

PERCENTAGES

Percentage (written %) means "**out of one hundred**" i.e. 12% means "twelve out of a hundred" or $\frac{12}{100}$

50% means "50 out of a hundred" or $\frac{50}{100}$

Fractions and decimals can easily be changed into percentages and vice-versa.

CHANGING FRACTIONS TO PERCENTAGES

RULE

To change a fraction into a %age, **multiply** the *fraction* by $\frac{100}{1}$ e.g. $\frac{3}{5}$ to a percentage becomes

$$\frac{3}{5} \times \frac{100}{1} = 60\%$$

Remember to use your rules of fractions, and to cancel where possible.

Example 1

Change $\frac{3}{7}$ to a percentage

$$= \frac{3}{7} \times \frac{100}{1} = \frac{300}{7}$$

$$= 42\frac{6}{7}\%$$

Example 2

Change $1\frac{2}{5}$ to a percentage

First change the mixed number to an improper fraction, then multiply by 100.

$$\frac{7}{5} \times \frac{100}{1} \quad (\text{cancel where possible})$$

$$= 140\%$$

Exercise 1

Change these fractions to percentages:

1. $\frac{4}{5}$ 2. $\frac{1}{3}$ 3. $\frac{3}{4}$ 4. $\frac{2}{7}$ 5. $1\frac{1}{5}$

CHANGING DECIMALS TO PERCENTAGES

RULE

To change a decimal to a percentage, **multiply** the *decimal* by **100**.

Example 1

Change 0.82 to a percentage

$$0.82 \times 100 = 82\%$$

Example 2

Change 0.175 to a percentage

$$0.175 \times 100 = 17.5\%$$

Example 3

Change 0.7 to a percentage

$$0.7 \times 100 = 70\%$$

(Remember that a 0.7 can be written as 0.70)

Example 4

Change 1.67 to a percentage

$$1.67 \times 100 = 167\%$$

Exercise 2

Change these decimals to percentages:

1. 0.65
2. 0.375
3. 0.89
4. 0.6
5. 2.34

CHANGING PERCENTAGES TO FRACTIONS

RULE

To change a percentage to a fraction **divide by 100**.

Example 1

Change 75% to a fraction

$$75\% = \frac{75}{100} = \frac{3}{4} \quad (\text{Cancel where possible})$$

Example 2

Change $12\frac{1}{2}\%$ to a fraction

First change the mixed number to an improper fraction,

$$\frac{25}{2}$$

then divide by 100

$$\frac{25}{200} = \frac{1}{8} \quad (\text{Cancel where possible})$$

Example 3

Change 120% to a fraction

$$\text{i.e. } \frac{120}{100} = \frac{6}{5} = 1\frac{1}{5}$$

Exercise 3

Change these to fractions:

1. 25% 2. 30% 3. 140% 4. $33\frac{1}{3}\%$ 5. $37\frac{1}{2}\%$

CHANGING PERCENTAGES TO DECIMALS (DECIMAL FRACTIONS)

RULE

To change a percentage to a decimal (decimal fraction) **divide** the *percentage* by **100**.

Example 1

Change 54% to a decimal fraction

$$54\% = \frac{54}{100} = 0.54$$

Example 2

Change 2.3% to a decimal fraction

$$2.3\% = \frac{2.3}{100} = 0.023$$

Example 3

Change 32.73% to a decimal fraction

$$32.73\% = \frac{32.73}{100} = 0.3273$$

Example 4

Change 6% to a decimal fraction

$$6\% = \frac{6}{100} = 0.06$$

Exercise 4

1. Convert these to decimals

a) 63% b) 4.7% c) 51.65% d) 3.1% e) 7%

2. Copy out this table, then fill in the gaps

fraction	decimal	%age
1	1.0	100%
$\frac{1}{2}$	0.5	
$\frac{3}{4}$		75%
$\frac{1}{4}$	0.25	
$\frac{1}{8}$		
$\frac{1}{3}$		
$\frac{2}{3}$		

PERCENTAGE CHANGE

A number can be increased or decreased by a given percentage, e.g., shoes in a sale, may be decreased (or reduced) by 10%, you may have to pay a deposit of 20% if you are buying a video.

Example 1

What is 20% of £50?

Remember that 'of' means multiply. 20% means 20 divided by 100, so

20% of £50 is now written

$$\frac{20}{100} \times \frac{\pounds 50}{1} = \pounds 10 \quad (\text{Cancel where possible})$$

So, 20% of £50 is £10

Example 2

What is $12\frac{1}{2}\%$ of £160?

$$\frac{12\frac{1}{2}}{100} \times 160$$

Multiply $12\frac{1}{2}$ and 100 by 2, so we now get

$$\frac{25}{200} \times \frac{\pounds 160}{1} = \pounds 20 \quad (\text{Remember to cancel where possible})$$

Exercise 5

1. 10% of 150
2. 30% of 70
3. 5% of 300
4. 12% of 30
5. 12.5% of 240

INCREASING A NUMBER BY A GIVEN PERCENTAGE

There are several methods which can be used, one of which is shown here.

Increase £50 by 6%

If we find 6% of £50, this gives the actual amount of the increase.
This increase must be then added to £50 to give the final price.

$$\text{So, } \frac{6}{100} \times \frac{\pounds 50}{1} = \pounds 3$$

The increase of £3 must now be added to £50, giving the answer £53.

DECREASING A NUMBER BY A GIVEN PERCENTAGE

Using the same figures as in the example above, we know that 6% of £50 is £3.

However, because we are looking for a decrease, the £3 must be subtracted from £50, giving the answer of £47.

Exercise 6

1. Increase £200 by 4%
2. Decrease £200 by 4%
3. Increase £420 by 10%
4. Decrease £420 by 10%

MAKING A NUMBER A PERCENTAGE OF ANOTHER NUMBER

Example 1

What is 43 out of 86 as a percentage?

This is written as

$$\frac{43}{86} \times \frac{100}{1} = 50\%$$

Example 2

What percentage is 50 of 150?

$$\frac{50}{150} \times \frac{100}{1} = 33.\bar{3}\%$$

Exercise 7

What percentage is:

1. 12 out of 48
2. £30 out of £150
3. £200 out of £700
4. 0.5 out of 2.5
5. 1000 out of 8000

COMMISSION

This is another way of using percentages.

Salesmen, representatives, etc, are sometimes paid commission on top of their basic pay. The commission is a percentage of the total value of the goods they have sold,

Example 1

A double-glazing salesman is paid commission of 2% on the goods that he sold. If he sells double-glazing to the value of £900 find the amount of commission.

Commission = 2% of £900 (of means x)

$$= \frac{2}{100} \times \frac{£900}{1}$$

= £18 This would then be added to his basic wage.

Example 2

A woman is paid a basic wage of £136 per week and a commission of 2.5% on the goods she sells.

How much commission will she be paid if she sells £1500 of goods and what will be her pay for that week?

Commission = 2.5% of £1500

$$= \frac{2.5}{100} \times \frac{£1500}{1}$$

$$= \frac{5}{200} \times \frac{£1500}{1}$$

$$= \frac{75}{2}$$

= £37.50 Commission

Pay for that week

= £136 + £37.50

= £173.50

Exercise 8

1. A car saleswoman sells a car for £4600. If she is paid a commission of 2%, what is total amount of commission she receives?
2. The basic wage of a sales rep for baby goods is £112 per week. In addition to this he is paid a commission of 3.5%. Calculate -
 - a) commission on £960
 - b) his wage for that week, assuming he sells £960 of baby goods.

HIRE PURCHASE

If we buy goods and pay for them by instalments, over a certain period of time, this is called buying goods on **hire purchase**.

Usually, a deposit is required first, and the difference or balance and any interest must be repaid in a number of instalments. "Interest free credit" means that no interest is charged on the balance.

Example 1

A couple buy a three-piece suite for £1500. They have to pay a deposit of 10% and the amount outstanding is charged at 12% per annum. The amount owing is then paid off in 12 monthly instalments. How much will each instalment be?

$$\text{Price of suite} = \text{£}1500$$

$$\begin{aligned} \text{Deposit of 10\%} &= \frac{\text{£}1500}{1} \times \frac{10}{100} \\ &= \text{£}150 \end{aligned}$$

$$\begin{aligned} \text{Outstanding balance} &= \text{£}1500 - \text{£}150 \\ &= \text{£}1350 \end{aligned}$$

$$\begin{aligned} \text{Interest charged at 12\% on this amount} &= \frac{\text{£}1350}{1} \times \frac{12}{100} \\ &= \text{£}162 \end{aligned}$$

$$\text{Total amount to be repaid} = \text{£}162 + \text{£}1350 = \text{£}1512$$

$$\text{Monthly Instalments} \frac{\text{£}1512}{12} = \text{£}126$$

Exercise 9

1. A man buys a car for £4500 and puts down a deposit of 20%. Interest is charged at 2.5% per annum on the outstanding balance. He repays this balance over 24 months. How much are the monthly instalment to the nearest pence?
2. An article can be purchased for £120. If bought on hire purchase, a deposit of 10% has to be paid first, then an interest rate of 2% is added to the outstanding balance. What is the difference between paying cash and buying the article on hire purchase if the balance is repaid in one year.

BANK LOANS

It is possible to borrow money from the bank on a 'personal loan'. The bank gives a rate of interest for the loan, and the loan plus the interest, is usually repaid over equal monthly instalments:

Example

A couple borrows £1000 from the bank. The interest rate is 24%. If the loan is repaid over 12 months, calculate the amount of each payment to the nearest pence.

$$\begin{aligned} \text{Loan} &= \text{£}1000 \\ \text{Interest rate} &= 24\% \end{aligned}$$

$$= \frac{24}{100} \times \frac{£1000}{1}$$

$$= £240$$

So, the total amount to be repaid = loan + interest = £1240

$$\text{Monthly Instalments} = \frac{12}{£1240}$$

$$= £103.33$$

Exercise 10

1. A man borrows £300 from the bank. Interest is charged at 15% and the loan plus interest is to be repaid over 12 months, what is his monthly repayment?
2. A bank lends a customer £650. Interest is charged at 13% per annum. If the loan is repaid, in equal monthly instalments, over 2 years, calculate the amount of each instalment to the nearest pence. (*HINT!* Loan to be repaid over two years.)

PROFIT AND LOSS

When a shopkeeper buys goods, he buys them at **Cost Price (CP)**.
When he sells the goods, this is called the **Selling Price (SP)**.

Profit or Gain is made when the selling price is bigger than the cost price. Loss is made when the selling price is smaller than the cost price.

So, to find profit (gain) or loss

$$\text{PROFIT (GAIN)} = \text{SP} - \text{CP}$$

$$\text{LOSS} = \text{CP} - \text{SP}$$

If SP is £20 and the CP is £15 find the actual profit.

$$\begin{aligned} \text{Profit} &= \text{SP} - \text{CP} \\ &= £20 - £15 \\ &= £5 \end{aligned}$$

If the selling price is £12 and cost price is £16, find the actual loss.

$$\begin{aligned} \text{Loss} &= \text{CP} - \text{SP} \\ &= £16 - £12 \\ &= £4 \end{aligned}$$

FINDING THE PERCENTAGE GAIN AND LOSS

You are often asked to find the percentage gain (profit) or loss. You simply use the formula given here.

$$\text{Percentage profit} = \frac{\text{Gain}}{\text{CP}} \times \frac{100}{1}$$

$$\text{Percentage profit} = \frac{\text{Loss}}{\text{CP}} \times \frac{100}{1}$$

Example 1

A woman buys a T-shirt for £4 and sells it at £5. What percentage profit does she make?

$$\text{CP} = \text{£}4, \text{SP} = \text{£}5$$

$$\text{Percentage profit} = \frac{\text{Gain}}{\text{CP}} \times \frac{100}{1}$$

$$\begin{aligned} \text{Percentage profit} &= \frac{\text{£}5 - 4}{4} \times \frac{100}{1} \\ &= \frac{1}{4} \times \frac{100}{1} \end{aligned}$$

Example 2

A man buys a computer for £300 and sells it for £250. Calculate his %age loss.

$$\begin{aligned} \text{Percentage loss} &= \frac{\text{CP} - \text{SP}}{\text{CP}} \times \frac{100}{1} \\ &= \frac{\text{£}300 - \text{£}250}{300} \times \frac{100}{1} \\ &= 16\frac{2}{3}\% \end{aligned}$$

Exercise 11

1. A car is bought for £2400. and sold for £1800. Calculate the percentage loss.
2. A shopkeeper buys 40 articles costing 2p each and sells them at 3p each. What is her percentage profit?
3. 1 kg of potatoes were bought at 10p and sold at 12p. What is the percentage profit?
4. A man weighed 100 kg. After a diet he then weighed 90 kg. What was his percentage loss in weight?
5. A plant was bought when it was 12 cm high. It grew to a height of 16 cm. Calculate the percentage growth of the plant.

DISCOUNT

In some shops, if you pay cash for an article, you may ask for a discount. This is a percentage of the selling price.

A television is priced in a shop at £300. For cash, a discount of 10% is offered. What would the price of the TV be with 10% off?

First of all find the **actual discount**, then, **subtract** it from the **original price**.

Find 10% of £300

$$\frac{10}{100} \times \frac{\overset{3}{\text{£}300}}{1} = \text{£}30$$

Take this £30 from the original price:

$$\text{£}300 - \text{£}30 = \text{£}270$$

So, you would pay £270 for the television.

VAT

This is the abbreviation used for Value Added Tax. At the time of writing, this stood at 17.5%. You pay VAT on many things, such as electrical goods, food in restaurants, clothes etc.

To calculate VAT simply work out 17.5% of the amount given.

Find the VAT on a meal costing £30, and give the cost of the meal when VAT is added.

17.5% of 30

$$\frac{17.5}{100} \times \frac{30}{1} = \text{£}5.25$$

This amount, £5.25 must be added to the bill, so your final bill should be:

$$\text{£}30 + \text{£}5.25 = \text{£}35.25$$

COST PRICE AND SELLING PRICE

Sometimes, you are asked to calculate the SP if you are given the CP and gain or loss, or the CP if you are given the SP and the gain or loss.

Questions of this type are straight forward and the least complicated method of finding what is needed, is given here.

Example 1

An item was bought for £10 and had to sold so that a profit of 20% was made. What should be the selling price be? '

First find 20% of £10

$$\frac{20}{100} \times \frac{\text{£}10}{1} = \text{£}2 \quad \text{This is the actual profit made.}$$

The selling price is, therefore, the cost price plus the profit:

$$\text{£}10 + \text{£}2 = \text{£}12$$

Example 2

A car was bought for £3000 and a loss of 15% was made on the sale. What was the car sold for?

First find 15% of £3000 - to give the actual loss.

$$\frac{15}{100} \times \frac{\text{£}3000}{1} = \text{£}450$$

Then take away this amount from the cost price:

$$\text{£}3000 - \text{£}450 = \text{£}2550$$

Example 3

If the selling price of an item is £22 and the profit made was 10%, what was the cost price? The cost price is 100%, so the selling price will be 100% + 10%.

$$\begin{aligned} \text{Cost price} &= \frac{100}{110} \times \frac{\text{£}22}{1} \\ &= \text{£}20 \end{aligned}$$

Example 4

If the selling price is £190 and a loss of 5% is made, what is the cost price?

Cost price = 100%

Selling price = 100% - 5% = 95%

$$\begin{aligned} \text{So, CP} &= \frac{100}{95} \times \frac{\text{£}190}{1} \\ &= \text{£}200 \end{aligned}$$

Exercise 12

1. A mirror has 10% discount off the marked price of £15, what is the actual discount?
2. A shop offers a discount of 6p in the £1. How much will be allowed on an item costing £300.
3. CP = £120, Gain = 5%
What is SP?

4. CP = £320, Loss = 12.5%
What is SP?
5. SP = £45, Profit = 20%
What is the CP?
6. SP = £400, Loss = 35%
What is CP to the nearest pence?

ANSWERS

Exercise 1

1. 80% 2. $33\frac{1}{3}\%$ 3. 75% 4. $28\frac{7}{7}\%$ 5. 120%

Exercise 2

1. 65% 2. 37.5% 3. 89% 4. 60% 5. 2.34%

Exercise 3

1. $\frac{1}{4}$ 2. $\frac{3}{10}$ 3. $1\frac{2}{5}$ 4. $\frac{1}{3}$ 5. $\frac{3}{8}$

Exercise 4

1. a) 0.63 b) 0.047 c) 0.5165 d) 0.031 e) 0.07

2.

fraction	decimal	%age
1	1.0	100%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{1}{4}$	0.25	25%
$\frac{1}{8}$	0.125	$12\frac{1}{2}\%$
$\frac{1}{3}$	0.333	$33\frac{1}{3}\%$
$\frac{2}{3}$	0.667	$66\frac{2}{3}\%$

Exercise 5

1. 15 2. 21 3. 15 4. 3.6 5. 30

Exercise 6

1. £208 2. £192 3. £462 4. £378

Exercise 7

1. 25% 2. 20% 3. $28\frac{4}{7}\%$ 4. 20% 5. $12\frac{1}{2}\%$

Exercise 8

1. £92
2. a) £33.60 b) £145.60

Exercise 9

1. £157.50 2. £2.16

Exercise 10

1. £28.75 2. £34.13

Exercise 11

1. 25% 2. 50% 3. 20% 4. 10% 5. $33\frac{1}{3}\%$

Exercise 12

1. £1.50 2. £18 3. £126 4. £280 5. £37.50
6. £615.38