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REMINDERS

ELEMENTARY DERIVATIVES

These derivatives are elementary in the sense that they form the **elements** of all the differentiation that you do. They will normally be provided in a formula sheet in the exams: but if you know most of them, it will help to speed up your working.

$f(x)$	$f'(x)$
c (a constant)	0
x	1
x^n	nx^{n-1}
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
e^x	e^x
$\ln(x)$	$\frac{1}{x}$

Rules

And here are the rules to use when the functions to be differentiated are some combination of the elementary functions. The first two rules are ones that you will have come across before and they are used constantly:

Sum Rule:

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

Constant Multiple Rule:

$$\frac{d}{dx}(cy) = c\frac{dy}{dx}$$

Example of the sum rule:

$$\frac{d}{dx}\left(x^2 + x^3\right) = 2x + 3x^2$$

Example of the constant multiple rule:

$$\frac{d}{dx}\left(4x^3\right) = 4\left(3x^2\right) = 12x^2$$

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Here are additional rules that appear in earlier maths packs:

Chain rule :	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Product rule:	$\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$
Quotient rule:	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

The rules given above will normally be on any formula sheet provided in an exam.

Example of the chain rule:

To find $\frac{dy}{dx}$ if $y = \sin(x)^2$

Put $u = x^2$ Then $y = \sin(u)$

$\frac{du}{dx} = 2x$ $\frac{dy}{du} = \cos(u)$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \cos(u)\ 2x = 2x\cos\left(x^2\right)$$

Example of product rule:

To find $\frac{dy}{dx}$ if $y = x\sin(x)$

Put $u = x$ $v = \sin(x)$ Then $y = uv$

$\frac{du}{dx} = 1$ $\frac{dv}{dx} = \cos(x)$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx} = \sin(x)\ 1 + x\cos(x)$$

$$= \sin(x) + x\cos(x)$$

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Example of quotient rule:

to find $\frac{dy}{dx}$ if $y = \frac{e^x}{x}$

Put $u = e^x$ $\quad v = x$ $\quad$ Then $y = \frac{u}{v}$

$\frac{du}{dx} = e^x$ $\quad \frac{dv}{dx} = 1$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{xe^x - e^x 1}{x^2}$$

$$= \frac{e^x(x-1)}{x^2}$$

Now recall your skills before you move on!

Exercise

Find expressions for $\frac{dy}{dx}$ in the following cases:

a) $y = \ln(3x^2 + 7)$

b) $y = (4x+1)\cos(x)$

c) $y = \frac{2x}{(x^2 - 5)}$

Now check your answers.

SUMMARY

The ‘elementary” derivatives given here form the elements of all the more complicated derivatives you will encounter.

The ‘rules’ given here help you to differentiate tough-looking expressions by reducing the problem to elementary derivatives.

ANSWERS

Exercise

a) $$\frac{dy}{dx} = \frac{6x}{3x^2+7}$$

$$y = \ln\left(3x^2+7\right)$$

Put $u = 3x^2+7$ then $y = \ln(u)$

$$\frac{du}{dx} = 6x \qquad \frac{dy}{du} = \frac{1}{u}$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{u}\;6x = \frac{6x}{3x^2+7}$$

b) $$\frac{dy}{dx} = 4\cos(x) - (4x+1)\sin(x)$$

$$y = (4x+1)\cos(x)$$

Put $u = 4x+1$ $v = \cos(x)$

$$\frac{du}{dx} = 4 \qquad \frac{dv}{dx} = -\sin(x)$$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx} = \cos(x)\;4 + (4x+1)\;(-\sin(x))$$

$$= 4\cos(x) - (4x+1)\sin(x)$$

c) $$\frac{dy}{dx} = \frac{-2x^2-10}{\left(x^2-5\right)^2}$$

$$y = \frac{2x}{x^2-5}$$

Put $u = 2x$ $v = x^2-5$

$$\frac{du}{dx} = 2 \qquad \frac{dv}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{\left(x^2-5\right)2 - 2x(2x)}{\left(x^2-5\right)^2}$$

$$= \frac{-2x^2-10}{\left(x^2-5\right)^2} \quad \text{or} \quad \frac{-2\left(x^2+5\right)}{\left(x^2-5\right)^2}$$

Now return to the text.