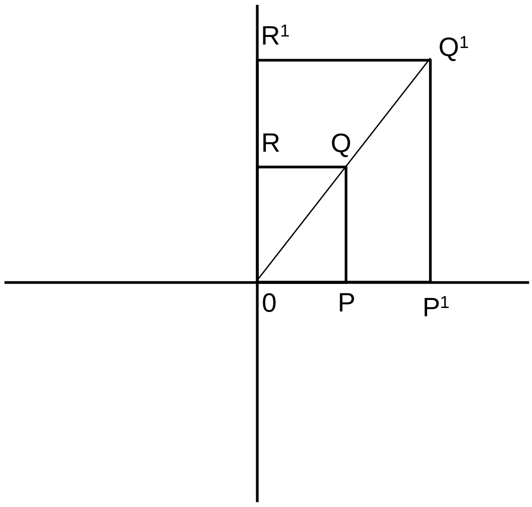
Identify the single transformation which is equivalent to each of the following pairs by:

Using a matrix method and checking by drawing a sketch, using a triangle ABC
a) Reflection in x axis followed by rotation of 180^0 about origin
b) Rotation of 180^0 about origin followed by reflection in x axis.
c) Reflection in x axis followed by reflection in y axis.
d) Reflection in y axis followed by reflection in y = x.
e) Rotation of 180^0 about origin followed by reflection in y = -x.

MATRICES AND ENLARGEMENT

Sketch the image when square OPQR, where O is (0,0) P is (1,1) and R is (0.1), is enlarged with scale factor 2 and centre the origin.

We find:



We find that: P (1.0) maps to P' (2,0)
Q (1.1) maps to Q' (2,2)
R (0.1) maps to R' (0,2)

i.e. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ changes to $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

i.e. $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ changes to $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

This suggests that the matrix $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ might be associated with this enlargement.

Check

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

In general, for an enlargement with a scale factor k and centre the origin, the associated matrix will be:

$$\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

Example 1

Using a matrix method find the image of the triangle ABC, where A is (1,1), B is (2,1) and C is (2,2), under an enlargement centre the origin and scale factor -3. Check with a sketch.

Solution

Matrix will be $\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

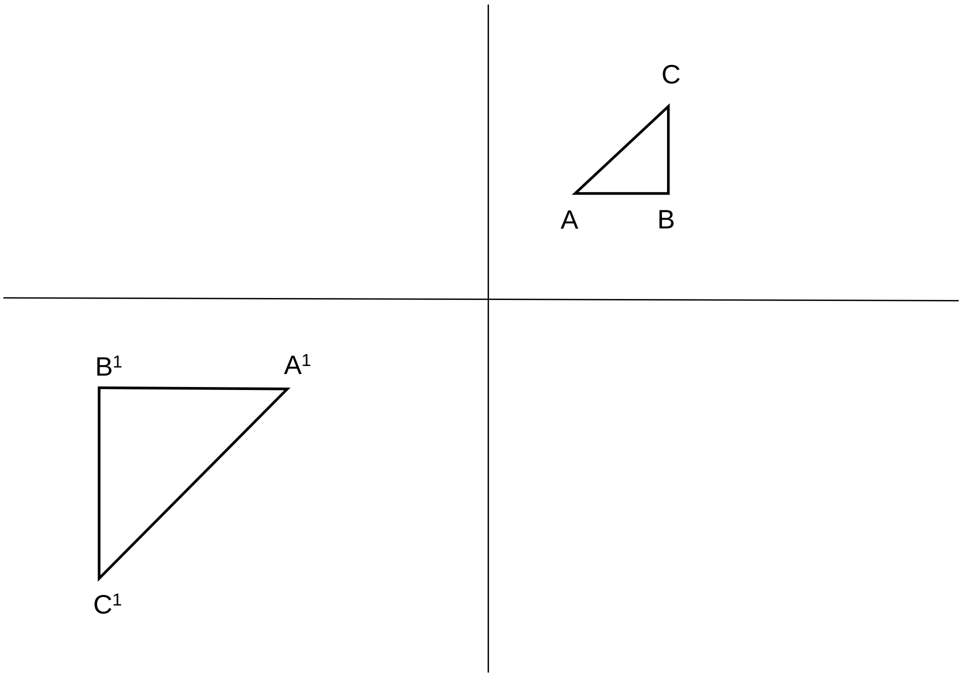
$$\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \end{bmatrix} \qquad \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -6 \\ -6 \end{bmatrix}$$

i.e. A → (-3,-3)

B → (-6,-3)
C → (-6,-6)

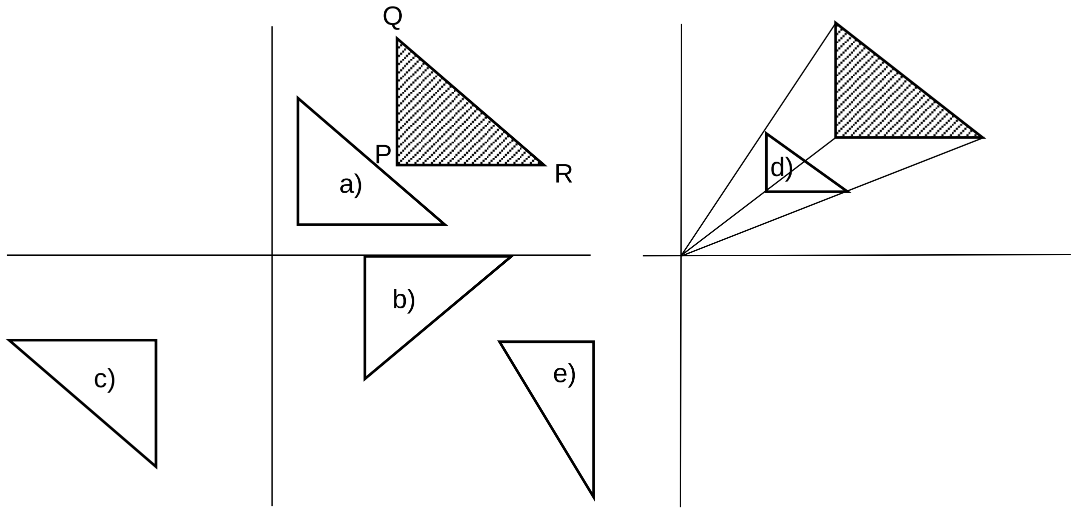
Sketch



ANSWERS

Exercise 1

1.



2. a) Rotation of 180^0 about 0.

   b) Reflection in x axis.

   c) Rotation of -90^0 about 0.

   d) Translation $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

Exercise 2

a) $\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

$\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

$\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$

b) $\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

Exercise 3

a) Reflection in y axis.

b) Rotation of 180° about 0.

c) No change.

d) Reflection in y = x.

e) Rotation of 90° about origin.

Exercise 4

a) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} +1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ Reflection in y axis.

b) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ Reflection in y axis.

Same as (i). This can happen.

c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ Reflection in y = -x

d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ Rotation -90^0 about 0.

e) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ Reflection in y = x

Exam Questions

1. a) On the graph paper draw the x axis with values of x from -3 to +4 and y axis with values of y from -4 to +4. Use a scale of 2 cm to represent 1 unit on both axis.(1 mark)

   b) On this graph draw the triangle PQR with vertices P(1,2), Q(3,1) and R (0,0). (1 mark)

c) The triangle PQR is transformed by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

to form the triangle P'Q'R'.

Draw this triangle P'Q'R' on your graph. (3 marks)

d) Draw on your graph the triangle P"Q"R" formed when the triangle

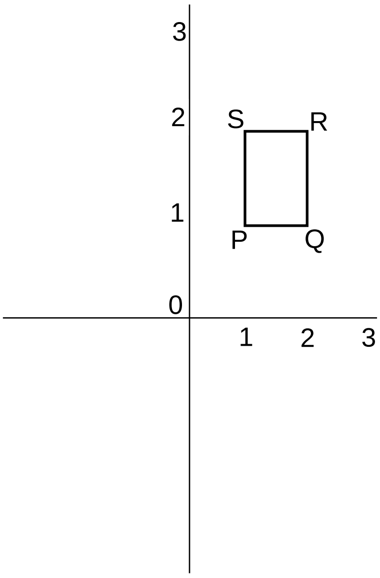
PQR is transformed by the matrix $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

The matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ transformed PQR onto P'Q'R'

The matrix $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ transform P"Q"R" onto PQR (3 marks)

e) What single matrix will transform P"Q"R" onto P'Q'R"? (5 marks)

2.



The diagram shows the square PQRS. The points P, Q, R, and S have position vectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ respectively.

a) A transformation, x is represented by the matrix and it maps P, Q, R, S on to A, B, C and D respectively

   i) Calculate the position vectors of A, B, C and D. (4 marks)

ii) Draw ABCD on your diagram. (2 marks)
iii) Describe geometrically the transformation X. (2 marks)

b) A transformation. Y. reflects the square PQRS in the x axis to form the square EFGH.
i) Write down the position vectors of E, F, G and H. (4 marks)
ii) Write down the matrix which represents the transformation Y. (1 mark)

c) Describe the transformation that maps E, F, G and H on to A, B, C and D respectively. Write down the matrix of this transformation. (2 marks)