CIRCLES IN COORDINATE GEOMETRY

Useful Data:

Equation of a circle centre (a, b) radius r is

$(x - a)^2 + (y - b)^2 = r^2$

The general form of equation for a circle is

$(x - a)^2 + (y - b)^2 = r^2$

This is a circle centre (a, b) radius r.

So $(x - 1)^2 + (y + 2)^2 = 9$ would be a circle centre (1, -2) radius 3.

Exercise 1

Find the centre and radii of the following circles;

a) $(x + 1)^2 + (y - 3)^2 = 25$

b) $(x - 4)^2 + (y - 1)^2 = 17$

Now check your answers.

Often the equation is not given in the form shown previously and we must then complete the square to find the standard form of the equation.

Example 1

Find the centre and radius of the circle with equation

$x^2 - 2x + y^2 + 4y - 4 = 0$

$(x - 1)^2 - 1 + (y + 2)^2 - 4 - 4 = 0$

$(x - 1)^2 + (y + 2)^2 = 9$

$\therefore$ centre (1, -2) radius 3

Exercise 2

Find the centre and radius of the circles with equations;

a) $x^2 + 2x + y^2 - 6y - 6 = 0$

b) $x^2 + 8x + y^2 + 6y + 16 = 0$

Now check your answers.

Remember if the coefficients of x^2 and y^2 are not unity (they must be the same) you should divide through each equation by the coefficient before completing the square.

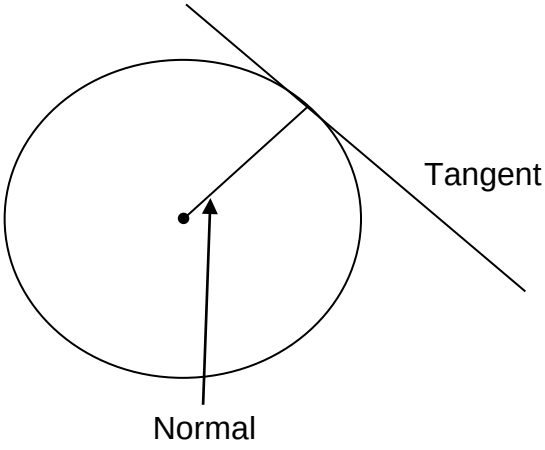
Exercise 3

Find the centre and radius of the circle with equation.

$2x^2 + 2y^2 + 4x + 2y + 1 = 0$

Now check your answer.

TANGENTS AND NORMAL'S TO A CIRCLE



By finding the gradients and a point we can find the equation of these straight lines.

Example 1

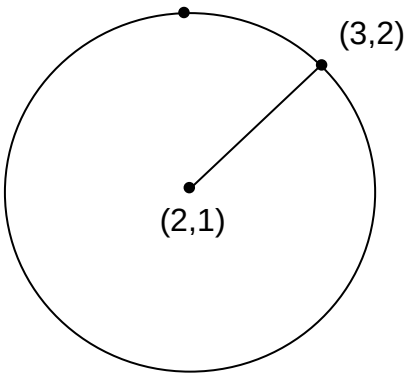
Find the equation of the tangent and normal to the circle

$(x - 2)^2 + (y - 1)^2 = 2$

at the point (3, 2)

the centre is (2, 1)

Let's draw a diagram.



The gradient of normal is:

$$\frac{2-1}{3-2} = \frac{1}{1} = 1 \qquad \text{(using } \frac{y_2 - y_1}{x_2 - x_1}\text{)}$$

So the equation of the **normal** is

$y - 2 = 1\ (x - 3)$ (using $y - y_1 = M\ (x - x_1)$)

The gradient of the tangent is -1 (product of gradient is -1)

Equation of **tangent** is $y - 2 + -1\ (x - 3)$

$$y = -x + 5$$

Exercise 4

Find the equation of the tangent and normal to the circle

$(x - 4)^2 + (y + 3)^2 = 20$ at the point (2, 1).

Now check your answers.

ANSWERS

Exercise 1

a) $(x + 1)^2 + (y – 3)^2 = 25$

Centre (-1, 3) radius 5

b) $(x – 4)^2 + (y +1)^2 =17$

Centre (4, -1) radius $\sqrt{17}$

There is no calculation involved the centre coordinates and the radius is, in each case, obtained by inspection.

Now return to the text.

Exercise 2

a) $x^2 + 2x + y^2 – 6y – 6 = 0$

$(x + 1)^2 – 1 + (y – 3)^2 – 9 – 6 = 0$

$(x + 1)^2 + (y – 3)^2 = 16$

Centre (-1, 3) radius 4

b) $x^2 + 8x + y^2 + 6y + 16 = 0$

$(x + 4)^2 – 16 + (y – 3)^2 – 9 + 16 = 0$

$(x + 4)^2 + (y + 3)^2 = 9$

Centre (-4, -3) radius 3

Now return to the text.

Exercise 3

$2x^2 + 2y^2 + 4x + 2y + 1 = 0$

First divide by 2

$x^2 + y^2 + 2x + y + \frac{1}{2} = 0$

$x^2 + 2x + y^2 + y + \frac{1}{2} = 0$

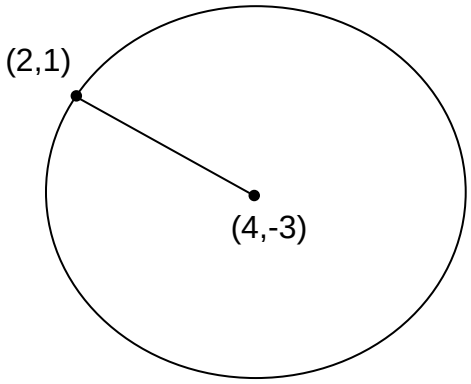
$(x + 1)^2 - 1 + (y + \frac{1}{2})^2 = \frac{3}{4}$

Centre (-1, $\frac{-1}{2}$) radius $\frac{\sqrt{3}}{2}$

Now return to the text.

Exercise 4

Draw a diagram! Centre (4, -3)



Gradient of normal $\frac{1-(-3)}{2-4} = \frac{4}{-2}$ = -2 (using $\frac{y_2 - y_1}{x_2 - x_1}$)

Equation of normal y – 1 = -2 (x – 2) (using $y - y_1 = M(x - x_1)$)

y = -2x + 5

Gradient of tangent = $\frac{-1}{\text{grad of normal}} = \frac{-1}{-2} = \frac{1}{2}$

Equation of tangent y – 1 = $\frac{1}{2}$ (x – 20

$$y = \frac{1}{2}x$$