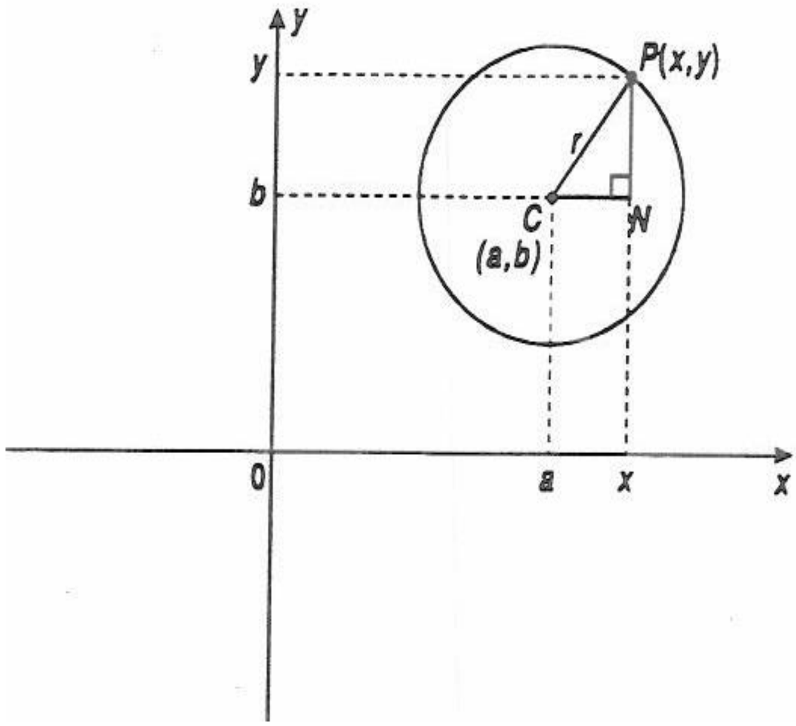
EQUATION OF A CIRCLE (CENTRE A, B)

A circle can have different centres as well as different radii. Let's now consider a more general equation for a circle with centre (a, b) and radius r



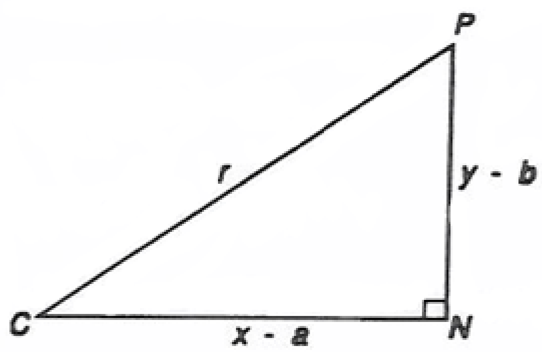
Exercise 1

Consider the diagram above. In terms of a, x, b and y what are the distances CN and PN.

Now check your answers.

Exercise 2

In the triangle below (which is taken from the previous diagram) find, using Pythagoras' Theorem, a relationship between r^2, x, y, a and b.



Now check your answer.

Now we have an equation for a circle with centre (a, b) and radius r.

So, comparing our standard equation to a given equation, we can determine the centre and radius of the circle.

Example

Find the centre and radius of the circle with equation.

$$(x - 3)^2 + (y - 4)^2 = 25$$

Compare with the standard form

$$(x - a)^2 + (y - b)^2 = r^2$$

$$(x - 3)^2 + (y - 4)^2 = 25$$

Can you see that a = 3, b = 4 and r = 5.

So this circle has centre (3, 4) radius 5.

Exercise 3

Find the centres and radii of the circle with equations:

a) $(x - 1)^2 + (y - 3)^2 = 16$

b) $(x = 3)^2 + (y - 2)^2 = 49$

Now check your answers.

Exercise 4

Write down the equation of the circles with the following centres and radii:

a) Centre (1, 2) radius 2

b) Centre (-1, 3) radius 4

c) Centre (-4, -3) radius $\sqrt{3}$

Now check your answers.

Exercise 5

Find the centres and radii of the following circles:

a) $(2x - 4)^2 + (2y - 2)^2 =25$

b) $(3x - 1)^2 + (3y + 4)^2 = 49$

Now check your answers.

Exercise 6

Find the centres and radii of the following circles:

a) $(5x + 10)^2 + (5y – 5)^2 = 125$

b) $(3x – 1)^2 + (2y +4)^2 = 16$ (Be careful!)

Now check your answers.

We've already discovered that the coefficients of x^2 and y^2 must be the same for an equation to be that of a circle. One other piece of information is available to us to give us a clue as to whether the equation is a circle or not.

Consider $(x – a)^2 + (y – b)^2 = r^2$

This is the equation of a circle centre (a, b) radius r.

Expands the brackets:

$$x^2 – 2ax + a^2 + y^2 – 2by + b^2 = r$$

or $$x^2 + y^2 – 2ax – 2by + a^2 + b^2 = r^2$$

The other thing apart from the coefficients of x^2 and y^2 being equal, that gives us a clue is that there is no xy term. So

$x^2 + y^2 + 3xy + 6 = 30$ **cannot** be a circle

where as $2x^2 + 2y^2 + 4x + 2y = 60$ **is** a circle

You may see the equation of a circle written in this way so its best to know **it is** a circle before you start.

To summarise.

For an equation to be a circle it **must** have the following:

a) coefficients of x^2 and y^2 which are the same;

b) there is **no** xy term;

c) r^2 must be positive.

Point c) is because you can't find the square root of a negative number.

Exercise 7

For each of the following equations state whether or not it is a circle and if not why not.

a) $(x - 1)^2 + (y - 2)^2 = 27$

b) $(2x + 3)^2 + (y - 1)^2 = 16$

c) $2x^2 + 2y^2 + 6x + 5y = 42$

d) $3x^2 + 3y^2 + 6xy + 4x + 4y = 17$

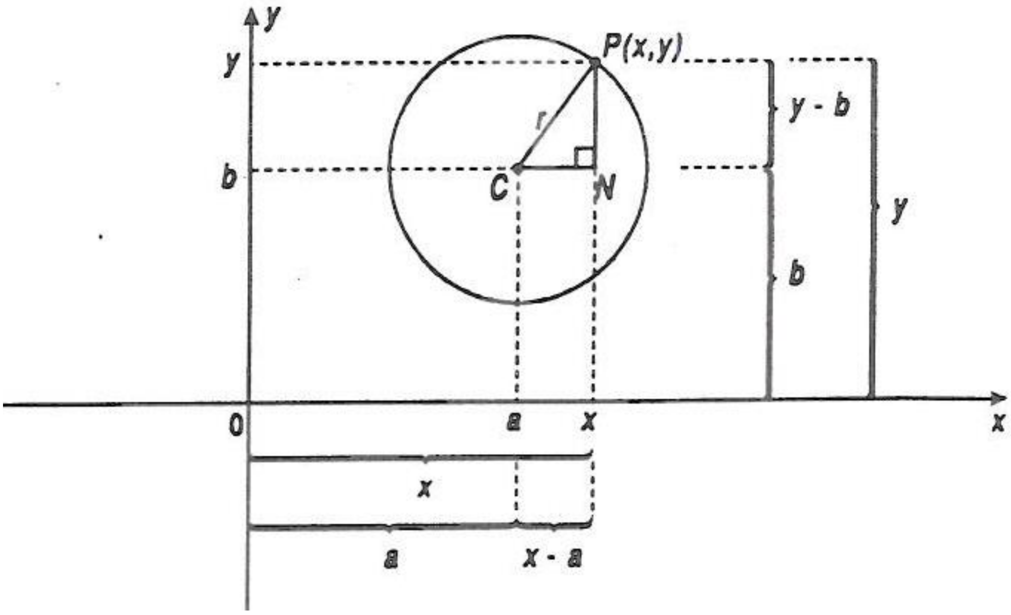
e) $x^2 + y^2 = -30$

Now check your answers.

ANSWERS

Exercise 1

CN is x –a and PN is y – b because



Now return to the text.

Exercise 2

$r^2 = (x - a)^2 + (y - b)^2$

If you expanded the right side to get

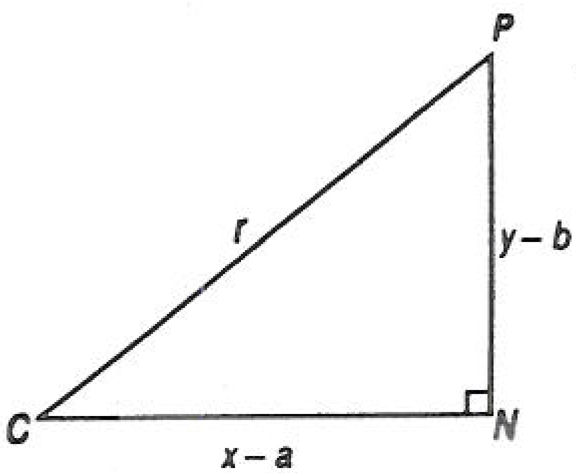
$r^2 = x^2 + y^2 - 2ax - 2ay + a^2 + b^2$

fine, but its usual to leave the equation as:

$r^2 = (x - a)^2 + (y - b)^2$

$(PC)^2 = (CN)^2 + (PN)^2$

$r^2 = (x - a)^2 + (y - b)^2$



Now return to the text.

Exercise 3

a) Centre (1, 3) radius 4

b) Centre (-3, 2) radius 7

Always compare with the standard equation.

a) $(x - a)^2 + (y - b)^2 = r$

$(x - 1)^2 + (y - 3)^2 = 16$

a = 1 b = 3 r = 4

b) $(x - a)^2 + (y - b)^2 = r^2$

$(x + 3)^2 + (y - 2)^2 = 49$

a = 3 [to give x- (-3) = x + 3] b = 2 r = 7

Now return o the text

Exercise 4

a) $(x - 1)^2 + (y - 2)^2 = 4$ a = 1 b = 2 r = 2

b) $(x + 1)^2 + (y - 3)^2 = 16$ a = -1 b = 3 r = 4

c) $(x + 4)^2 + (y + 3)^2 = 3$ a = -4 b = -3 r = $\sqrt{3}$

Now return to the text.

Exercise 5

a) If you got the centre (4, 2) and radius 5, hard luck. The coefficients of x^2 and y^2 must be unity. Take a look at the solution and try part b) before looking at the answer.

$(2x - 4)^2 + (2y - 2)^2 = 25$

To make the coefficients of x^2 and y^2 unity we must factorise both brackets by 2, but remember, when the 2 comes outside the bracket it will be 2^2 because of the squared bracket.

So: $2^2 (x - 2)^2 + 2^2 (y - 1)^2 = 25$

$4 (x - 2)^2 + 4 (y - 1)^2 = 25$

$(x - 2)^2 + (y - 1)^2 = \frac{25}{4}$ divide through by 4

The equation is now our standard form, So:

Centre (2, 1) radius $\frac{5}{2}$

b) The correct answer is: centre $\{\frac{1}{3}, -\frac{4}{3}\}$ radius $\frac{7}{3}$

$(3x - 1)^2 + (3y + 4)^2 = 49$

Factorise each bracket by 3 to get the coefficients equal to 1

$3^2 [x - \frac{1}{3}]^2 + 3^2 [y + \frac{4}{3}]^2 = 49$

$$9 [x - \frac{1}{3}]^2 + 9 [y + \frac{4}{3}]^2 = 49$$

$[x - \frac{1}{3}]^2 + [y + \frac{4}{3}]^2 = 49$

Now it's the standard form so:

Centre $[\frac{1}{3}, -\frac{4}{3}]$ radius $\frac{7}{3}$

Please note that, as before, the coefficients of *x* and *y* in the brackets must be the same.

Now return to the text.

Exercise 6

a) $(5x + 10)^2 + (5y - 5)^2 = 125$

$5^2 (x + 2)^2 + 5^2 (y - 1)^2 = 125$

$25 (x + 2)^2 + 25 (y - 1)^2 = 125$

$(x + 2)^2 + (y - 1)^2 = 5$

Centre (-2, 1) radius $\sqrt{5}$

b) This equation is not a circle! The coefficients of *x* and *y* are not the same so the equation cannot be rearranged to give us our standard form.

Now return to the text

Exercise 7

a) It is a circle. It's of the form $(x - a)^2 + (y - b)^2 = r^2$

b) It is not a circle, the coefficients of x and y are different.

c) It is a circle. It can be converted onto the form of
$(x - a)^2 + (y - b)^2 = r^2$

d) It is not a circle. It contains an xy term.

e) It is not a circle, r^2 is negative.