MENSURATION

Mensuration is the measurement of lines, areas, and volumes.
Before, you start this pack, you **need** to know the following facts.

When you see "kilo", it indicates 1000 in length, mass and capacity.

LENGTH

METRIC UNITS

mm = millimetre
cm = centimetre
m = metre
km = kilometre

IMPERIAL UNITS

ins = inches
ft = feet
yd = yards
mile = mile

LEARN

10mm = 1cm
1000mm = 1m
100cm = 1m
1000m = 1km

12ins = 1ft
(look at your ruler) 3ft =1yd
1760yd = 1mile

Example

Convert these

a) 5.21 km to metres

1km = 1000m

so, 5.21 = 1000 x 5.21

= 5210m

b) 0.056 km to metres

1km = 1000m

0.056 km = 1000 x 0.056

= 56m

c) 368 m to kilometres

1000m = 1km

$1\text{km} = \frac{1}{1000}\text{km}$

$368 = \frac{1}{1000} \times 368$

= 0.368km

d) 6.54 m to cm

1m = 100

6.54 = 100 x 6.54

= 654cm

e) 36.8m to mm

1m = 100 cm

36.8m = 100 x 36.8mm

= 36800 cm

f) 248mm to m

1000mm = 1m

$1\text{mm} = \frac{1}{1000}$

$248 = \frac{1}{1000} \times 248$

= 0.248m

MASS

METRIC UNITS	IMPERIAL UNITS
g = grams	oz = ounces
kg = kilogram	lb = pounds
	st = stones
	cwt = cwt
	t = ton

LEARN

1000 g = 1 kg	16 oz = 1 ib
1000 kg = 1 tonne	14 lbs = 1 st
	8 st = 1 cwt
	20 cwt = 1 ton

Example

Convert the following:

a) 540 g to kg

100 g = 1 kg

$$1 \text{ g} = \frac{1}{1000}$$

$$540 = \frac{1}{1000} \text{ x } 540 \text{ kg}$$

$$= 0.54 \text{ kg}$$

b) 2 kg to g

1 kg = 1000 g
2 kg = 1000 x 2g
= 2000 g

Learning Development

University of Northampton

CAPACITY

METRIC UNITS		IMPERIAL UNITS	
cl	= centilitres	pts	= pints
l	= litre	galls	= gallons
ml	= millilitres	8 pts = 1 gall	

100 cl = 1 l

1000 ml = 1 l

NB
100 cl = 1000 cm^3 = 1000 ml = 1 l

Example

Convert the following :

a) 5. 6 l to ml

1l = 1000 ml
5.6 l = 1000 x 5.6 ml
= 5600 ml

b) 4600 cm^3 to l

1000 cm^3 = l

$$4600 = \frac{1}{1000} \times 4600$$

= 4.6 l

c) 4600 ml to l

Method of working is the same as for example 2, and the answer is:

= 4.6 l

Exercise 1

1.	123 mm to cm	2.	247cm to m
3.	5469 m to km	4.	0.632 km to m
5.	7.5 km to m	6.	6.3 kg to g
7.	734 g to kg	8.	4 l to cm^3
9.	5437 cl to l	10.	1643 cl to l

Now you can proceed, using the information to calculate the measurements of lines, areas and volumes.

HINT - draw a diagram, where possible, to help you see the problem.

LINE

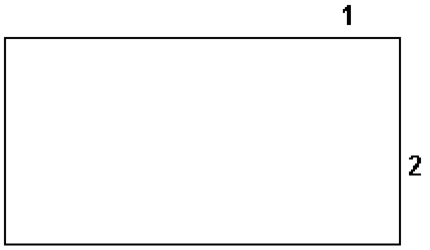
Measurement of **one dimension** only i.e., length.

Unit of measurement will be mm, cm, m, kin, ins, ft. yds, mile...

You will be asked to find the **perimeter** of figures with straight lines or the **circumference** of circles.

Both of these measure the **distance around a figure.**

Area

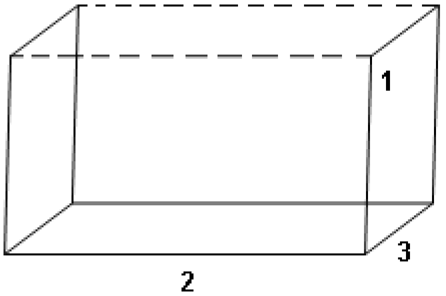


Measurement of **two dimensions** i.e., length multiplied by width (or breadth).

Unit of measurement is **"square"** units, i.e., mm^2, cm^2, in^2, kin^2, ins^2, ft^2, yd^2, $mile^2$.

You are measuring the area of a **flat surface** or **plane.**

VOLUME



Measurement of **three dimensions** i.e., length, width (or breadth) and height.

Unit of measurement is **"cubic"** units, i.e., mm^3, cm^3, m^3, km^3, ins^3, ft'^{3}, yd^3, $mile^3$.

You are measuring a solid.

Length --- One Dimension
Area --- Two Dimensions “Square Units”
Volume --- Three Dimensions “Cubic Units”

Shapes You Need To Know

There are several shapes which you will be asked to consider, when dealing with length area and volume. Here you will be given:

square
rectangle
parallelogram
rhombus
trapezium
triangle
circle

List Of Abbreviations Used
(you may need to refer to these in this and following packs).

A = area
b = breadth
b = base
C = circumference
d = diameter
h = perpendicular, height or altitude
l = length
l = slant height
π = “pi” - Greek letter (pronounced “pie”) with the value 3.14 or $\frac{22}{7}$ or $3\frac{1}{7}$

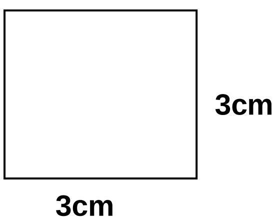
(Pi is the ratio of the circumference of a circle to its diameter)

P = perimeter
r = radius
θ = “theta” - Greek letter used to represent an unknown angle.
V = volume
w = width

Now read the following very carefully, making notes if necessary.

SQUARE AND RECTANGLE

a) Square



b) Rectangle

Perimeter is the distance all the way round the shape.

Perimeter = l + l + b + b
= 2l + 2b
= 2(l + b)

Square

P = 2(l + b)

= 2(3 + 3) cm

= 2(6) cm

P = 12 cm

Rectangle

P = 2(l + b)

= 2(6 + 4) cm

= 2(10) cm

P = 20 cm

Area = length x breadth

Square

A = lb

= l x b

= 3 x 3 cm^2

= 9 cm^2

Rectangle

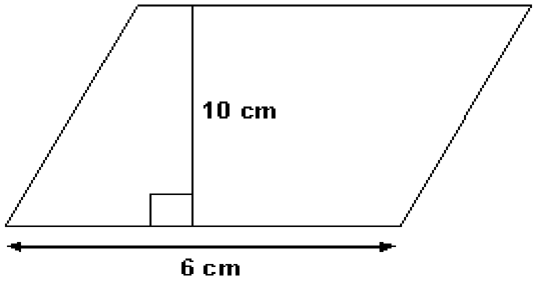
A = lb

= 1 x b

= 6 x 4 cm^2

= 24 cm^2

PARALLELOGRAM

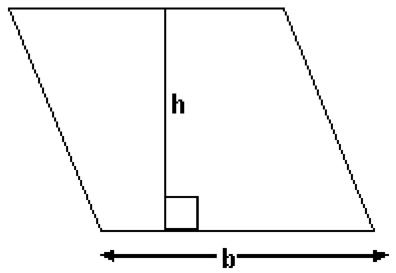


Area = base x perpendicular height

= 6 cm x 10 cm

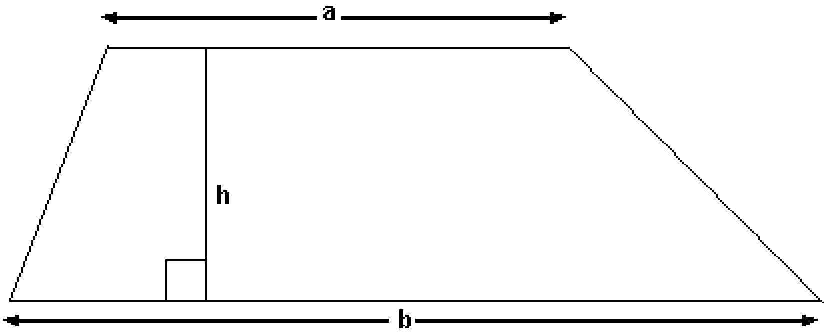
= 60 cm^2

RHOMBUS (All side equal in length)



Area = b x h

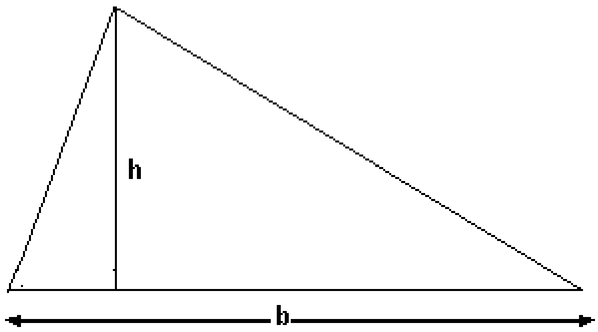
TRAPEZIUM



a and b are the lengths of the parallel sides, and h is the perpendicular height or altitude between the sides a and b.

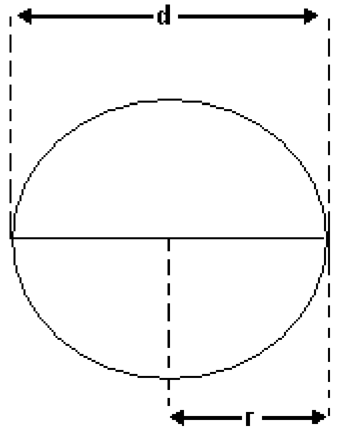
Area = $\frac{(a+b)}{2}h$

TRIANGLE



Area = $\frac{b \times h}{2}$

CIRCLE (distance around)



Circumference

$C = \pi d$

Or $C = 2\pi r$

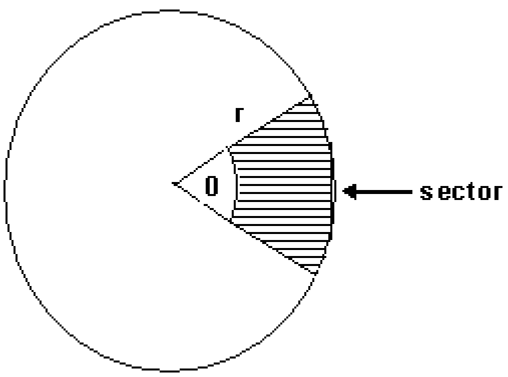
NB Diameter = 2 x radius

Therefore radius = $\dfrac{\text{Diameter}}{2}$

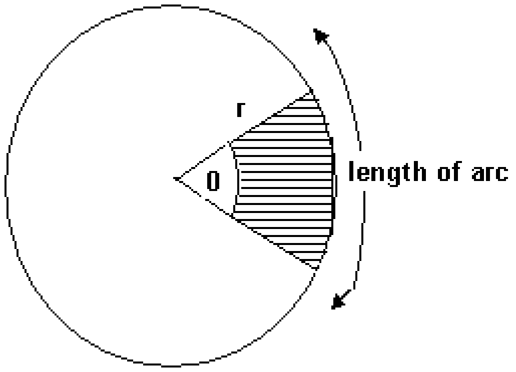
Area

$a = \pi r^2$

Sector of a Circle



Area of a sector: $\pi r^2 \times \dfrac{\theta}{360}$



$$\text{Length of arc} = 2\pi r \times \frac{\theta}{360}$$

VOLUME

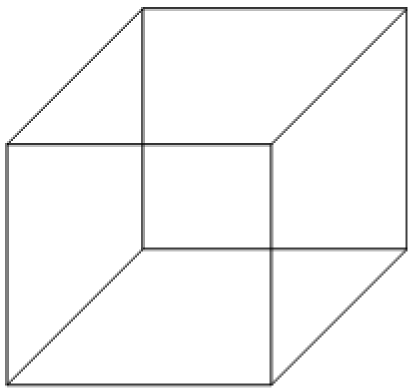
Once you have understood the steps taken to find areas, you can easily go on to find volumes of regular solids, because

VOLUME = lbh

i.e. Volume = the length or height of a solid multiplied by its cross-sectional area.

Look at these examples:

CUBE



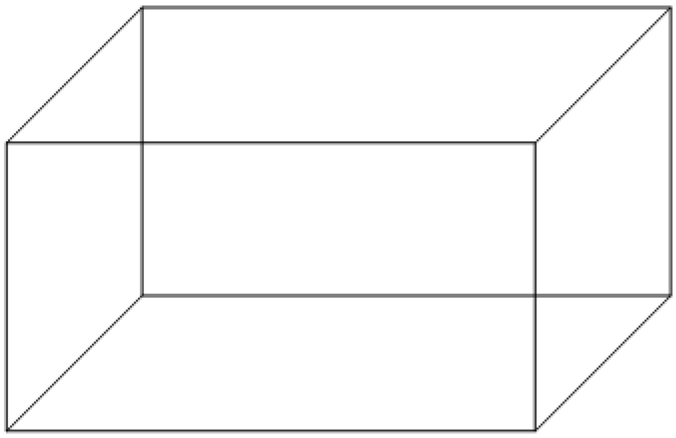
Volume of cube = area of end x height (or length)

The end is a square, so:

area of square= lb

volume of cube = lbh

CUBOID



Volume = area of end **multiplied by** height or length of solid.

The end is a rectangle

area of rectangle = lb

volume of cuboid = lbh

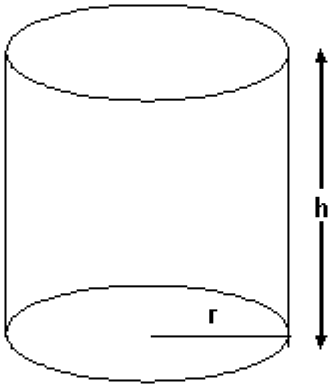
PRISM

Volume = area of cross-section x perpendicular height

If the solid is a regular shape, it is possible to find its volume, simply by multiplying the area of the cross-section by the height or length of the solid.

Here are some special kinds of prism.

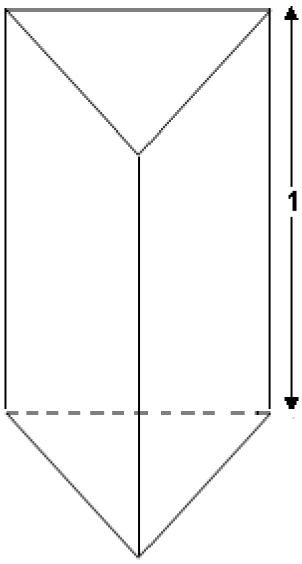
CYLINDER (circular prism)



Area of circle = πr^2

Volume of cylinder = $\pi r^2 h$

TRIANGULAR PRISM



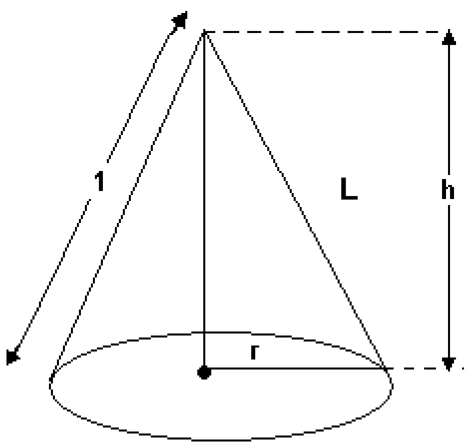
End is a triangle

Area of triangle = $\frac{b \times h}{2}$

Volume of prism = $\frac{b \times h}{2}$ x1

NOTE: The following volumes and surface areas must also be understood, even though they are not found by the method given previously.

CONE



Remember l = slant height

h = altitude or perpendicular height

Curved surface area of cone = πrl

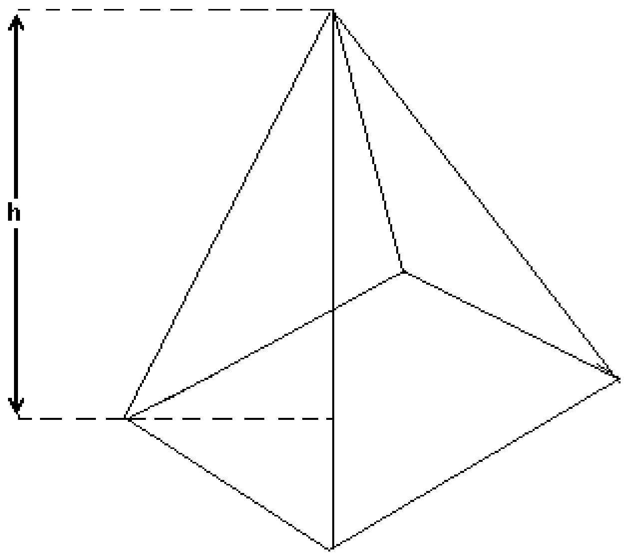
Volume of cone = $\frac{1}{3}\pi^2$

Sphere ("Globe shape")

Surface Area of sphere = 4 π^2

Volume of sphere = $\frac{4}{3}\pi r^3$

PYRAMID



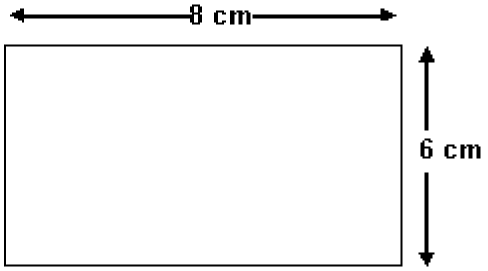
Surface Area of pyramid = **SUM** of the area of triangles which form the sides **PLUS** the area of the base (A)

Volume of pyramid = $\frac{1}{3}$Ah

You **do not** need to remember **all** these formulae some are given to you in the examination on a reference leaflet.
.
Read through the following example.

Example 1

Find the area of a rectangle whose sides are 6 cm and 8 cm respectively.



Area = length x breadth

= 6 cm x 8 cm

= 48 cm ^{2}

Example 2

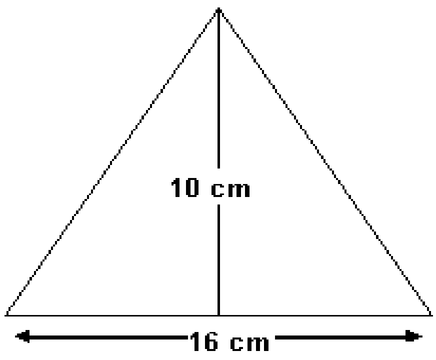
Find the area of a triangle whose base is 16 cm, and the altitude is 10 cm.
(Altitude is the perpendicular height).

$$\text{Area} = \frac{b \times h}{2}$$

$$\text{Area} = \frac{16\text{ cm} \times 10\text{ cm}}{2}$$

$$\text{Area} = \frac{160\text{ cm}^{2}}{2}$$

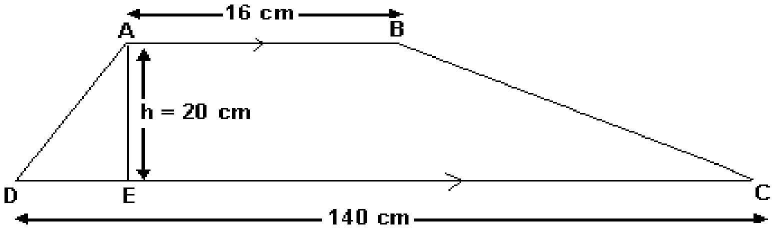
Area = 80 ^{2}



Example 3

Find the area of a trapezium ABCD, the parallel sides of which measures 16 cm and 1.4 cm, and the perpendicular distance between them is 0.2 cm.

First of all, change all the units to cm!



AB = 16 cm

DC = 1.4 cm = 140 cm

AE = 0.2 cm = 20 cm

Area of ABCD = $\frac{l + b}{2}$

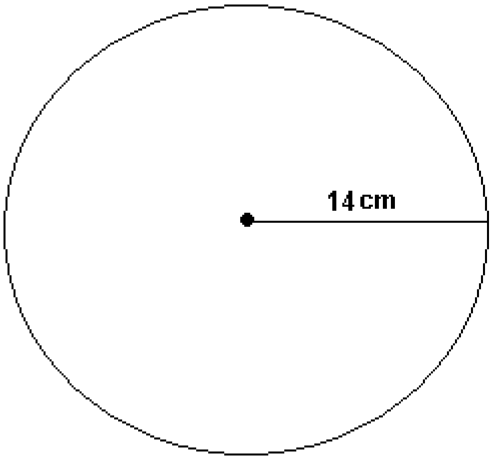
$= \frac{140 + 16}{2}$ x 20

= 78 x 20 cm^2

=1560 cm^2

Example 4

Find the area of a circle with radius 14 cm.



Take π as $\frac{22}{7}$

Area = πr^2

$= \frac{22}{7} \text{ x } \frac{14}{1} \text{ x } \frac{14}{1} \text{ cm}^2$

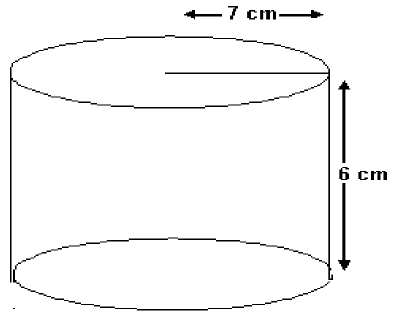
CANCEL WHERE POSSIBLE

$= 44 \times 14 cm^2$

$= 616\ cm^2$

Example 5

Find the total surface area of a cylinder of height 6 cm and radius of base 7 cm.



Total surface $= 2\pi rh + 2\pi r^2$

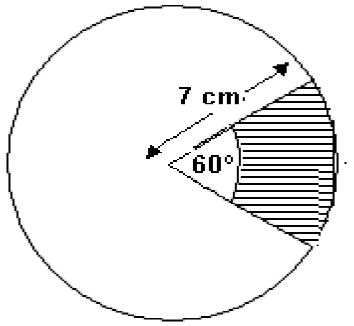
$$= \left(\frac{2}{1} \times \frac{22}{7} \times \frac{7}{1} \times \frac{6}{1}\right) + \left(\frac{2}{1} \times \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1}\right)$$

$= 264 + 308\ cm^2$

$= 572\ cm^2$

Example 6

If r is the radius and θ is the angle subtended at the centre by an arc, find the length of the arc, when r = 7 cm, and θ = 60º.



$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$

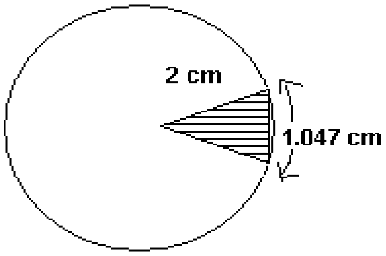
$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 7$$ Use your calculator

$= \frac{22}{3}$

length of arc = $\mathbf{7\frac{1}{3}}$ cm

Example 7

If l is length of an arc, r is the radius and θ is the angle subtended by the arc, find θ when r = 2 cm and l = 1.047 cm.



$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$

$$1.047 = \frac{\theta}{360} \times \frac{2}{1} \times \frac{22}{7} \times \frac{2}{1}$$

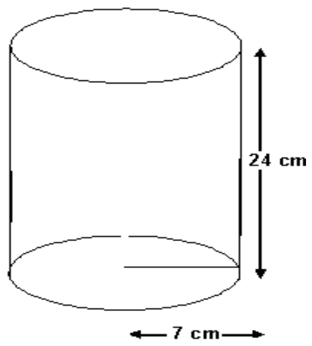
$$\theta = \frac{1.047 \times 360 \times 7}{2 \times 22 \times 2}$$ Transposition of formula! If in doubt ask.

$= 29.98^{o}$

θ is approximately 30^{o}

Example 8

Find the volume of a cylinder, whose base radius is 7 cm and height is 24 cm.



$\pi = 3\frac{1}{7}$

Volume = $\pi r^2 h$

$= \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1} \times \frac{24}{1}$

= 3696 cm^3

Example 9

Find the volume of a sphere of radius 6 mm. (Take π to be 3.142 and give your answer to the nearest whole number).

Volume = $\frac{4}{3} \pi r^3$

$= \frac{4}{3} \times 3.142 \times 6 \times 6 \times 6$

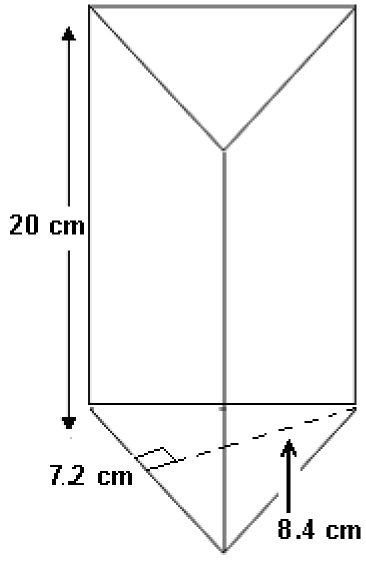
= 288 x 3.142

= 904.896 mm^3

= 905 mm^3

Example 10

Find the volume of the triangular prism shown below.



Volume = area of cross section x perpendicular height

Volume = $\frac{1}{2} \times 7.2 \times 8.2 \times 20$

= 604.8 cm^3

Example 11

What us the radius of spherical balloon, if its volume is 24 cm³.

Take π as $\frac{22}{7}$

Volume of balloon = $\frac{4}{3}\pi r^3$

$$24 = \frac{4}{3}\pi r^3$$

$$24 = \frac{4}{3} \times \frac{22}{7} \times \frac{r^3}{1}$$

$$24 = \frac{88}{21} r^3$$

$$\frac{24 \times 21}{88} = r^3$$

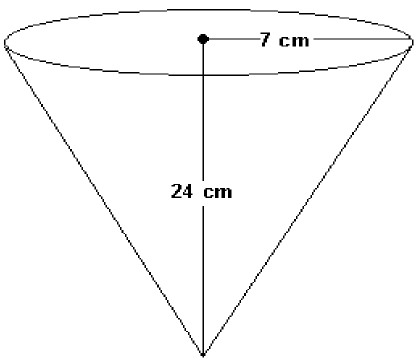
This means the cube root

$$\sqrt[3]{\frac{21 \times 21}{88}} = r$$

1.789 cm = r^3 (using calculator)

Example 12

Find the volume of a cone of radius 7 cm and height 24 cm.



Volume = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1} \times \frac{24}{1}$$

$$= 1232 \text{ cm}^3$$

NOTE
Show all working out - it will help your tutor to see how you are thinking.

AND
In your examination **marks will be awarded for working out, so it is important that you set out your answers in a clear manner.**

Do not put numbers down without an explanation of what they are.

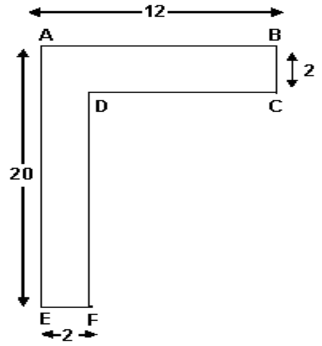
Do not be afraid to write a sentence giving the important steps and your answer!

Exercise 2

Draw a diagram for each problem. Show all working out.

1. The area of a rectangle is 200 mm^2. If the width is 25 mm, find the length.

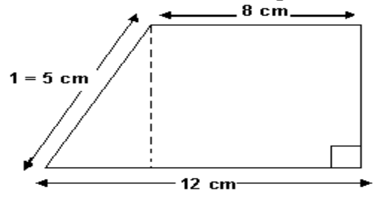
2. Find the area of the shape.



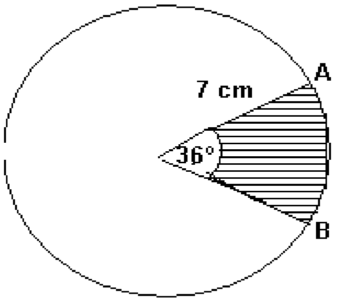
3. Find the area of a triangle with base 8 cm, and perpendicular height 6 cm.

4. Find the area of a parallelogram, whose base is 8 cm and whose altitude is 5 cm.

5. Find the area of this figure. **HINT** – you must find the altitude first.



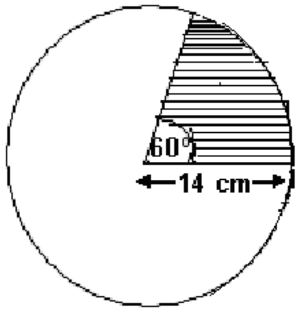
6. Find the length of the arc **AB**.



7. Find the area of the circle, whose diameter is 14 cm. **HINT –** find radius first. Take π as $\frac{22}{7}$

8. In a circle the length of an arc of a sector is 18.8 cm. the radius is 9 cm. If π = 3.142, what is the size of the angle?

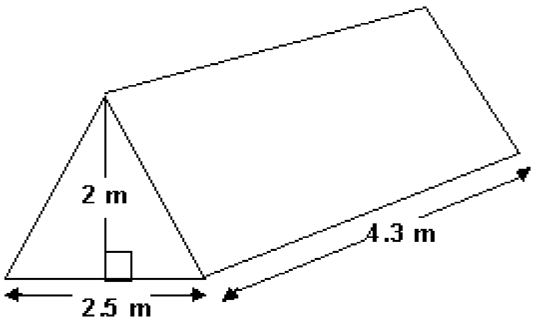
9. What is the area of the sector. Take π as $\frac{22}{7}$



10. A cone has a diameter of 70 cm and a height of 10 cm. What is the volume?

11. In a cylinder, the height is 12.2 cm, and the radius is 3.7. Find its volume?

12. Find the volume of this triangular prism.



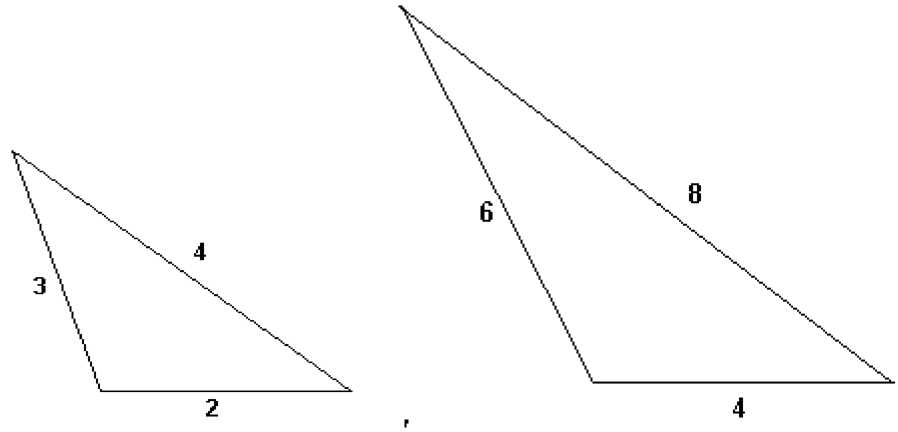
13. In a sphere, the diameter, is 15.68 cm. What is its volume?

14. A pyramid has a square base of side 6 cm. the perpendicular height of the pyramid is 10 cm, and the slant height is 2.4 cm. Find its volume.

SIMILAR SOLIDS

Similar means the same shape, but having different sizes.

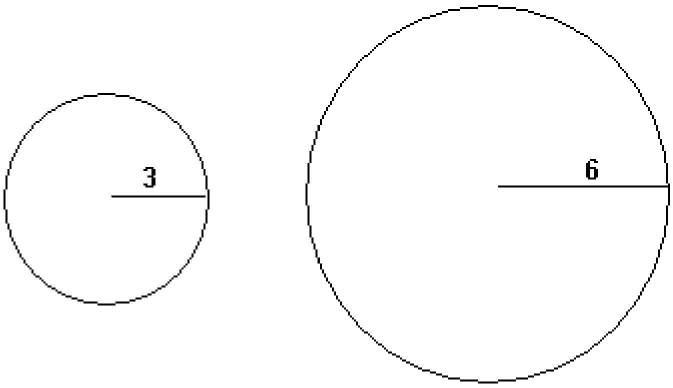
Look at these diagrams



Ratio of Sides

To find the ratio of the sides, you compare the lengths of the sides. In the example above we say that the sides are the ratio 3:6 which cancels to 1:2.

Ratio of sides = 1:2



Ratio of radius = 1:2

Ratio of Areas

The ratio of the volume is the cube of the ratio of the corresponding sides:

Ratio of volumes = $1^3:2^3$

= 1:8

NOTE:
If two solids are similar and the lengths in one are five time the corresponding lengths in the other.

State

1. The ratio of their corresponding areas, and
2. The ratio of their volumes.

Comparing the **sides** gives:

Ratio is 1:5

Comparing the **areas** gives:

Ratio is $1^2:5^2$
1 :25

Comparing the **volumes** gives:

Ratio is $1^3:5^3$
1:125

REMEMBER:

- **the ratio of the areas of similar figures is the square of the ratio of the corresponding sides.**
- **the ratio of the volumes of similar figures is the cube of the ratio of the corresponding sides.**

Example 1

A photograph measuring 8 ins by 11 ins, is enlarged to 16 ins x 22 ins

i) What is the ratio of the sides?

Sides are in the ratio 11:22
1:2

ii) What is the ratio of the areas?

Areas are in ratio $1^2:2^2$
1:4

Example 2

A poster measuring 35cm by 15cm is reduced to 7cm x 3 cm.

i) What is the ratio of the sides?

Ratio of sides is 35:7
5:1

ii) What is the ratio of the areas?

25:1

Example 3

Two similar jars of coffee A and B, contain 125 g and 1000 g respectively.
The height of the larger jar is 24 cm.

NB Coffee jars therefore dealing with volume

What is the height of the smaller jar?

Ratio of the heights is x : 24

Ratio of the volumes is $x^3 : 24^3$

OR $x^3 = \frac{125 \times 24^3}{1000} = \frac{125 \times 24 \times 24 \times 24}{1000}$

giving x = 12 cm (using calculator)

Example 4

In a scale model of the "Old Building", the area of the mathematics workshop is:

$\frac{1}{100}$ of the actual area. Calculate the ratio of the volume **model** of the maths workshop to the volume of the **actual** maths workshop.

Area is $\frac{1}{100}$ (given)

Ratio is 1:100 or $1^2:10^2$

Therefore
Ratio of the sides is 1:10

Ratio of the volumes is $1^3:10^3$
1:1000

Exercise 3

1. Two cuboids have sides 3 cm, and 6 cm respectively.

   i) Find the ratio of the areas,

   ii) Find the ratio of the volumes.

2. A cylinder has height 2 cm, and volume 8 cubic centimetres. A similar cylinder has height 3 cm. What is the volume of the second cylinder?

3. The volume of a scale model of a train to the volume of actual train has a ratio :

$$\frac{1}{1000}$$

   i) Find the ratio of the sides.

   ii) Find the ratio of the areas.

4. Two similar biscuit boxes have heights of 6 cm and H cm respectively. The first box holds 250 g of biscuits and the second box holds 2 kg of biscuits. Find H.

ANSWERS

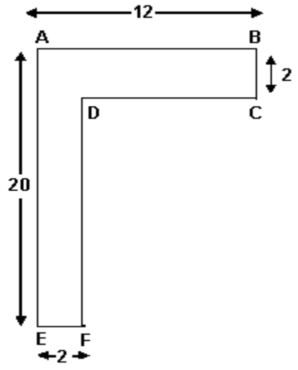
Exercise 1

1. 123 mm to 12.3 cm
2. 247 cm to 2.47 m
3. 5469 m to 5.469 km
4. 0.632 km to 632 m
5. 7.5 km to 7500 m
6. 6.3 kg to 6300 g
7. 734 g to 0.734 kg
8. 4 l to 4000 cm^3
9. 5437 cl to 54.37 l
10. 164 cl to 16.43 l

Exercise 2

1. 8 mm

2.



Area = 24 + 36

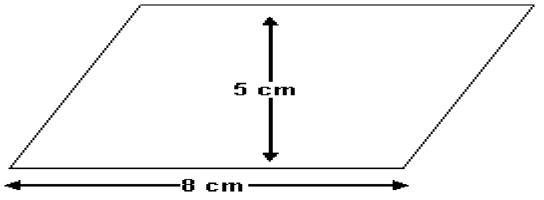
= 60 square units

3. Base = 8 cm

Height = 6 cm

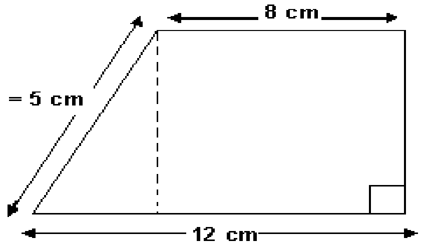
Area = 24 cm^2

4.



Area = 40 cm^2

5.



$$\text{Area} = \frac{(8+12)}{2} \times 3$$

$$= 30\ cm^2$$

6. $$\pi = \frac{22}{7}$$

$$\text{Length of arc} = \frac{36}{360} \times \frac{2}{1} \times \frac{22}{7} \times \frac{7}{1} = \frac{44}{10}$$

$$= 4.4\ cm$$

7. $$\text{Area} = \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1}$$

$$= 154\ cm^2$$

8. $$\frac{\theta}{360} \times \frac{2}{1} \times \frac{3.142}{1} \times \frac{9}{1} = 18.8$$

$$\theta = \frac{18.8 \times 360}{2 \times 3.142 \times 9} = \frac{6768}{56.556}$$

$$\theta = 119.67$$

9. $$\frac{60}{360} \times \frac{22}{7} \times \frac{14}{1} \times \frac{14}{1} = \frac{308}{3}$$

Area of sector = 102.67 cm^2

10. $$\text{Volume} = \frac{1}{3} \times \frac{22}{7} \times \frac{35}{1} \times \frac{35}{1} \times \frac{10}{1}$$

$$= \frac{38500}{3} = 12833\frac{1}{3}$$

OR = 12833.33 cm^3

11. Vol = 3.142 x 3.7 x 3.7 x 12.2

    = 524.77 cm^3

12. Vol = area of base x length

    = 2.5 cm^2 x 4.3 cm

    = 10.75 cm^3

13. Vol = $\frac{4}{3}$ x 3.142 x 7.84 x 7.84 x 7.84

    = $\frac{6056.3973}{3}$ cm^3

    = 2018.8 cm^3

14. Vol = $\frac{1}{3}$ base area x height

    = $\frac{1}{3}$ x 36 x 10 cm^3

    = 120 cm^3

Exercise 3

RATIO OF SIMILAR AREAS AND VOLUMES

1. Sides 3 : 6
   1 : 2

   Areas $1^2:2^2$ = 1 : 4

   Vol $1^3:2^3$ = 1 : 8

2. Height 2 : 3

   Vol $2^3:3^3$ = 8:27

   $$\frac{8}{27} = \frac{8}{x}$$

   x = 27 cm^3

3. $1 : 1000 \ (1 : 10^3)$

Sides 1 :10

Areas $1 : 10^2 = 1 : 100$

4. Height 6 : H
Vol in ratio of $6^3 : H^3$

$$\frac{6^3}{H^3} = \frac{250}{2000}$$

$$H^3 = \frac{2000 \times 6 \times 6 \times 6}{250}$$

H = 12 cm