Learning Development

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SIMULTANEOUS EQUATIONS

Before you start on this pack, remember that +c can be written as +1c. The number in front of the c term is called the **COEFFICIENT**.

i.e. 1 is the coefficient of c
3 is the coefficient of 3c
14 is the coefficient of 14y

You will also need to remember how to use directed numbers. If you need any revision, see the study pack on Directed Numbers.

SIMULTANEOUS EQUATIONS

In this type of equation there are **two unknown quantities**.

e.g. $2c - 5y = 8$
$4c + 9y = 17$

i.e. We do not know the value of c or y. We have to find one value for c and one value for y which fits both equations at the same time.

(1) and (2) will be used to distinguish equation 1 from equation 2.

Look at these examples, making notes if necessary:

e.g. $c + y = 5$ (1)
$c - y = 1$ (2)

We have to find the value of c and the value of y to solve these equations.
The coefficients of c and y are the same in both equations – they are all 1.

If you add (1) to (2) the result is $2c = 6$

$$c = \frac{6}{2} = 3$$

You have eliminated y from the equations –

$(+y) + (-y)$
$= +y - y = 0$

Now that the value of c is known, you can **substitute** $c = 3$ in either equation to obtain the value of y.

SUBSTITUTE $c = 3$ in (1)

$3+y = 5$
$y = 5 - 3$
$\underline{y = 2}$

As a check, substitute the values of c and y in either equation. This time use (2) as you substituted in (1)

$c - y = 1$
$3 - 2 = 1$

which you know is correct.

The equation has now been solved and the **two missing values have been found**.

$c = 3$

$y = 2$

REMEMBER – if you are asked to solve simultaneous equations **you must have two solutions in your answer**, one for c and one for y.

E.g. $4c - 3y = 1$ (1)

$c + 3y = 19$ (2)

The coefficients of c are **NOT** the same, i.e. 4 and 1. However, the coefficients of y are the same – they are both 3 and, because as in the first example the signs are **UNLIKE,** we can, again, **ADD** (1) to (2) giving:

$5c = 20$ $(-3y + 3y = 0)$

$c = \frac{20}{5} = 4$

Substitute into (1) the value c = 4 to find value of y

$4(4) - 3y = 1$

$16 - 3y = 1$

$16 - 1 = 3y$

$15 = 3y$

$\frac{15}{3} = y$

$\underline{5 = y}$

Check by substituting the values of c and y into either equation. This time use (2) as you substituted in (1).

$c + 3y = 19$

$4 + 3(5) = 19$

$4+15 = 19$

Therefore, values of c and y are correct.

NB. If the signs of the terms with the **same** coefficient are **UNLIKE,** you **ADD** the two equations.

If the signs of the terms with the **same** coefficient are the **SAME**, i.e. (both + or both -) you **SUBTRACT** the two equations.

What happens when the coefficients are not the same?
What happens when the signs are “like” signs?

Look at this example:

e.g. $10c - 3y = 50$ (1)

$3c - 4y = -16$ (2)

All the coefficients are different. We must **ALTER** them, so that one of the terms, e.g. y, has the same coefficient in both equations.
(**It does not matter whether we choose c or y**)

If we **multiply all terms in** (1) by 4 we get
$40c - 12y = 200$ (3)

Now if we **multiply all terms** in (2) by 3 we get
$9c - 12y = -48$ (4)

Now we have the same coefficient for y i.e. –12y.

Before going on, however, remember that with **UNLIKE SIGNS** we **ADDED** the equations.

With **LIKE SIGNS**, we must **subtract** the **equations.**
$40c - 12y = 200$ (3)
$9c - 12y = -48$ (4)

SUBTRACT (4) from **(3)**

$40c - 9c = 200 - -48$
$31c = +248$
Remember $(+200) - (-48)$
$= +200 + 48 = +248$

IF IN DOUBT WRITE IT OUT!

So $c = \frac{248}{31}$

$c = 8$

Substitute this value of c = 8 into either equation. This time use (1) because the numbers are slightly more straight forward.

$10(8) - 3y = 50$
$80 - 3y = 50$
$-3y = 50 - 80$
$-3y = -30$

MULTIPLY BOTH sides by –1, to make y term positive, giving

$3y = +30$
$y = \frac{30}{3} = 10$

Check these values in (2)

$3c - 4y = -16$
$3(8) - 4(10) = -16$
$24 - 40 = -16$

CORRECT!

Work out the following questions:

1. $c + y = 12$
   $c - y = 6$

2. $2c + y = 10$
   $c - y = 2$

3. $4c + y = 10$
   $3c + y = 9$

4. $2c + 3y = 11$
   $4c + y = 12$

5. $3c + 4y = 25$
   $4c - 3y = 0$

6. $2c + 5y = 16$
   $3c - 2y = 5$

7. $a + 3b = 7$
   $a + b = 3$

8. $a + 5b = 21$
   $a + 2b = 9$

9. $3c - 2y = 10$
   $c + 2y = 6$

10. $a - 2b = 5$
    $5a - 2b = 9$

11. $3a - 5b = 10$
    $a - 2b = 4$

12. $3a - 6b = 33$
    $a - 3b = 16$

13. $5a - 3b = 16$
    $4a + 2b = 4$

14. $2a + 5b = 11$
    $7a + 3b = -5$

15. $2a - 3b = 1$
    $a + b = 3$

ANSWERS

1. c = 9, y = 3
2. c = 4, y = 2
3. c = 1, y = 6
4. c = 2.5, y = 2
5. c = 3, y = 4
6. c= 3, y = 2
7. a = 1, b = 2
8. a = 1, b = 4
9. c = 4, y = 1
10. a = 1, b = -2
11. a = 0, b = -2
12. a = 1, b = -5
13. a = 2, b = -2
14. a = -2, b = 3
15. a = 2, b = 1