

RATIO | PROPORTION | CHANGE OF UNITS

RATIO

A ratio gives us a way of comparing two or more quantities. However, two or more quantities can only be compared **when they are in the same units**.

Example 1

Find the ratio of 2 cm to 6 cm.

First check that the units are the same (in this example they are in centimetres), and then write down the numbers as shown below.

2 : 6 (No need to write the units)
= 1 : 3 (Numbers cancelled by 2)

This is said "one to three" and can be written as a fraction $\frac{1}{3}$

It tells us that one measurement is 3 times greater than the other (i.e. 6cm is 3 times greater than 2 cm).

Example 2

Express the ratio 75p to £1 in its simplest form.

First change everything to pence.

Ratio becomes 75: 100

and cancel 3 : 4

Therefore, the number is 3: 4

Said "in the ratio of three to four", and can be written as a fraction $\frac{3}{4}$.

REMEMBER - care must be taken to write down the part of the ratio which is required first.

Example 3

Express 4: $\frac{1}{3}$ in its lowest terms.

These terms must be in same units - we cannot have mixed numbers and fractions.

So, multiply one-third by 3 to make it into a whole number = 1.

The 4 has also to be multiplied by 3 giving 12.

So, we now have

$$4 \times 3 : 3 \times \frac{1}{3}$$

12 : 1 this is the answer.

Exercise 1

- Express the following ratios in their lowest terms:
 - $2 : 4$
 - $8 : 12$
 - $60 : 150$
 - $18 : 15$
 - $4 : \frac{1}{2}$
- Express the ratio £5 to 75p in its lowest terms.
- Express the ratio 400 m to 2 km as a fraction in its lowest terms.

DIVIDING AN AMOUNT INTO PROPORTIONAL PARTS

Example 1

Divide £500 into two parts in the ratio 2: 3

- Find the total number of parts

$$2 : 3 = 5 \text{ parts}$$

- The total number of parts is equal to the total amount of money

$$5 \text{ parts} = £500$$

- To find 1 part divide the total amount of money by the total number of parts

$$\frac{£500}{5} = £100$$

- To find the value of 2 and 3 parts, multiply 1 part by 2, then multiply 1 part by 3

$$2 \text{ parts} = £100 \times 2 = £200$$

$$3 \text{ parts} = £100 \times 3 = £300$$

$$\text{To check, add 2 and 3 parts together } £200 + £300 = £500$$

The required ratio = **£200: £300**

Example 2

A line 30 cm long is to be divided into 3 parts in the ratio 2: 3 : 5. Find the length of the **longest** part.

$$2 : 3 : 5 = 10 \text{ parts}$$

$$10 \text{ parts} = 30 \text{ cm}$$

$$1 \text{ part} = \frac{30}{10} = 3 \text{ cm}$$

The longest part = 5 parts

Therefore, $5 \times 3 \text{ cm} = 15 \text{ cm}$

The longest part is **15 cm**

Example 3

Two amounts of money are in the ratio 4: 3. If the first amount is £24, what is the second amount?

This time we are told that the first amount of money is £24, we are not given the total.

4 parts = £24

1 part = £6 Therefore,

3 parts = £6 x 3
= £18

Exercise 2

1. Divide £300 in the ratio of 2: 1.
2. Divide £60 in the ratio 5: 7
3. A line is divided into 3 parts 2: 3 : 7. If the line is 84 cm long, calculate the length of each part.
4. A sum of money is divided in the ratio of 2: 3. If the larger amount is £18, what is
 - a) the other amount and
 - b) the total sum of money?
5. £600 is divided amongst three children in the ratio of their ages. John 5 years old, Claire is 7 years old and Robert is 8 years old. How much money does Claire receive?

DIRECT PROPORTION

If two quantities increase or decrease at the same rate, they are said to **vary directly to one another**, which means that if 2 ice-lollies cost 24p, then we would know that 4 ice-lollies would cost 48p, 6 would cost 72p and 1 would cost 12p.

Double the amount - double the cost
Treble the amount - treble the cost
Half the amount - half the cost

Example 1

If 3 kg of apples cost £1.20, how much will 5 kg cost? If 3 kg = £1.20

If 3 kg = £1.20

$$1 \text{ kg} = \frac{£1.20}{3} = 40\text{p}$$

So, 5 kg will cost 5 x 40p

= 200p or £2.00

Example 2

A car travels 100 km in 2 hours. How long will it take to travel 250 km?

100 km in 2 hours

$$1 \text{ km in } \frac{2}{100} = \frac{1}{50}$$

Therefore, for 250 km, it would take

$$\frac{250}{1} \times \frac{1}{50}$$

= 5 hours (Usually cancels out here)

Exercise 3

1. 7 pears cost 84p. What is the cost of 5 pears?
2. 5 kg of potatoes cost 40p. What is the cost of 8 kg?
3. A train travels 300 km in 5 hours. How long will it take to complete a journey of 450 km?
4. Three metres of wood costs £2.25. What is the cost of 7m?
5. Two bottles of wine fill 8 wine glasses (big glasses!). How many glasses of wine can be poured from 5 bottles?

FOREIGN CURRENCY AND EXCHANGE

Every country has its own currency. There is a "rate of exchange" between countries.

e.g. £1 = 200 (Spanish) pesetas
 £1 = 4 Deutschmarks
 £1 = 10 (French) francs

The DIRECT PROPORTION method can be used to solve many foreign exchange questions.

Example

If £1 = 200 pesetas

- a) How many pesetas do you get for £15
- b) Change 1850 pesetas into £'s

a) £1 = 200 pesetas
 so £15 = 200 x 15 = 3000 pesetas

b) 200 pesetas = £1

$$1 \text{ pesetas} = \frac{1}{200}$$

$$\begin{aligned} \text{Therefore, } 1850 \text{ pesetas} &= \frac{1850}{1} \times \frac{1}{200} \\ &= \text{£}9.25 \end{aligned}$$

Exercise 4

10 francs = £1 (French)
 200 pesetas = £1 (Spanish)
 4 Deutschmarks = £1 (German)
 2000 lire = £1 (Italian)

1. A man takes £200 to France. How many francs does he receive in return?
2. A businessman spends 600 francs on travelling in France. How much does he spend in £'s.
3. A woman took to Germany 750 DM. How much did she have in £'s.
4. If a couple going to Italy took £200 to spend, how many lire did they receive?
5. If you went to Spain with £150
 - a) How many pesetas would you have?
 - b) If you came back with 1000 pesetas, how much will you have spent abroad? (Give this answer in £'s)

IMPERIAL AND METRIC UNITS

At present in this country a mixture of imperial and metric units are in everyday use. You are probably familiar with most of the following:-

Imperial Units of length such as inches, feet, yards, miles.

Metric Units of length such as centimetres, metres, kilometres etc.

Imperial Units of weight like ounces, pounds, stones, tons etc.

Metric Units of weight like grams, kilograms, and tonnes.

Imperial Units of capacity for measuring liquids such as pints, gallons, fluid ounces.

Metric units of capacity like millilitres and litres.

While we still use units from both systems it is often useful to be able to convert from one system to the other.

Some approximate conversions are

Length: 1 inch is approximately 2.5cm

So 40 inch is approximately 1 m

or 1 yard is slightly less than 1 metre

Also

8 km is about 5 miles

∴ 1 km is about $\frac{5}{8}$ miles

1 mile is about $\frac{8}{5}$ km

Weight: 1 kg is about 2.21b

Capacity: 1 gallon = 8pts is about 4.4 litres

Example 1

Convert a waist measurement of 24" to centimetres.

inches	=	centimetres
1"	=	2.5 cm
24"	=	2.5 x 24 = 60cm

Example 2

A child's height is 130 cm. Change this to inches.

cm inches

2.5cm = 1

1 cm = $\frac{1}{2.5}$

$$130 \text{ cm} = \frac{1}{2.5} \times 130 = 52 \text{ inches}$$

Example 3

Convert 100 litres to gallons

$$4.4 \text{ litres} = 1 \text{ gallon}$$

$$1 \text{ litre} = \frac{1}{4.4} \text{ gallon}$$

$$\begin{aligned} 100 \text{ litre} &= 100 \times \frac{1}{4.4} \text{ gallon} \\ &= 22.73 \text{ gallons (2 dp)} \end{aligned}$$

Exercise 5

Use the conversions given above to answer the following:

1. Convert 32 kg to pounds.
2. Change 180 cms to inches.
3. A household gets 2 pints of milk daily. How many litres is this per week?
4. The speed limit in towns in Britain is 30 mph. Convert this to km per hour.
5. Convert 500 yard to metres (1 yard = 36 inches).
6. A motorist buys petrol at 40p per litre. How much is this per gallon?

INVERSE PROPORTION

If an increase in one quantity produces a decrease in another, then this is said to be a case of **inverse proportion**.

Example 1

If 10 men take 8 days to build a wall, how long will it take 4 men to do the same job

10 men take 8 days

1 man takes $10 \times 8 = 80$ days

So, 4 men take $\frac{80}{4} = 20$ days

Example 2

15 women can pack 2000 articles into boxes in 3 days, how long will it take 10 women to pack the same quantity?

15 women take 3 days
1 woman takes $3 \times 15 = 45$ days

10 women take $\frac{45}{10} = 4.5$ days

Another way of solving this type of problem is shown below.

Example 3

A College building can be decorated by 12 men working 8 hours a day for 5 days. If 10 men worked 6 hours a day, how many days would it take them?

Men	Hours	Days
12	8	5
10	6	?d

Set the problem out as an equation as shown below.

$$12 \times 8 \times 5 = 10 \times 6 \times d$$

$$480 = 60 \times d$$

$$\frac{480}{60} = d$$

$$8 = d$$

Therefore, it would take 10 men 8 days to do the work.

Exercise 6

- 4 men can decorate a house in 3 days. How long does it take 2 men?
- An amount of money is divided amongst 8 children. Each child receives £9. If the same amount of money was divided amongst 12 children, how much would each child receive?
- 20 men produce 1000 articles in 5 days. How long would it take 25 men to produce the same number of articles?

SCALES

You are expected to know the following:

1000 mm = 1m
100 cm = 1m
1000m = 1 km

mm means millimetres
cm means centimetres
km means kilometres

If a map has a scale of 1 : 500 it means that 1 cm on the map **represents** 500cm on the ground. If the scale is 1 : 100000, it means that 1 cm on the map **represents** 100000cm on the ground. Scales can also be written **as representative fractions** e.g.

$$1 : 500 \text{ can be written } \frac{1}{500} \text{ and } 1 : 100000 \text{ as } \frac{1}{100000}$$

Example

The scale of a map is 1 : 25000. Find

- the actual length of a field represented by 3 cm on the map.
- the actual distance between 2 towns, which are 5 cm apart on the map.
- the distance on the map between two churches which are 5 km apart.

- a) Scale 1: 25000

1 cm represents 25000 cm

3 cm represents $3 \times 25000 \text{ cm} = 75000 \text{ cm}$

As you know, 100 cm = 1m

The length of the field is $\frac{7500}{100} = 750\text{m}$

- b) Scale 1: 25000

1cm represents 25000 cm

5 cm represent $5 \times 25000 \text{ cm} = 125000 \text{ cm}$

which is 1250m **OR** 1.25 km

- c) Here you are given the **actual** distance and have to find the distance on the map, so you have to 'reverse' the above method.

25000 cm represents 1 cm on the map

1 cm represents $\frac{1}{25000}$ on the map

The distance between the churches is given in kilometres, so this must be changed to centimetres to correspond with the scale.

1 cm = 100 000 cm

5 km = 500 000 cm

Scale = 25000: 1

500 000 represents $\frac{1}{25000} \times \frac{500000}{1} = 20 \text{ cm}$

ANSWERS

RATIO

Exercise 1

- 1: 2
 - 2: 3
 - 2: 5
 - 6: 5
 - 8: 1
- 20: 3
- 1: 5 as a fraction $\frac{1}{5}$

Exercise 2

- £200: £100
- 25: 35
- 14 cm
21 cm
49 cm
- £12
 - £30
- £210

DIRECT PROPORTION

Exercise 3

- 60p
- 64p
- 7.5 hours or 7 hours 30 mins
- £5.25
- 20 glasses

FOREIGN CURRENCY

Exercise 4

- 2000 francs
- £60
- £187.50
- 400 000 lire
- 30000
 - £145.00

IMPERIAL AND METRIC UNITS

Exercise 5

1. $32 \times 2.2 = 70.4$ pounds
2. $180 + 2.5 = 72$ inches
3. $14 \text{ pints} = \frac{14}{8} \text{ gallons} = \frac{14}{8} \times 4.4 = 7.7 \text{ litres}$
4. $30 \times \frac{8}{5} = 48 \text{ km/hr}$
5. $500 \text{ yds} = 500 \times 36 \text{ inches} = \frac{500 \times 36}{40} = 450 \text{ m}$
6. $1 \text{ gallon} = 4.4 \text{ litres} \setminus \text{cost per gallon} = 4.4 \times 40 = \text{£}1.76$

INVERSE PROPORTION

Exercise 6

1. 6 days
2. £6
3. 4 days