FUNCTIONS

This not new to maths, it's just a different way of writing things down.
Suppose $y = x^2 - 3x + 4$. Different vales of x give different values of y. y is then said to be a `function` of x. It can be written like this: $y = f(x) = x^2 - 3x + 4$.

If $y = x^2 - 6zx + 5z^2$ then y depends on two variables x and z. y is a function of x and z: $y = f(x, z)$

$$f(x, z) = x^2 - 6zx + 5z^2$$

Sometimes y doesn't enter the picture: you may see just

$$f(x) = 4x^3 - 3x + 7$$

or $f(x) = a x^2 + bx + c$

and instead of y, the text will refer to `the function` or just `$f(x)$`.

Notice in the last example that, although there are several letters on the right-hand-side, the function is $f(x)$ **not** $f(x, a, b, c)$. That that means that a, b and c are constants.
The fact that they come from early in the alphabet supports that.

Here are two examples:

The total interest I on an account which gives interest at a rate of $R\%$ per year depends on the principal P (the starting summing) and the time T in years:

$$I = f(P, R, T)$$

If the interest is simple then $f(P, R, T) = \frac{PRT}{100}$

The power P dissipated in a resistance R depends on the current I through it:

$$P = f(I, R)$$

$$f(I, R) = I^2 R$$

Exercise 1

Use the function notation in the following examples, first to show that the first variable is a function of the rest, then to state what the function is. In the case of I, P, R and T, the answer would be:

$$I = f(P, R, T)$$

$$f(P, R, T) = \frac{PRT}{100}$$

a) C, r Circumference and radius of a circle.

b) A, r Area and radius of a circle.

c) *A, l, b* Area, length and breadth of a rectangle.

d) *I, E, R* Current, voltage and resistance.

e) *H, I, E* Happiness, income, expenditure.

f) *R, M* Retail Price Index, Money supply.

Now check your answers.

OTHER FUNCTIONS

The letter `*f*` is most often used for a function, but any letter can be used. It's common practice to use `*f*` for the first function, then `*g*` and `*h*` for other functions if necessary.

For example: $f(x) = 3x + 5$

$g(x) = x^2 - 5x + 7$

$h(x) = 3x^4$

Capital letters are also allowed. Normally $F(x)$ is not the same as $f(x)$.

DERIVATIVES

When the function is written as $f(x)$, then the derivative is written as $f`(x)$. This is spoken as `eff dashed exe ` or ( sometimes ) `eff dash exe `.

If $y = f(x) = x^3$ then $\frac{dy}{dx} = f`(x) = 3x^2$

Of course, if the function is $g(x)$ then the derivative is $g`(x)$.

Exercise 2

To get used to the idea, write down a few functions and derivatives like this:

$f(x) = 5x^3 - 8x$

$f`(x) = 15x^2 - 8$

a) $f(x) = 6x^2$

b) $f(x) = x^7 + 5x^4 - 9x + 2$

c) $f(x) = 2x^{3/2} - 4x^{1/2}$

d) $g(x) = x^{-5} - 3x^{-2}$

Now check your answers.

USEFUL SHORTHAND

The function notation also allows you to abbreviate such statements as When $x = 7$, then $y = -9.2$. This becomes:

$f(7) = -9.2$.

So for example if $f(x) = x^2$ then $f(0) = 0$

$f(2) = 4$

$f(5) = 25$

$f(-3) = 9$ and so on.

The same can be done with the derivative. If $f(x) = x^2$

Then $f`(x) = 2x$. So $f(3) = 9$ $f`(3) = 6$

$f(7) = 49$ $f`(7) = 14$

$f`(9) = 18$

$f`(14) = 28$

The essential steps in finding (for example) $f`(7)$ are first to find the formula of $f`(x)$, **then** put $x = 7$ into the formula.

Exercise 3

1. $f(x) = 3x^2 - 5$ Write down the values of

   a) $f(0)$ b) $f(2)$

   c) $f`(1)$ d) $f`(-4)$

2. $g(x) = 8x^{-1} + 2x$ Write down the values of

   a) $g`(1)$ b) $g`(2)$

   c) $g(2)$ d) $g(3)$

Now check your answers.

SUMMARY

$y = f(x)$ means y is a function of x. That means that the formula for y involves the variable x and some constants.

Equations like $y = 3x^2 - 7$ can be written in two parts:

$$y = f(x)$$

$$f(x) = 3x^2 - 7$$

Sometimes y is left out altogether. The statement of the function is simply $f(x) = 3x^2 - 7$

$f(2)$ means `the value of the function when $x = 2$ `. So if $f(x) = 3x^2 - 7$, then $f(2) = 5$

Any letter, small or capital, can be used to label a function: $f(x)$, $g(x)$, $b(x)$, $F(x)$ are all allowed.

$y = f(x, z)$ means the formula for y includes two variables x and z, and some constants.

The derivative of the function $f(x)$ is indicated by $f`(x)$.

If $f(x) = 3x^2 - 7$, then $f`(x) = 6x$. If $y = f(x)$ then $f`(x)$ and $\frac{dy}{dx}$ have the same meaning.

$f`(3)$ means the value of $f`(x)$ when $x = 3$. If $f(x) = 3x^2 - 7$, then $f`(x) = 6x$, and $f`(3) = 18$.

ANSWERS

Exercise 1

a) $C = f(r)$ $f(r) = 2\pi r$

b) $A = f(r)$ $f(r) = \pi r^2$

c) $A = f(l, b)$ $f(l, b) = l\,b$

d) $I = f(E, R)$ $f(E, R) = \frac{E}{R}$

e) $H = f(I, E)$

f) $R = f(M)$

The last two formulas would have to be guesswork.

Now return to the text.

Exercise 2

a) $f(x) = 6x^2$ $f`(x) = 12x$

Read the text again if you have any problems.

b) $f(x) = x^7 + 5x^4 - 9x + 2$ $f`(x) = 7x^6 + 20x^3 - 9$

c) $f(x) = 2x^{3/2} - 4x^{1/2}$ $f`(x) = 3x^{1/2} - 2x^{-1/2}$

If you made a slip, remember and use the basic rule:

$$f'(x) = 2\left(\frac{3}{2}\right)x^{\frac{3}{2}-1} - 4\left(\frac{1}{2}\right)x^{1/2-1}$$

$$= 3x^{1/2} - 2x^{-1/2}$$

d) $g(x) = x^{-5} - 3x^{-2}$ $g`(x) = -5x^{-6} + 6x^{-3}$

If you got that right, then you remembered that when you differentiate x^{-5} you DON'T get $-5x^{-4}$, you get $-5x^{-5-1} = -5x^{-6}$

Now return to the text.

Learning Development

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Exercise 3

1. $f(x) = 3x^2 - 5$ $f'(x) = 6x$

a) $f(0) = -5$

b) $f(2) = 7$

c) $f'(1) = 6$

d) $f'(-4) = -24$

The important thing here is first to differentiate to get $f'(x)$. Then use the formula for $f(x)$ or $f'(x)$ as needed.

2. $g(x) = 8x^{-1} + 2x$ $g'(x) = -8x^{-2} + 2$

a) $g'(1) = -8 + 2 = -6$

b) $g'(2) = \frac{-8}{4} + 2 = 0$

c) $g(2) = \frac{8}{2} + 4 = 8$

d) $g(3) = \frac{8}{3} + 6 = 8\frac{2}{3}$ or $\frac{26}{3}$

Now return to the text.