

AREAS AND VOLUMES

Area of a triangle = $\frac{1}{2}$ x base x height

Area of a circle = πr^2

Area of a sector = $\frac{1}{2} r^2 \theta$ in radians

Volume of a cylinder = $\pi r^2 h$

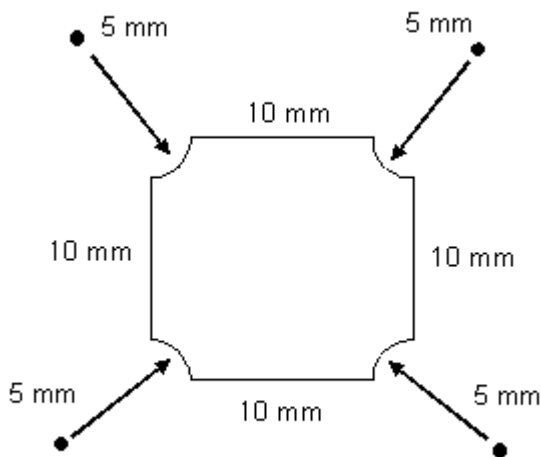
Volume of a cone = $\frac{1}{3} \pi r^2 h$

Volume of a sphere = $\frac{4}{3} \pi r^3$

It is often the case that we are required to find the areas of the shapes that are “unusual”, that is, they have parts cut out of them in the following example.

Example

Find the area of the following shape



It's necessary to split the shape into areas that we are able to find.

Here we have a square with 4 quarter circles cut out of it.

So if I find the area of the square (as if the $\frac{1}{4}$ circles were not cut out of it) and

then find the area of the $\frac{1}{4}$ circles, by subtraction I can get the area of the shape.

$$\text{Area of square} = 20 \times 20 = 400 \text{ mm}^2$$

$$\text{Area of one } \frac{1}{4} \text{ circles} = \frac{1}{4} \times \pi \times 5^2$$

$$\text{So area of four } \frac{1}{4} \text{ circles} = 4 \times \frac{1}{4} \times \pi \times 5^2$$

$$(\text{Taking } \pi \text{ as } 3.142) = \pi \times 5^2$$

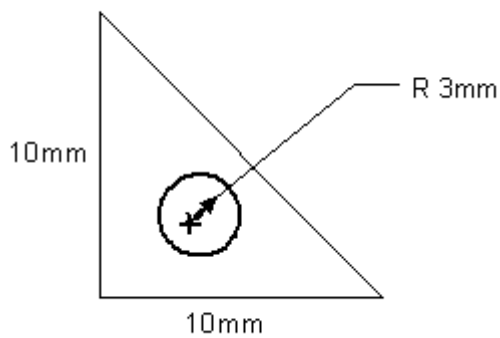
$$= 78.6 \text{ mm}^2 \text{ to 3 significant figures}$$

$$\text{Therefore, the area of the shape} = 400 - 78.6 = \mathbf{321.4 \text{ mm}^2}$$

Exercise 1

Find the area of the following shape.

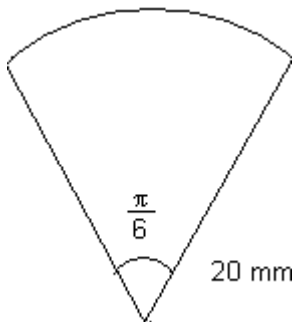
Give your answer to 3 significant figures.



Now check you answer.

Exercise 2

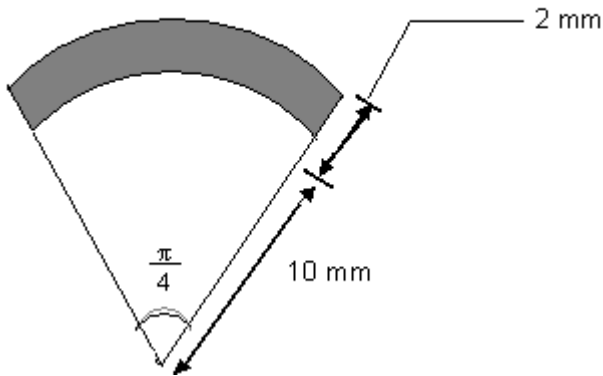
Calculate the area of the sector shown below.
The angle is in radians.



Now check you answer.

Exercise 3

Calculate the shaded area of the following shape



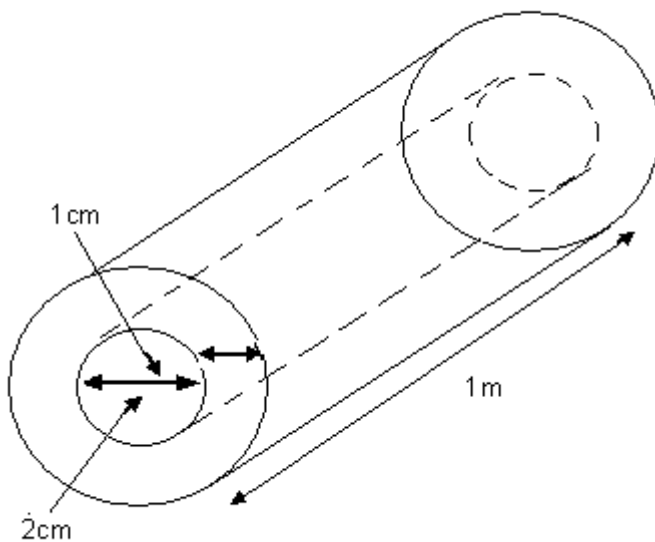
Now check your answer

Volumes, like areas, can be found by applying the correct formula for the shape.

Exercise 4

A tube is shown in the diagram below.

Find the volume of the metal in the cm^3 .



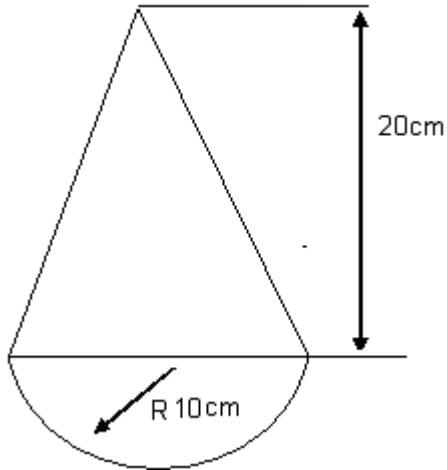
Be careful with the units!

Now check your answer.

Exercise 5

A toy is made by a cone on top of a hemisphere as shown below.

Calculate the volume of the toy.



Now check your answer.

ANSWERS

Exercise 1

$$\text{Area of triangle} = \frac{1}{2} \times 10 \times 10 = 50 \text{ mm}^2$$

$$\text{Area of the circle} = \pi \times 3^2 = 28.3 \text{ mm}^2 \text{ to 3 significant figures}$$

$$\bullet \bullet = 50 - 28.3 = 21.7 \text{ mm}^2$$

Now return to the text.

Exercise 2

$$\text{Area of sector} = \frac{1}{2} r^2 \theta \quad \text{where } \theta \text{ radians}$$

$$= \frac{1}{2} \times 20^2 \times \frac{\pi}{6} = \frac{100}{3} \pi \text{ mm}^2$$

or 104.7² if you work out π

Now return to the text.

Exercise 3

First find the area of the sector radius 12mm and then subtract the area of the sector radius 10mm.

$$\text{Area of sector} = \frac{1}{2} \times 12^2 \times \frac{\pi}{4} = 18\pi \text{ mm}^2$$

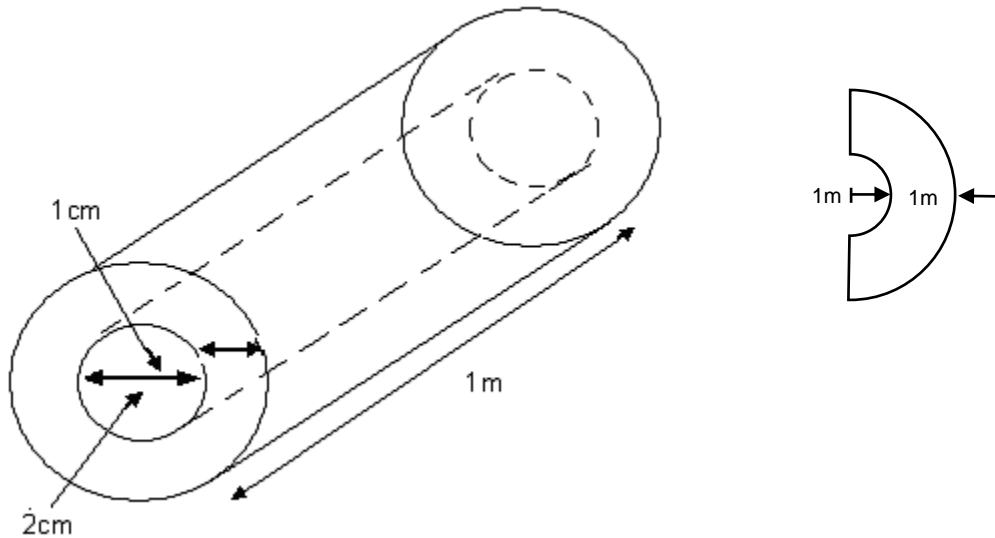
$$\text{Less } \frac{1}{2} \times 10^2 \times \frac{\pi}{4} = \frac{25}{2} \pi \text{ mm}^2$$

$$\text{Area of shaded part} = 18\pi - \frac{25}{2}\pi = \frac{11}{2}\pi \text{ mm}^2$$

Now return to the text.

Exercise 4

As the question asks for the answer in cm^3 convert all the units to cm before finding the volume.



Volume of cylinder

Without the bore is $\pi \times 2^2 \times 100 = 400\pi \text{ cm}^3$

Volume of bore is $\pi \times 1^2 \times 100 = 100\pi \text{ cm}^3$

∴ Volume of metal = $40\pi - 100\pi = 300\pi \text{ cm}^3$ or 942.6 cm^3

Now return to the text.

Exercise 5

Consider the hemisphere separately.

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 10^2 \times 20 = 2094.7 \text{ cm}^3$

Volume of sphere = $4\pi r^3$

So volume of hemisphere = $2\pi r^3 = 2 \times \pi \times 10^3 = 2094.7 \text{ cm}^3$

∴ Volume of toy = $2094.7 + 2094.7 = 4189.4 \text{ cm}^3$