Learning Development

University of Northampton

EQUATION OF A CIRCLE (CENTRE ORIGIN)

Linear equations of the form $y = mx + c$, for example $y = 2x + 1$, will always be a straight line when plotted on a graph paper. So if you see the equation $2y = x – 3$, you should realise that this is a straight line. From the equation you can also tell the gradient of the line and the point the line crosses the y axis.

Exercise 1

Tick which one of the following is not a straight line?

a) $y = 3x – 2$

b) $x + y = 2$

c) $xy =1$

d) $x = 4y +7$

Now check your answers

Just as a straight line can be described by an equation so can curves. The circle is no exception. It has an equation which identifies it as a circle and from which we can get information about the circle.

Just as the equation $y = 2x + 1$ describes a connection or link between y and x (y is always twice x add 1), so a link can be described for a circle.

Whereabouts a circle is drawn on graph paper depends on two things:

i) It’s centre.

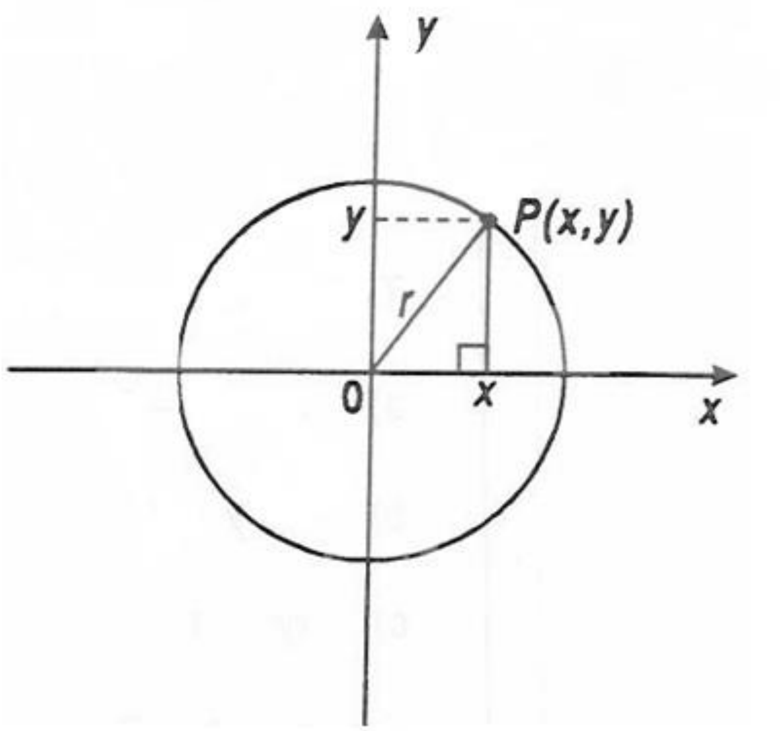
ii) It’s radius.

For the time being we will only consider circles whose radius is the origin (0, 0).

What we must try to do is describe any point on the circumference of the circle.

Exercise 2

Let the radius be r.



Using Pythagoras' Theorem, find r in terms of x and y.

Now check your answer.

So the equation of a circle **centre (0, 0)** radius r is:

$x^2 + y^2 = r^2$

so for example the equation

$x^2 + y^2 = 25$

is a circle centre (0, 0) radius 5 (r^2 is 25 so r = 5)

Exercise 3

Find the radii of the following circles.

a) $x^2 + y^2 = 16$

b) $x^2 + y^2 = 1$

c) $x^2 + y^2 = 2$

d) $2x^2 + 2y^2 = 50$

Now check your answers.

The last part of Exercise 3 showed that the equation of a circle must be rearranged sometimes to get the standard form $x^2 + y^2 = r^2$, just as we need to rearrange
$2y = 4x + 1$ to get the standard form $y = mx + c$.

Please remember that at the moment we are only considering circles with centre (0, 0). That was the assumption when we proved the equation in Exercise 2.

Exercise 4

Rearrange where necessary and find the radii of the following circle. One of the equations may not be that of a circle.

a) $2x^2 + 2y^2 = 100$

b) $x^2 + y^2 = 49$

c) $5x^2 + 5y^2 = 125$

d) $2x^2 + 3y^2 = 30$

Now check your answers.

Part of Exercise 4 gave us some more information about the equation of a circle. We now know that the coefficients of x^2 and y^2 must be the same.

Exercise 5

Tick which of the following are circles:

a) $x^2 + y^2 = 25$

b) $2x^2 - 2y^2 = 16$

c) $4x^2 + 4y^2 = 12$

d) $2x^2 + 3y^2 = 15$

Now check your answers.

Exercise 6

Find the co-ordinates of the points where the circle $x^2 + y^2 = 16$ crosses the x axis.

Now check your answers.

Exercise 7

Find the co-ordinates of the points where the circle $3x^2 + 3y^2 = 27$ crosses the y axis.

Now check your answers.

ANSWERS

Exercise 1

(c) is the correct answer because it is not a straight line. No way can $xy = 1$ be re-arranged in the form $y = mx + c$

(a) is already in the form $y = mx + c$ and, therefore, gives a straight line.

(b) $x + y + 2$ can be rearranged as $y = x + 2$ which is in $y = mx + c$ form. Therefore $x + y = 2$ also gives a straight line.

(d) $x = 4y + 7$ can be rearranged as $y = \frac{1}{4x} - \frac{7}{4}$ again this is in $y = mx + c$ form and, therefore, $x = 4y + 7$ gives a straight line.

The steps in the last one are: $x = 4y + 7$

$$x - 7 = 4y$$

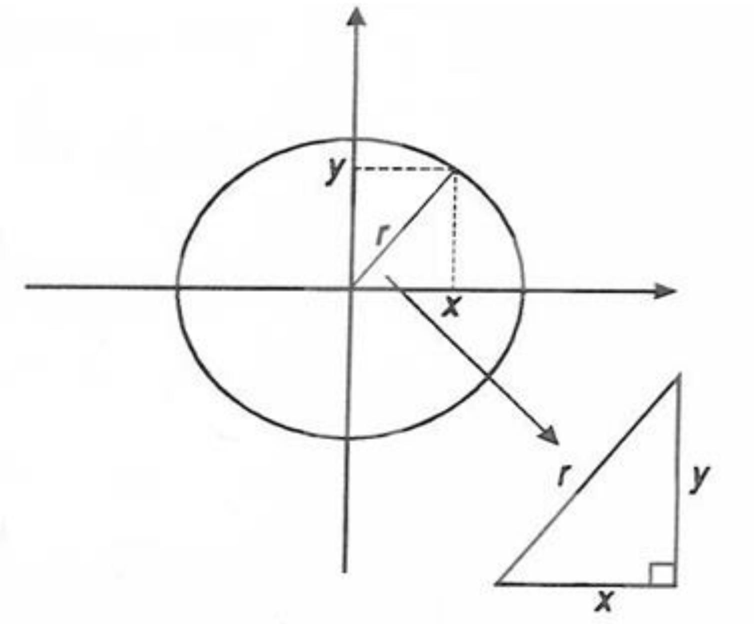
$$\frac{x}{4} - \frac{7}{4} = y$$

$$\therefore \quad y = \frac{x}{4} - \frac{7}{4}$$

Now return to the text.

Exercise 2

Hope you got $x^2 + y^2 = r^2$ (or $r = \sqrt{x^2 + y^2}$) because:



by Pythagoras $x^2 + y^2 = r^2$

Now return to the text.

Exercise 3

a) radius = 4 $r^2 = 16$ so r = 4

b) radius = 1 $r^2 = 1$ so r = 1

c) radius = $\sqrt{2}$ $r^2 = 2$ so r = $\sqrt{2}$

d) radius = 5 If you put $\sqrt{50}$ you can understand why!

The equation has x^2 and y^2 with coefficients of 1 so $2x^2 + 2y^2 = 50$ must be rearranged so that the coefficients of x^2 and y^2 are unity.

Therefore: $2x^2 + 2y = 50$

becomes: $x^2 + y^2 = 25$ (divide through by 2)

and: $r^2 = 25$ so r = 5

Now return to the text.

Exercise 4

a) radius = $\sqrt{50}$ $2x^2 + 2y^2 = 100$

$x^2 + y^2 = \underline{50}$ (÷ by 2)

b) radius = 7 no rearranging necessary

c) radius = 5 $5x^2 + 5y^2 = 125$

$x^2 + y^2 = 25$ (÷ 5)

d) This is not a circle. It cannot be rearranged to give the standard form, (what do we divide by, 2 or 3?)

Now return to the text.

Exercise 5

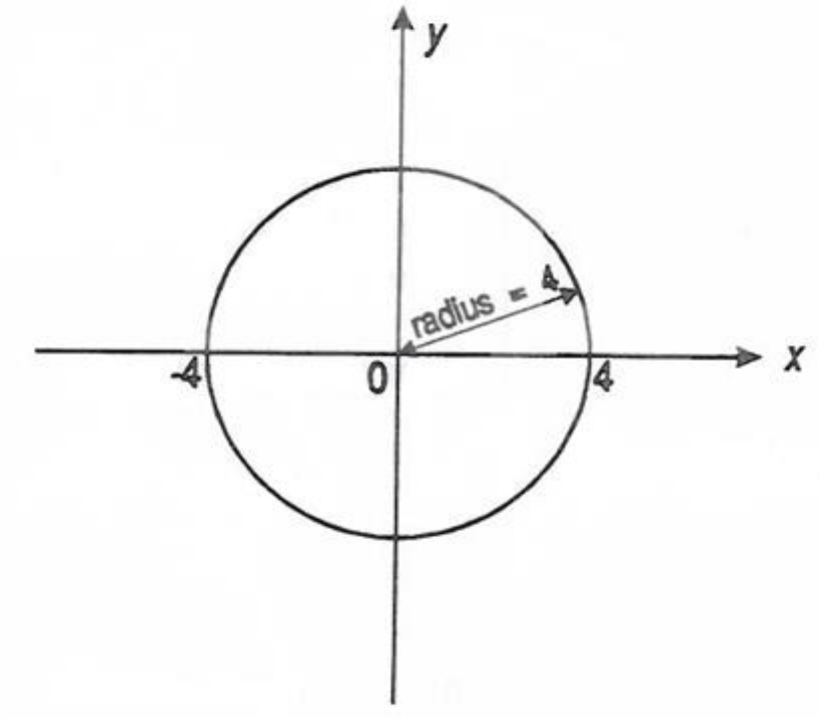
a) and c) are both circles because they can be rearranged where necessary in the form of $x^2 + y^2 = r^2$.

b) is not a circle because the coefficients of x^2 and y^2 are not the same.

Now return to the text.

Exercise 6

The circle crosses the x axis at (4, 0) and (-4, 0) because the radius of the circle is 4 and its centre is (0, 0) so



Now return to the text.

Exercise 7

The circle crosses the y axis at (0, 3) and (0, -3). Remember the equation has to be rearranged to give $x^2 + y^2 = 9$.

So the radius is 3 and the centre is (0, 0).