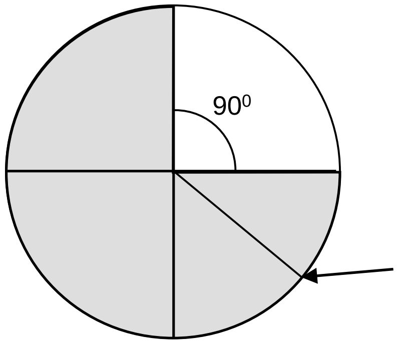
Learning Development

University of Northampton

CIRCLES, SECTORS AND RADIANS

SECTORS

The non-shaded area of the circle shown below is called a **SECTOR.**



In this example the **sector** subtends a right-angle (90^0) at the centre of the circle. The non-shaded area would still be a **sector** if the angle at the centre of the circle was larger, or smaller, than a right-angle (90^0).

We can see that the non-shaded **sector** is a quarter of the circle, so its area is one quarter of the total area of the circle.

Area of a sector = $\frac{1}{4}(\pi R^2)$ **for this example**

$$= \frac{1}{4} \times \pi \times 10^2$$

$$= \frac{1}{4} \times 100\pi$$

$$= 25\pi cm^2$$

Since, in this example, the angle subtended by the sector at the centre of the circle is 90^0 and the angle for a full circle 360^0 we can calculate the area of the sector as follows.

Area of sector = $\frac{90°}{360°} \times (\pi R^2)$

$$= \frac{90°}{360°} \times \pi 10^2$$

$$= \frac{1}{4} \times 100\pi$$

$= 25\pi$ cm^2 same as before.

The same argument applies for angles other than 90^0 and we can state a general formula as:

Area of sector = $\frac{\phi}{360}(\pi R^2)$

Where ϕ is the angle (in degrees) subtended by the sector at the centre of the circle.

Exercise 1

Complete the following table:

	Radius	ϕ	Area of sector
a)	10cm	60^0	
b)	25mm	200^0	
c)	10mm		50π mm^2
d)		30^0	75π mm^2

Now check your answers

So far we have measured the angle, subtended by the sector, in degrees.

RADIANS

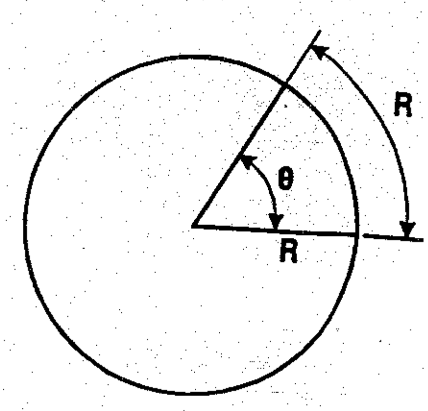
Another unit of angular measure, used frequently in engineering, is the **RADIAN**.

We are now going to discover how we can calculate the area of a sector when the angle it subtends is measured in radians.

Let's remind ourselves what a radian is.

A radian is defined as:

> The angle (ϕ) subtended at the centre of a circle by an arc of the circle equal in length to the radius.



Now, how many radians are there in a complete circle you may ask yourself? Well, the circumference of a circle is 2π times the radius that is $2\pi R$, and the angle subtended by one radian is equal to one radius R. So the number of radians in a complete circle is $\frac{2\pi R}{R}$ = 2π radians, or to put it another way, 2π radians = 360^0

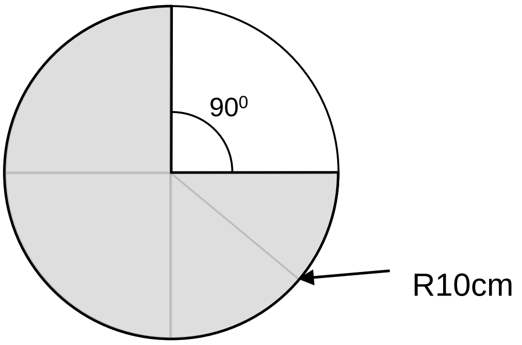
Exercise 2

Complete the table

a)	2π radians	360^0
b)	π radians	………^{0}
c)	………radians	90^0
d)	………radians	45^0
e)	1 radian	……..^{0}

Now check your answers.

Area of the non-shaded sector is:



Area $= \dfrac{90°}{360°}$ x ($\pi\, 10^2$)

$= \dfrac{1}{4} \times 100\pi = 25\pi$ cm^2

But we have previously discovered that $90^0 = \dfrac{\pi}{2}$ radians

And $360^0 = 2\pi$ radians

So we can also say

Area = $\dfrac{\frac{\pi}{2} radians}{2\pi radians} \times (\pi 10^2)$

$= \dfrac{1}{4} \times 100\pi = 25cm^2$ the same as before.

So it would seem reasonable to assume that:

$$\text{Area} = \frac{\phi radians}{2\pi radians} \times \pi R^2$$

$$= \frac{\phi}{2\pi_1} \times \frac{1}{\pi} R^2$$

$$= \frac{1}{2} R^2 \phi$$ when ϕ is in radians.

Area of sector = $\frac{1}{2} R^2 \phi$ **when** ϕ **is in radians.**

Exercise 3

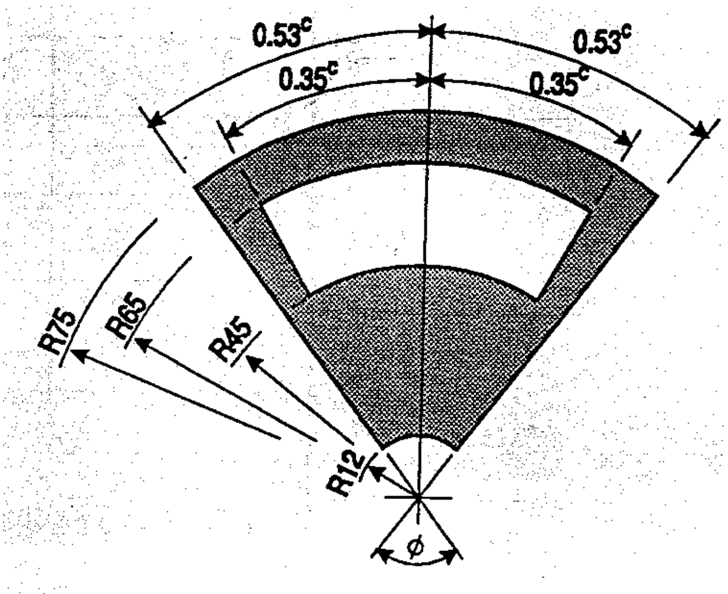
Complete the following table:

	Angle ϕ	Radius	Area of sector
A	0.8 rads	20mm	……..mm^2
B	…..rads	10mm	50π mm^2
C	$\frac{\pi}{2}$ rads	…..mm	400π mm^2

Now check your answers.

Exercise 4

Calculate the shaded area of the optical shutter blade and convert $\angle\phi$ to degrees of arc. The angles given are radians (^{c}) . Dimensions in millimetres.



Now check your answers.

ANSWERS

Exercise 1

	Radius	ϕ	Area of sector
a)	10cm	60^0	**16.67 π cm^2** **52.37 π cm^2**
b)	25mm	200^0	**347.2 π mm^2** **1091mm^2**
c)	10mm	**180^0**	50 π mm^2
d)	**30mm**	30^0	75 π mm^2

The Answers are in **bold**.

a) Area of a sector $= \frac{\phi}{360}$ x (π R^2)

$= \frac{60}{360} X \pi 10^2$

$= \frac{1}{6}$ x 100π

$= 16.67\pi$ cm^2

b) Area of a sector $= \frac{\phi}{360}$ x (π R^2)

$= \frac{200}{360} \times \pi 25^2$

$= \frac{200}{360} \times 625\pi$

$= 347.2\pi mm^2$

c) Area of sector $= \frac{\phi}{360}$ x (π R^2)

$50\pi = \frac{\phi}{360} X(\pi 10^2)$

$$\frac{50\pi \times 360}{100\pi} = \phi$$

$\phi = 180^0$

d) Area of sector $= \frac{\phi}{360}$ x (π R^2)

$$75\pi = \frac{30}{360} \times \pi R^2$$

$$\frac{75\pi}{\pi} \times \frac{360}{30} = R^2$$

$R^2 = 900$

R = 30mm

Now return to the text.

Exercise 2

a)	2π radians	360^0
b)	π radians	…**180^0**
c)	**…$\frac{\pi}{2}$……radians**	90^0
d)	**……$\frac{\pi}{4}$…radians**	45^0
e)	1 radian	…**57.3**.^{0}

The Answers are in **bold**.

a) To start you off, you have been given 2π radians = 360^0

b) If 2π radians = 360^0

Then π radians = $\frac{360°}{2}$ = 180^0

c) If 180^0 = π radians

Then 90^0 = $\frac{\pi}{2}$ radians

d) If 180^0 = π radians

Then 45^0 = $\frac{\pi}{4}$ radians

Similarly 60^0 = $\frac{\pi}{3}$ radians

These are useful to remember

30^0 = $\frac{\pi}{6}$ radians

e) If π radians = 180^0

then 1 radian = $\frac{180^0}{\pi}$ = 57.3^0

57.3^0 is an easy figure to remember and is accurate for most practical purposes. Where greater accuracy is required, use conversion tables or a scientific calculator.

Now return to the text.

Exercise 3

	Angle ϕ	Radius	Area of sector
A	0.8 rads	20mm	**160**mm^2
B	….π **.rads**	10mm	50 π mm^2
C	$\frac{\pi}{2}$ rads	….**40**.mm	400 π mm^2

The Answers are in **bold.**

a) Area of a sector $= \frac{1}{2}R^2\phi$

$= \frac{1}{2} \times 20^2 \times 0.8$

$= \frac{1}{2} \times 400 \times 0.8$

= 160mm^2

b) Area of a sector = $\frac{1}{2}R^2\phi$

50π mm^2 $= \frac{1}{2} \times 10^2 \times \phi$

$\frac{50\pi}{(\frac{1}{2} \times 10^2)} = \phi$

$\frac{50\pi}{50} = \phi$

$\phi = \pi$ radians (or 180^0)

c) Area of a sector = $\frac{1}{2}R^2\phi$

$$400\pi \text{ mm}^2 = \frac{1}{2} \times R^2 \times \frac{\pi}{2}$$

$$\frac{400\pi}{(\frac{1}{2} \times \frac{\pi}{2})} = R^2$$

$$400 \times 4 = R^2$$

$$1600 = R^2$$

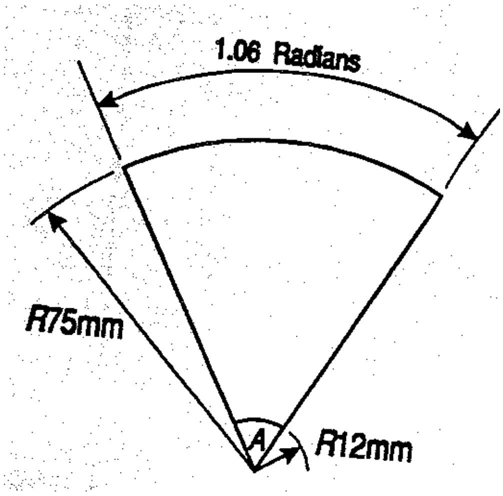
$$R = 40 \text{ mm}$$

Now return to the text.

Exercise 4

The shutter blade is made up from a number of sectors with a common centre and it is symmetrical about its centre lines.

a) First let's find the overall blank area.



Total area = $\frac{1}{2}R^2\phi$

$$= \frac{1}{2} \times 75^2 \times 1.06$$

$$= \frac{1}{2} \times 5625 \times 1.06$$

$$= 2981.25\text{mm}^2$$

Area A = $\frac{1}{2}R^2\phi$

$= \frac{1}{2}\times 12^2 \times 1.06$

$= \frac{1}{2}\times 144 \times 1.06$

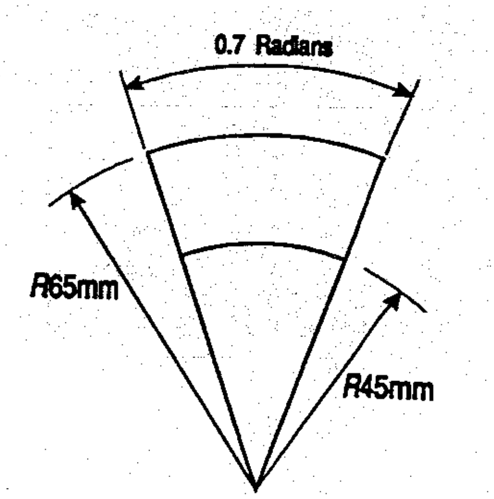
= 76.32mm^2

Shutter blank area is the difference between these two areas.

Shutter blank area = 2981.25 – 76.32

= 2904.93 mm^2

b) Now let's find the area of the "window". Again, this is the difference of two sectors



Area of larger sector

$= \frac{1}{2}R^2\phi$

$= \frac{1}{2}\times 65^2 \times 0.7$

$= \frac{1}{2}\times 4225 \times 0.7$

= 1478.75mm^2

Area of smaller sector

$$=\frac{1}{2}R^2\phi$$

$$=\frac{1}{2}\times 45^2\times 0.7$$

$$=\frac{1}{2}\times 2025\times 0.7$$

= 708.75mm^2

The “window” area is the difference between these areas.

Window area = 1478.75 – 708.75 = 770mm^2

c) To find the shaded area of the optical shutter, we take the window area from the shutter blank area.

Shaded are = 2904.93 – 770

= 2134.93mm^2

d) Finally we have to convert ϕ to degrees of arc. (ϕ is given as 1.06 radians) Remember we have found that 1 radian = 57.3^0

To convert radians to degrees, multiply by 57.3

To convert degrees to radians, divide by 57.3

So 1.06 radians = 1.06 x 57.3 = 60.74^0