VARIATION

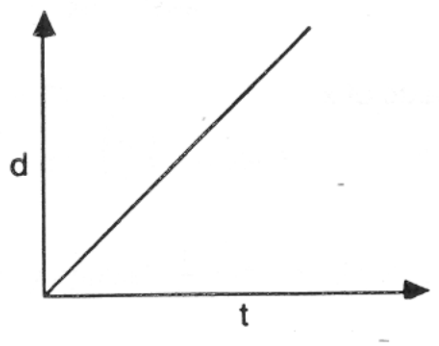
In Physics and Chemistry there are many laws where one quantity varies in some way with another quantity. We will be studying three types of variation direct, inverse and joint.

1. DIRECT VARIATION

If a car is travelling along a road at constant speed, the distance travelled is directly proportional to the time for which it travels. We say distance travelled varies directly as the time. We write this in symbols as:- $d \propto t$ eqn (1)

If a graph of d against t is drawn the result will be a straight line passing through the origin. The gradient of the graph will be constant and equal to $\frac{d}{t}$.

i.e. $\frac{d}{t} = k$ where k is a constant of proportionality equal to the gradient.



Any situation in which one variable varies directly as another can be described by an equation involving a constant of proportionality.

Notice we may replace ∞ in equation (1) by "= k" to give equation (2)

So if y is directly proportional to x, this can be written as $y \propto x$. The $\propto$ sign is then changed to an equals sign and a constant has to be written with it.

i.e. $y = kx$ where k is a constant

If we know one pair of matching values of x and y we can find the value of k. This will give us a general rule which can be used to find other matching pairs of values.

Example 1

If y is directly proportional to x and $x = 4$ when $y = 2$, find x when $y = 8$.

Solution

Step 1. First find k using matching values.

$y \propto x$

y = kx where k is a constant

Substituting x = 4 and y = 2

2 = 4k

$\frac{2}{4}$ = k

k = $\frac{1}{2}$ Now substitute to give general rule.

y = $\frac{1}{2}x$

Step 2. Use rule to find missing value of x.

So when y = 8, 8 = $\frac{1}{2}x$

Therefore x = 16

Example 2

The extension (e) produced in a stretched spring varies directly as the tension (T) in the spring. If a tension of 6 units produces an extension of 2 cm, what will be the extension produced by a tension of 15 units?

Solution

$e \propto T$

e = kT where k is a constant

But, e = 2 when T = 6

2 = k6

$\frac{1}{3} = \frac{2}{6}$ = k

e = $\frac{1}{3}$ T

When T = 15 units

$$e = \frac{1}{3} \times 15$$

$$e = 5 \text{ cm}$$

2. INVERSE PROPORTION

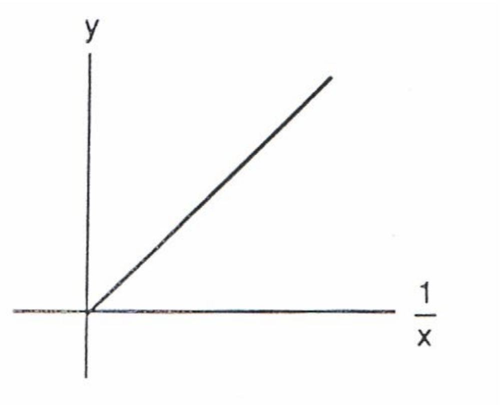
y is **inversely** proportional to x, this can be written as $y \propto \frac{1}{x}$

i.e. as x increases, y decreases.

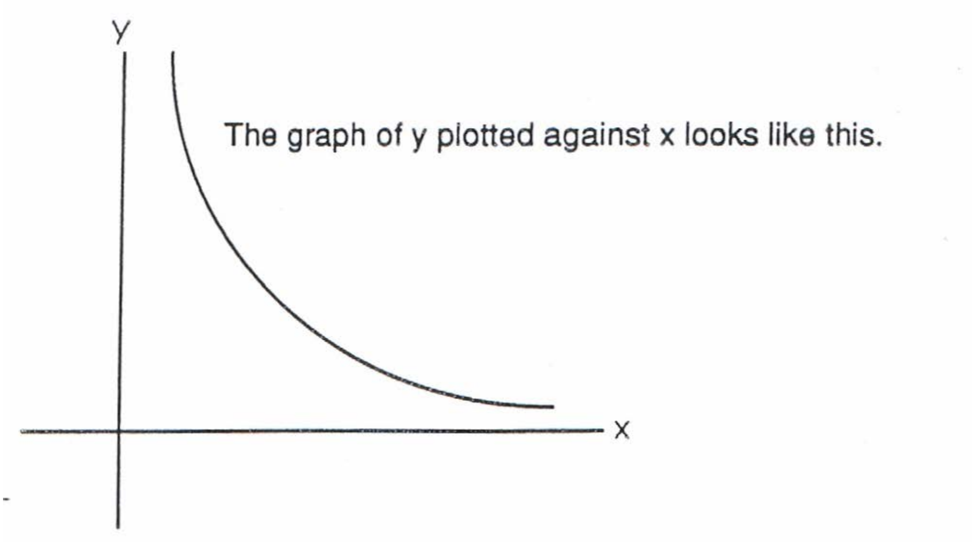
Again replace $\propto$ by = k

Then $y = k\frac{1}{x}$ or $\left(\frac{k}{x}\right)$ where k is a constant.

Notice that if we plot y against $\frac{1}{x}$ we would get a straight line graph.



$\frac{1}{x}$



Example 1

If y inversely proportional to x and x = 1 when y = 10, find x when y = 20

Solution

$y \propto \frac{1}{x}$

$y = k\frac{1}{x}$

Substituting x = 1 and y = 10

$10 = \frac{1}{1}k$

$10 = k$

$y = \frac{10}{x}$

When y = 20, $20 = \frac{10}{x}$ (see Solving Equations unit)

$20x = 10$

$x = \frac{10}{20} = \frac{1}{2}$

Example 2

The illumination (I) of a bulb varies inversely as the square of the distance (d). If the illumination is 5 at a distance of 3 m, what is the illumination at a distance of 2 m?

Solution

$$I \propto \frac{1}{d^2}$$

$$I = \frac{k}{d^2}$$

But, 1 = 5 when d = 3

$$5 = \frac{k}{3^2}$$

$$5 = \frac{k}{9}$$

k = 45

$$\therefore \quad I = \frac{45}{d^2}$$

when d = 2 $\quad I = \frac{45}{d^2}$

$$I = \frac{45}{4}$$

$$I = 11\frac{1}{4}$$

3. JOINT VARIATION

V is jointly proportional to a^2 **and** to b, this can be written as

$$V \propto a^2b$$

$\therefore$ $V = ka^2b$ where k is a constant

It may also state that V is directly proportional to the **product** of a^2 and b. **Product** means **Multiply.**

Example 1

A varies directly as b **and** inversely as c. If a = 4 when b = 1 and c = 3, find a when b = 4 and c = $\frac{1}{2}$

a $\propto$ b$\frac{1}{c}$ i.e. a $\propto$ b$\frac{b}{c}$

$\therefore$ a = $\frac{kb}{c}$ where k is a constant

Substituting a = 4, b = 1 and c = 2

4 = k$\frac{1}{2}$

8 = k

$\therefore$ a = $\frac{8b}{c}$

When, b = 4 and c = $\frac{1}{2}$

a = $\frac{8x4}{\frac{1}{2}}$

$\therefore$ a = 8 x 4 x 2 (dividing by a fraction – turn upside down and multiply)

$\therefore$ a = 64

Example 2

The electrical resistance (R ohms) of a piece of wire with a circular cross section varies directly as the length (l cm) and inversely as the square of the radius (r cm). If the resistance of a 4.5 cm length is 2 ohms and the radius of the wire is 0.3 cm, what length of wire will have a resistance of 10 ohms?

Solution

$$R \propto \frac{\ell}{r^2}$$

$$R = k\frac{\ell}{r^2}$$ Where k is constant.

When $r = 0.3$, $\ell = 4.5$ and $R = 2$

$$2 = k\frac{4.5}{(0.3)^2}$$

$$2 = \frac{k4.5}{0.09}$$

$$2 \times 0.09 = 4.5k$$

$$\frac{2x0.09}{4.5} = k$$

$$\frac{1}{25} = k$$

$$R = \frac{\ell}{25r^2}$$

When $R = 10$, $r = 0.3$

$$10 = \frac{\ell}{25(0.3)^2}$$

$$\ell = 10 \times 25 \times 0.09$$

ℓ **= 22.5 cm**

Exercises

1. p is directly proportional to q and when p is 2, q is 4. Find the value of q when p is 7.

2. Voltage varies directly with current. When the voltage is 10 volts, the current is 0.05 amperes. Find the current when the voltage is 35 volts.

3. x is inversely proportioned to y. When x is 5, Y = 10.
   Find the value of x when y = 30.

4. A law states that v is inversely proportional to the square of t. When v = 2, it also equals 2. Find the value of v when t = 8.

5. p varies directly with q and r. Given that p = 4 when q = 5 and r = 8, find p when q = 15 and r = 6.

6. T is inversely proportional to s^2 but directly proportional to p. If T = 4 when s = 5 and p = 15, find T when s = 2 and p = 10.

ANSWERS

1. 14
2. 0.175
3. $\frac{5}{3} = 1\frac{2}{3}$
4. $\frac{1}{8}$
5. 9
6. $\frac{50}{3} = 16\frac{2}{3}$