

## DIRECTED NUMBERS

### POSITIVE NUMBERS

These are numbers such as:

3 which can be written as +3

46 which can be written as +46

14.67 which can be written as +14.67

a which can be written as +a

### RULE

Any number or letter, which is written without a sign is a **positive number**. Positive numbers may contain a plus sign, but it is common to see them with no sign at all.

### NEGATIVE NUMBERS

These are numbers (and letters) which have a minus sign in front of them:

**minus** 3 is written – 3

**minus** 46 is written – 46

**minus** 14.67 is written -14.67

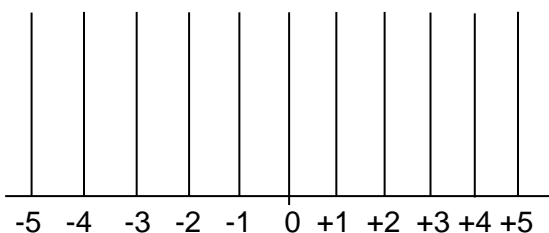
**minus** a is written –a

A negative number or letter **always** has a **minus** sign in front of it.

## ADDING AND SUBTRACTING DIRECTED NUMBERS

As you can see all numbers have a **direction** – positive or negative. This is best shown, at this stage, by using a number line and doing addition and subtraction along the number line.

### Number Line



The number line is **infinitely long**, because the set of positive and negative numbers has no end. The line drawn on page 1 is just a short part of the number line.

### ADDITION (using the number line)

#### Example 1

If you start on 0 and add 3, you move 3 places to the right, and your answer is +3.

### Example 2

Start at +1 and add 3. Your answer is +4.

### Example 3

Start at -1 and add 3. Your answer is +2.

### Example 4

Start at -2 and add 3. Your answer is +1.

When you **add**, you **move** to the **right** along the number line, when you **subtract**, you move to the left along the number line.

## SUBTRACTION

### Example 5

Start at 0 and subtract 1. Your answer is -1.

### Example 6

Start at 0 and subtract 2. Your answer is -2.

### Example 7

Start at -1 and subtract 2. Your answer is -3.

### Example 8

Start at +1 and subtract 2. Your answer is -1.

Draw a number line which goes from -20 through 0 up to +20. Make sure that the distance between each point is the same (one unit).

Try the examples shown above, then some of your own to prove that it really works!

## IS THERE A FASTER WAY OF ADDING AND SUBTRACTING DIRECTED NUMBERS?

**Yes!** There are certain rules, which when followed, will make these operations easy. They apply **every time** with numbers and letters.

### Example 1

Another way of writing 0 add 3 is given here. Brackets are placed round each number and the corresponding sign,

(0) + (+3) and  
written out fully is 0 + 3

Answer = 3.

Notice that the brackets have been removed in the second line. You are advised to write the second line for every example to avoid any confusion.

Notice the + sign in front of the second bracket.

Also, as with a number or a letter, if a **bracket** has no sign in front (like the first bracket) you must assume that it is positive.

### Example 2

$$\begin{array}{l} +1 \text{ add } 3 \\ \text{becomes } (+1) + (+3) \\ 2^{\text{nd}} \text{ line} = +1 + 3 \\ = +4 \end{array}$$

### Example 3

$$\begin{array}{l} -1 \text{ add } 3 \\ \text{becomes } (-1) + (+3) \\ 2^{\text{nd}} \text{ line} = -1 + 3 \\ = +2 \end{array}$$

### Example 4

$$\begin{array}{l} -2 \text{ add } 3 \\ \text{becomes } (-2) + (+3) \\ 2^{\text{nd}} \text{ line} = -2 + 3 \\ = +1 \end{array}$$

## REMEMBER

Look carefully at the way the examples have been 'translated' and written down using brackets.

Look carefully at the signs. You will see that where there is a **plus** sign before a bracket the sign **inside the bracket remains the same**. This is an important rule which you must remember.

In the second line there are only numbers with their signs (no brackets, no extra signs).

**If the signs are different, write down in the answer the sign of the bigger number and subtract the numbers.**

**But if the signs are the same, write that sign in your answer and add the numbers.**

a)  $(+5) + (+4)$   
 $= +5 + 4$

**Signs** are the same (like signs). Put that sign down and ADD the numbers.  
 $= +9$

b)  $(+10) + (+7)$   
 $= +10 + 7$

**Signs** are like. Put that sign down and ADD the numbers.  
 $= +17$

$$\begin{aligned} \text{c) } & (+14) + (-4) \\ & = +14 - 4 \end{aligned}$$

**Signs** are different (unlike signs). Put the sign of the greater number down and SUBTRACT the numbers.

$$= +10$$

$$\begin{aligned} \text{d) } & (-20) + (+10) \\ & = -20 + 10 \\ & = -10 \end{aligned}$$

**Signs** are unlike. Put down the sign of the greater number. Subtract the numbers. Take special note of:

$$\begin{aligned} \text{e) } & (+10) + (-10) \\ & = +10 - 10 \\ & = 0 \end{aligned}$$

$$\begin{aligned} \text{f) } & (-12) - (-12) \\ & = -12 + 12 \\ & = 0 \end{aligned}$$

### Example 5

0 minus 1  
becomes  $(0) - (+1)$

**watch carefully!**  
 $= 0 - 1$

**The minus sign in front of the second bracket changes the sign inside the second bracket.**  
(-becomes + and + becomes -).

The same rules apply now with regard to like and unlike signs so the answer is -1.

### Example 6

0 minus 2  
becomes  $(0) - (+2)$   
 $= 0 - 2$   
 $= -2$

### Example 7

-2 subtract 2  
 $= (-2) - (+2)$   
 $= -2 - 2$   
 $= -4$

### Example 8

1 take away 2  
 $= (+1) - (+2)$   
 $= +1 - 2$

= -1

## SUMMARY OF RULES

1. Positive numbers, letters or brackets need not have a **plus sign** in front of them, e.g.

$$\begin{aligned} 3 &= +3 \\ d &= +d \\ (+16) &= +(+16) = +16 \end{aligned}$$

2. Negative numbers, letters or brackets always have a minus sign in front of them, e.g.

$$\begin{aligned} -d \\ -(+7) &= -7 \end{aligned}$$

3. A plus sign in front of a bracket allows the sign **inside** the bracket to remain the same.
4. A minus sign in front of a bracket means that the sign **inside** the bracket must change (+ to – and – to +).
5. If numbers have different (unlike) signs then write the sign of the **bigger** number in the answer and **subtract** the numbers.

## TWO TERMS OR MORE

You may have more than two terms in questions. There are two ways of tackling this type of question

$$(+12) - (+4) + (-10)$$

Always write out the second line as follows:

$$+12 \quad -4 \quad -10$$

+12 -14 = -2 add together numbers with the same sign **OR** proceed along the line one number at a time

$$\begin{aligned} +12 -4 &= +8 \text{ (first two terms)} \\ +8 - 10 &= -2 \text{ (answer from first line, then third term)} \end{aligned}$$

The rules do not change if there are more than two terms in the question. You carry on in an orderly way, one step at a time, to solve any problem.

Armed with these hints, try these for yourself. Do not be afraid to do these on your number line and then write them out as shown above.

### Exercise 1

- |                  |                  |
|------------------|------------------|
| 1. (+3) + (+9)   | 2. (+10) + (-5)  |
| 3. (-15) + (+2)  | 4. (-20) + (-20) |
| 5. (+13) - (+10) | 6. (+24) - (-12) |
| 7. (-21) - (+21) | 8. (-21) - (-21) |

9.  $(+12) + (-12) - (-12)$       10.  $(+100) - (-50) + (+20)$

Before going on to multiplication of directed numbers:-

**REMEMBER** – when you see a bracket, everything **INSIDE** the bracket is multiplied by the number or letter with the sign which is **outside** the bracket.

## MULTIPLICATION AND DIVISION OF DIRECTED NUMBERS

### MULTIPLICATION

**POSITIVE** multiplied by **POSITIVE** = +  
**NEGATIVE** multiplied by **NEGATIVE** = +

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**POSITIVE** multiplied by **NEGATIVE** = -  
**NEGATIVE** multiplied by **POSITIVE** = -

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$+ \times + = +$  and  $- \times - = +$

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$+ \times - = -$  and  $- \times + = -$

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So,  $(+3) \times (+4) = +12$   
 $(-3) \times (-4) = +12$

$(+3) \times (-4) = -12$   
 $(-3) \times (+4) = -12$

**REMEMBER**  $(+3)^2$  and  $(-3)^2$  BOTH EQUAL +9

Written out fully,  $(+3)(+3)$  and  $(-3)(-3)$

(Remember ( ) ( ) means times)

$(+10)^2$  and  $(-10)^2 = 100$

Written out fully,  $(+10) (+10)$  and  $(-10) (-10)$ .

This is important.

### DIVISION

**POSITIVE** divided by **POSITIVE** = +  
**NEGATIVE** divided by **NEGATIVE** = +

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**POSITIVE** divided by **NEGATIVE** = -  
**NEGATIVE** divided by **POSITIVE** = -

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$$\frac{+}{+} = + \quad \text{and} \quad \frac{-}{-} = +$$


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$$\frac{+}{-} = - \quad \text{and} \quad \frac{-}{+} = -$$


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$$\text{So } \frac{+10}{-5} = -2 \quad \frac{-10}{+5} = -2$$

### SUMMARY OF RULES

1. When multiplying or dividing **like signs**, the answer will be positive.
2. When multiplying or dividing **unlike signs**, the answer will be negative.
3. Multiply or divide numbers as normal.

### Exercise 2

- |                   |                      |
|-------------------|----------------------|
| 1. $3 \times 4$   | 2. $-3 \times -4$    |
| 3. $(-10)(-4)$    | 4. $(+3)^2$          |
| 5. $(-4)^2$       | 6. $(-12)(+3)$       |
| 7. $+15 \div +5$  | 8. $-15 \div -5$     |
| 9. $+15 \div -5$  | 10. $+1000 \div -10$ |
| 11. $+12 \div -6$ | 12. $-36 \div +6$    |
| 13. $+24 \div -6$ | 14. $+16 \div -3$    |
| 15. $+14 \div -3$ | 16. $-125 \div -5$   |

## ANSWERS

### Exercise 1

#### ADDING AND SUBTRACTING

1. Becomes  $+3 + 9 = +12$
2. Becomes  $+10 - 5 = +5$
3. Becomes  $-15 + 2 = -13$
4. Becomes  $-20 - 20 = -40$
5. Becomes  $+13 - 10 = +3$
6. Becomes  $+24 + 12 = +36$
7. Becomes  $-21 - 21 = -42$
8. Becomes  $-21 + 21 = 0$
9. Becomes  $+12 - 12 + 12 = +12$
10. Becomes  $+100 + 50 + 20 = +170$

### Exercise 2

#### MULTIPLICATION AND DIVISION

- |        |                     |                     |         |
|--------|---------------------|---------------------|---------|
| 1. +12 | 2. +12              | 3. +40              | 4. +9   |
| 5. +16 | 6. -36              | 7. +3               | 8. +3   |
| 9. -3  | 10. -100            | 11. -2              | 12. -6  |
| 13. -4 | 14. $-5\frac{1}{3}$ | 15. $-4\frac{2}{3}$ | 16. +25 |