EQUATION OF A CIRCLE (CENTRE ORIGIN)

Linear equations of the form $y = mx + c$, for example $y = 2x + 1$, will always be a straight line when plotted on a graph paper. So if you see the equation $2y = x – 3$, you should realise that this is a straight line. From the equation you can also tell the gradient of the line and the point the line crosses the $y$ axis.

Exercise 1

Tick which one of the following is not a straight line?

a) $y = 3x – 2$

b) $x + y = 2$

c) $xy = 1$

d) $x = 4y + 7$

Now check your answers

Just as a straight line can be described by an equation so can curves. The circle is no exception. It has an equation which identifies it as a circle and from which we can get information about the circle.

Just as the equation $y = 2x + 1$ describes a connection or link between $y$ and $x$ ($y$ is always twice $x$ add 1), so a link can be described for a circle.

Whereabouts a circle is drawn on graph paper depends on two things:

i) It’s centre.

ii) It’s radius.

For the time being we will only consider circles whose radius is the origin ($0$, $0$).

What we must try to do is describe any point on the circumference of the circle.
Exercise 2

Let the radius be \( r \).

Using Pythagoras' Theorem, find \( r \) in terms of \( x \) and \( y \).

Now check your answer.

So the equation of a circle centre \((0, 0)\) radius \( r \) is:

\[ x^2 + y^2 = r^2 \]

so for example the equation

\[ x^2 + y^2 = 25 \]

is a circle centre \((0, 0)\) radius 5 (\( r^2 \) is 25 so \( r = 5 \))

Exercise 3

Find the radii of the following circles.

a) \( x^2 + y^2 = 16 \)
b) \( x^2 + y^2 = 1 \)
c) \( x^2 + y^2 = 2 \)
d) \( 2x^2 + 2y^2 = 50 \)

Now check your answers.
The last part of Exercise 3 showed that the equation of a circle must be rearranged sometimes to get the standard form $x^2 + y^2 = r^2$, just as we need to rearrange $2y = 4x + 1$ to get the standard form $y = mx + c$.

Please remember that at the moment we are only considering circles with centre $(0, 0)$. That was the assumption when we proved the equation in Exercise 2.

**Exercise 4**

Rearrange where necessary and find the radii of the following circle. One of the equations may not be that of a circle.

a) $2x^2 + 2y^2 = 100$

b) $x^2 + y^2 = 49$

c) $5x^2 + 5y^2 = 125$

d) $2x^2 + 3y^2 = 30$

Now check your answers.

Part of Exercise 4 gave us some more information about the equation of a circle. We now know that the coefficients of $x^2$ and $y^2$ must be the same.

**Exercise 5**

Tick which of the following are circles:

a) $x^2 + y^2 = 25$

b) $2x^2 - 2y^2 = 16$

c) $4x^2 + 4y^2 = 12$

d) $2x^2 + 3y^2 = 15$

Now check your answers.

**Exercise 6**

Find the co-ordinates of the points where the circle $x^2 + y^2 = 16$ crosses the $x$ axis.

Now check your answers.

**Exercise 7**

Find the co-ordinates of the points where the circle $3x^2 + 3y^2 = 27$ crosses the $y$ axis.

Now check your answers.
**ANSWERS**

**Exercise 1**

(c) is the correct answer because it is not a straight line. No way can \(xy = 1\) be re-arranged in the form \(y = mx + c\)

(a) is already in the form \(y = mx + c\) and, therefore, gives a straight line.

(b) \(x + y + 2\) can be rearranged as \(y = x + 2\) which is in \(y = mx + c\) form. Therefore \(x + y = 2\) also gives a straight line.

(d) \(x = 4y + 7\) can be rearranged as \(y = \frac{1}{4x} - \frac{7}{4}\) again this is in \(y = mx + c\) form and, therefore, \(x = 4y + 7\) gives a straight line.

The steps in the last one are:

\[
x = 4y + 7
\]

\[
x - 7 = 4y
\]

\[
\frac{x}{4} - \frac{7}{4} = y
\]

\[
\therefore y = \frac{x}{4} - \frac{7}{4}
\]

Now return to the text.

**Exercise 2**

Hope you got \(x^2 + y^2 = r^2\) (or \(r = \sqrt{x^2 + y^2}\)) because:

![Graph of a circle showing the relationship between x, y, and r](image)

by Pythagoras \(x^2 + y^2 = r^2\)

Now return to the text.
Exercise 3

a) radius = 4 \quad r^2 = 16 \quad so \quad r = 4

b) radius = 1 \quad r^2 = 1 \quad so \quad r = 1

c) radius = \sqrt{2} \quad r^2 = 2 \quad so \quad r = \sqrt{2}

d) radius = 5 \quad If \ you \ put \ \sqrt{50} \ you \ can \ understand \ why!

The equation has \( x^2 \) and \( y^2 \) with coefficients of 1 so \( 2x^2 + 2y^2 = 50 \) must be rearranged so that the coefficients of \( x^2 \) and \( y^2 \) are unity.

Therefore: \( 2x^2 + 2y^2 = 50 \)

becomes: \( x^2 + y^2 = 25 \) (divide through by 2)

and: \( r^2 = 25 \) \quad so \quad r = 5

Now return to the text.

Exercise 4

a) radius = \sqrt{50} \quad 2x^2 + 2y^2 = 100

\quad \quad \quad \quad \quad x^2 + y^2 = 50 \quad (\div \ 2)

b) radius = 7 \quad no \ rearranging \ necessary

c) radius = 5 \quad 5x^2 + 5y^2 = 125

\quad \quad \quad \quad \quad x^2 + y^2 = 25 \quad (\div \ 5)

d) This is not a circle. It cannot be rearranged to give the standard form, (what do we divide by, 2 or 3?)

Now return to the text.

Exercise 5

a) and c) are both circles because they can be rearranged where necessary in the form of \( x^2 + y^2 = r^2 \).

b) is not a circle because the coefficients of \( x^2 \) and \( y^2 \) are not the same.

Now return to the text.
Exercise 6

The circle crosses the $x$ axis at $(4, 0)$ and $(-4, 0)$ because the radius of the circle is 4 and its centre is $(0, 0)$ so

Now return to the text.

Exercise 7

The circle crosses the $y$ axis at $(0, 3)$ and $(0, -3)$. Remember the equation has to be rearranged to give $x^2 + y^2 = 9$.

So the radius is 3 and the centre is $(0, 0)$. 