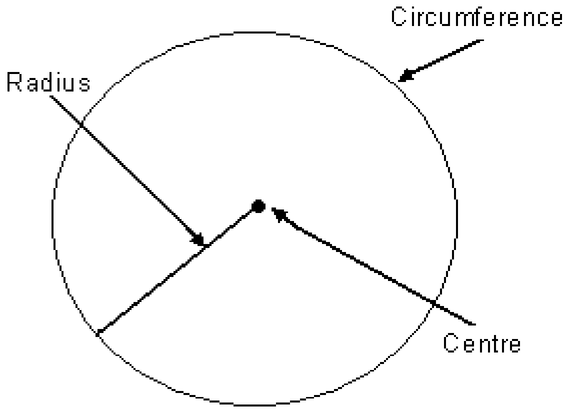
CIRCLES - DEVELOPMENT

A circle is defined as the path a point takes so that its distance from another (fixed) point remains constant.

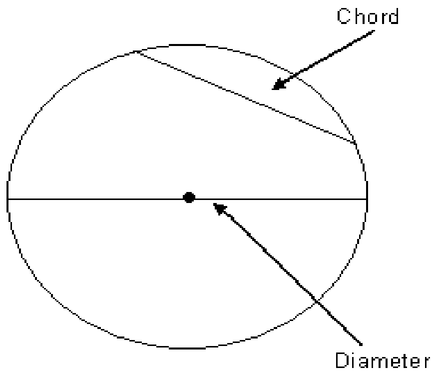
The fixed point is called the **centre** of the circle and the path that is drawn is called the **circumference** of the circle.



The distance between the centre of the circle and the circumference is called the **radius.** These are all shown on the accompanying figure.

Any line joining two points on the circumference is called a **chord** and any chord that passes through the centre we give a special name to: it's called a **diameter**.

These are both shown on the accompanying figure.

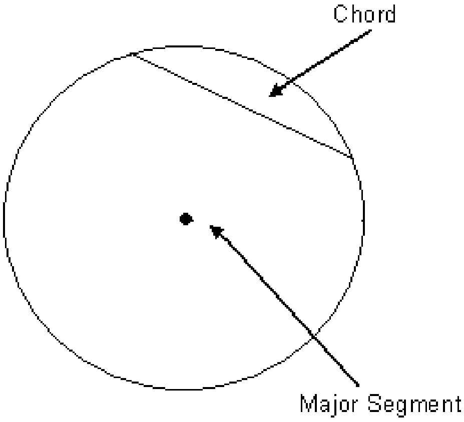


Exercise 1

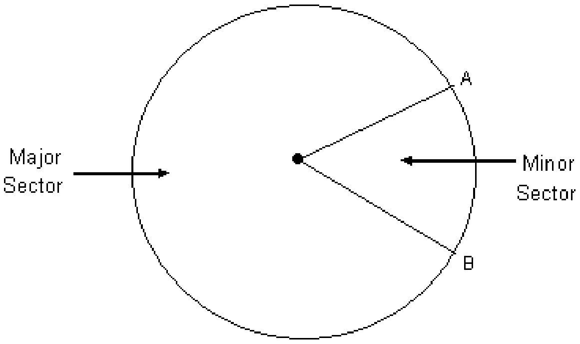
If the radius of a circle is 5cm, what is it's diameter?

Now check your answer.

A chord splits a circle into two **segments.** The smaller one is called the **minor segment** and the larger one is called the **major segment**. If the chord passes through the centre of a circle (i.e. the chord is a diameter) then the segments are both the same size. Segments are shown in the following figure.



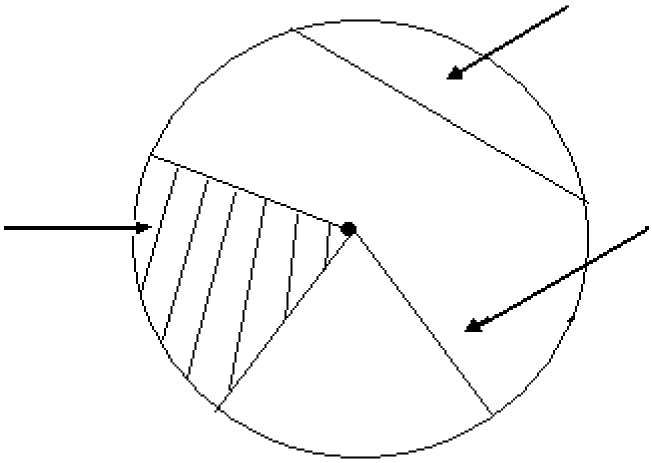
The area enclosed by two radii is called a **sector**. Here again we have a **major** and a **minor sector**. These are shown in the following figure.



The part of the circumference between the points ***A*** and ***B*** is called an ARC. Once again the short route from ***A*** to ***B*** is called the **minor** ARC and the long route the **major arc.**

Exercise 2

Label the accompanying diagram.



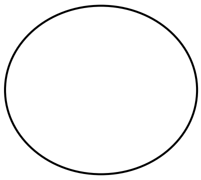
Now check your answer.

The Greeks noticed an interesting property of circles which we'll investigate in the next exercise.

Exercise 3

Using a piece of string find the length of the circumference for each of the circles. Don't worry about being too accurate.

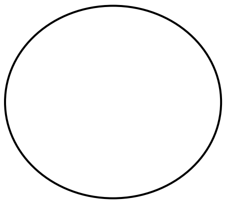
a)



diameter = 2.1cm

circumference = ________

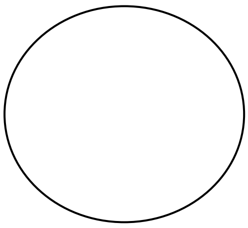
b)



diameter = 2.4cm

circumference = _________

c)



diameter = 2.8cm

circumference = _________

Exercise 4

Use your answers from Exercise 3 to work out the ratio

circumference ÷ diameter

in each of the following cases.

a) $\frac{\quad}{2.1}$

b) $\frac{\quad}{2.4}$

c) $\frac{\quad}{2.8}$

Now check your answers.

If the last two activities are done accurately the ratio always comes to the same answer - (three and a bit) regardless of the size of the circle. The Greeks noticed this and gave the ratio one of their letters. They called π (pi).

The value of π was worked out very accurately and your calculator will give you the value. Mine gives the value of π to be 3.141592654. π a number that goes on forever; it can never be written out entirely.

An approximation of π is $\frac{22}{7}$ but throughout this book use 3.142 or the button on your calculator. Also always give your answer to three significant figures.

$$\text{The ratio } \frac{\text{circumference}}{\text{diameter}} = \pi = 3.142$$

AREA AND CIRCUMFERENCE OF A CIRCLE

Now a link has been found between the circumference and diameter of a circle the following result can be stated.

Circumference of a circle = π ***d*** (Where ***d*** is the diameter)

The Greeks also discovered another relationship involving π namely:

Area of a circle = πr^2 (Where *r* is the radius)

Exercise 5

Calculate the area and circumference of a circle whose diameter is 10cm.

Now check your answer.

Exercise 6

Complete the table below.

Diameter (cm)	Radius (cm)	Circumference (cm)	Area (cm^2)
5			
15	7.5	47.1	177
			314
30			

Exercise 7

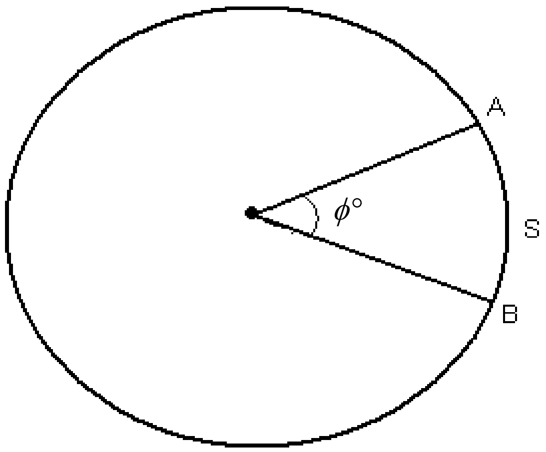
By writing d = 2 r (diameter is twice radius)

In equations for area and circumference derive alternative expressions for

$c = \pi d$ and $A = \pi r^2$.

Now check your answers.

We can also find the length of arc of a circle.

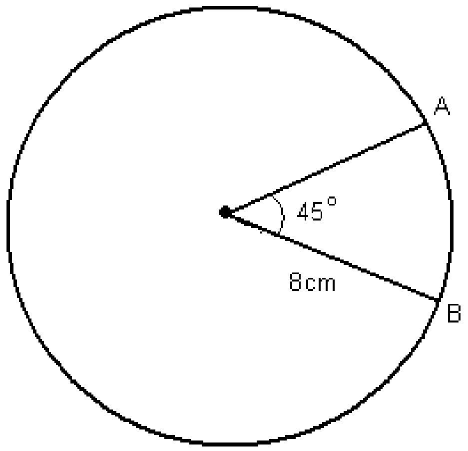


Let the length arc AB have length S. the angle at the centre is 0.As there are 360º in a circle then the AB is $\frac{\phi}{360}$ th of the whole circumference.

Length of arc(S) = $\frac{\phi°}{360°}$ x πd

Exercise 8

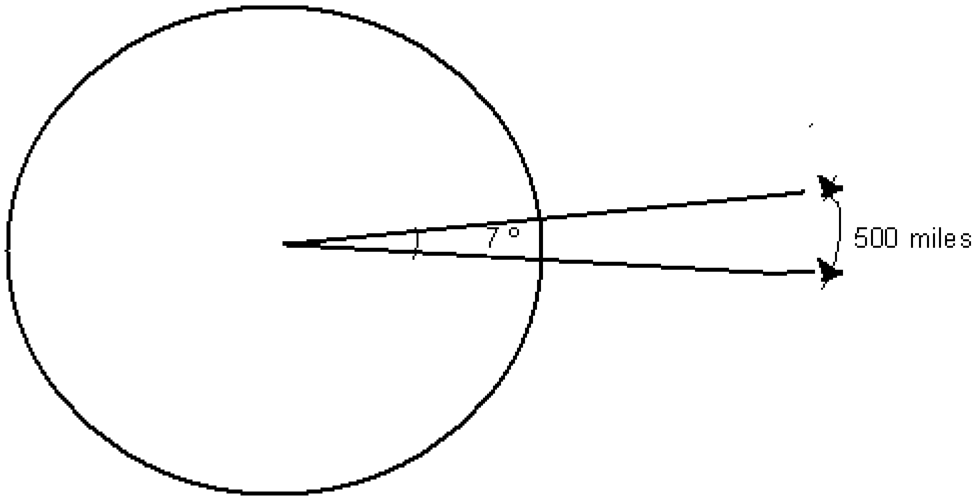
Find the length of the minor arc *AB.*



Now check your answers.

Exercise 9

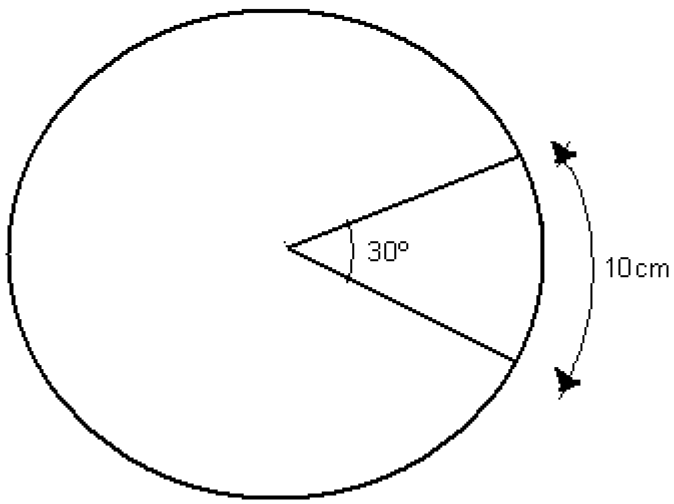
From the diagram below find the complete circumference of the circle shown below and hence fine the diameter.



Now check your answers.

Exercise 10

Calculate the area of the circle shown below.

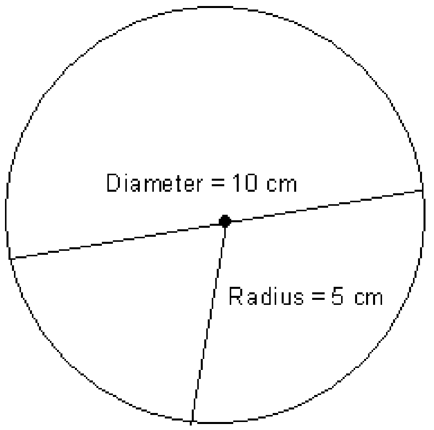


Now check your answers.

ANSWERS

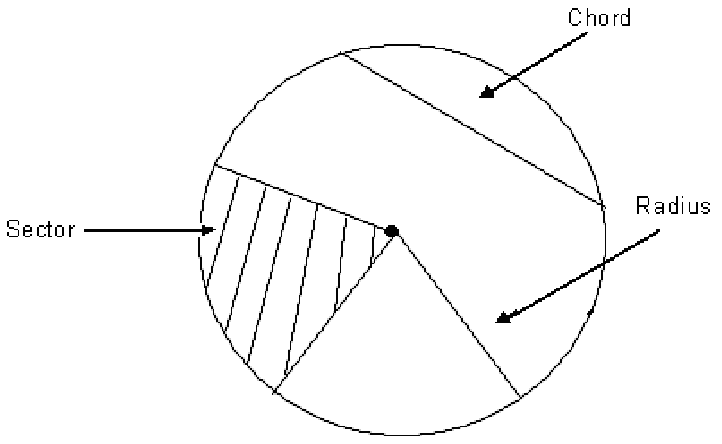
Exercise 1

10cm is the correct answer. If you didn't get this answer you might find it useful to look at the original diagram again and the one shown below. The diameter connects two points on the circumference and passes through the centre. The radius joins the centre and the circumference, so the diameter is twice the length of the radius.



Now return to the text.

Exercise 2



Now return to the text.

Exercise 3

As long as your answers are reasonably close it's O.K. It's very difficult to measure the circumference by running string around it.

a)6.8cm
b)	7.6cm
c)	8.9cm

Now return to the text.

Exercise 4

a) $\frac{6.8}{2.1}$ = 3.238

b) $\frac{7.6}{2.4}$ = 3.167

c) $\frac{8.9}{2.8}$ = 3.179

You should have got 3 point something for each one. If you didn't, check the length of your circumference again.

Now return to the text.

Exercise 5

Area of the circle is 78.5cm^2. If you didn't get this answer did you get 314cm^2 or 247cm^2? In the first case you forgot to halve the diameter to get the radius. In the second case you multiplied the radius by it and then squared the answer, whereas you should have squared the radius and then multiplied by π.

Area of circle = π x5^2 = π x 25 = 78.53981634

= 78.5cm^2 to 3 significant figures

Circumference of circle = π X 10 = 31.41592654

= 31.4cm

Be careful with the units. Remember area is in square units.

Now return to the text.

Exercise 6

The Answers are in bold.

Diameter (cm)	Radius (cm)	Circumference (cm)	Area (cm^2)
5	**2.5**	**15.7**	**19.6**
15	7.5	47.1	177
20	**10**	**62.8**	314
30	**15**	94.2	**707**

The third row needs some explanation. We know that the area is 314cm²

So that $\pi r^2 = 314$

And $r^2 = \frac{314}{\pi} = 100$ (3 significant figures) $r = \sqrt{100} = 10$cm

Now you can find the diameter (2r) and the circumference ($2\pi r$).
If you had any difficulty consult your tutor before moving on.

Now return to the text

Exercise 7

The correct answer to the first part is:

$C = 2\pi r$

This is because $C = \pi d$ and $d = 2r$

So, substituting 2r for d in $C = \pi d$, we get $C = 2\pi r$

The correct answer to the second part is:

$$A = \frac{\pi d^2}{4}$$

This is because $d = 2r$ so $r = \frac{d}{2}$

So, substituting in $A = \pi r^2$

We get $A = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$

Now return to the text.

Exercise 8

The answer is 16cm.

The radius is 8cm so the diameter is 16cm.

So the length of arc AB is

$\frac{45°}{360°} \times \pi \times 16 = 6.28$ cm (to 3 significant figures)

Now return to the text.

Exercise 9

For the first part we don't need the formula for arc length.

If 7º gives us 500 miles then 1º is

$\frac{500}{7}$ miles.

As there are 360º in a circle then the complete circle is:

$360 \times \frac{500}{7}$ = 25714 miles

if the circumference is 25714 miles then

πd = 25714 miles

So $d = \frac{25714}{\pi}$ = 8185 miles

This was how Eratosthenes of Cyrene (276 – 196 B.C.) calculated the dimensions of the Earth. His results were remarkably good.

Now return to the text.

Exercise 10

To find the area of the circle we need the radius.

We know the arc length so we can find the diameter and the radius.

So: $\frac{30}{360} \times \pi \times d = 10$

$\frac{1}{12} \times \pi \times d = 10$

Re-arranging: $\pi \times d = 10 \times 12 = 120$

$d = \frac{120}{\pi}$ = 38.2 cm (to 3 significant)

So the radius is 19.1 cm;

And the area is $\pi \times (19.1)^2 = 1146.08\text{cm}^2$

$= 1150\text{cm}^2$ (to 3 significant)

Remember $A = \pi r^2$
It's more a test of your powers of reasoning and logic than your ability to "do sums".