VOLUMES

VOLUME OF A CYLINDER

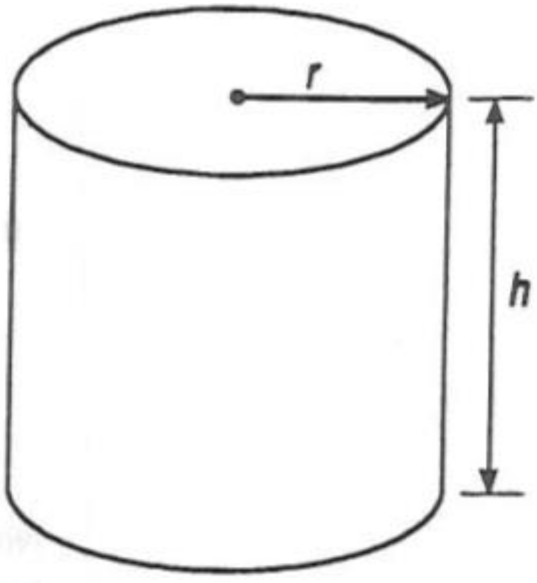
The volume of a cylinder is determined by multiplying the cross sectional area by the height.

$V = \pi r^2 h$

Where: V = volume

r = radius

h = height



Exercise 1

Complete the table (π = 3.142)

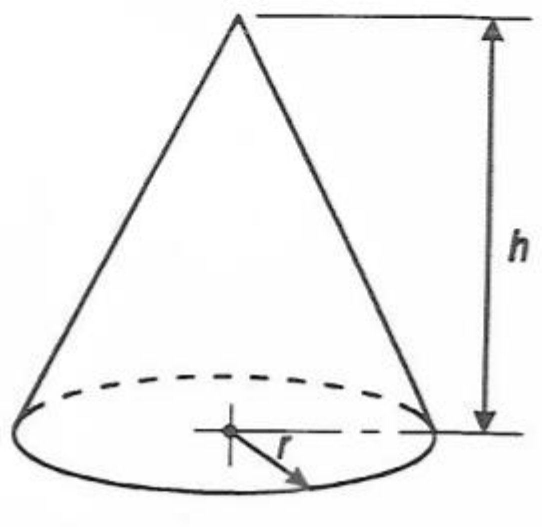
	r	h	V
a)	10 mm	25 mm	
b)	20 cm	12 mm	
c)		5 m	62.84 m^3
d)	12 mm		45.25 cm^3

Now check your answers.

VOLUME OF A CONE

The volume of a cone is $\frac{1}{3}$ the volume of a cylinder into which the cone would fit exactly.

$$V = \frac{1}{3}\pi r^2 h$$



Where: V = volume

r = radius

h = height (perpendicular)

Note that the height is measured perpendicularly (at right angles) to the base.

Exercise 2

Complete the table (π = 3.142)

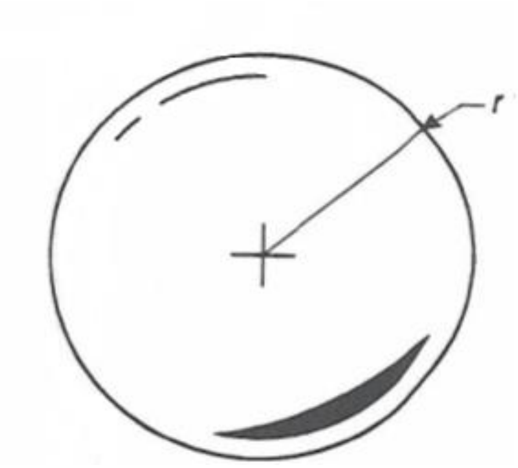
	r	h	V
a)	20 mm	50 mm	
b)	10 cm	0.5 m	
c)		5 m	33 m^3
d)	25 mm		65.46 cm^3

Now check your answers.

VOLUME OF A SPHERE

$$V = \frac{4}{3}\pi r^3$$

Where: r = radius



r

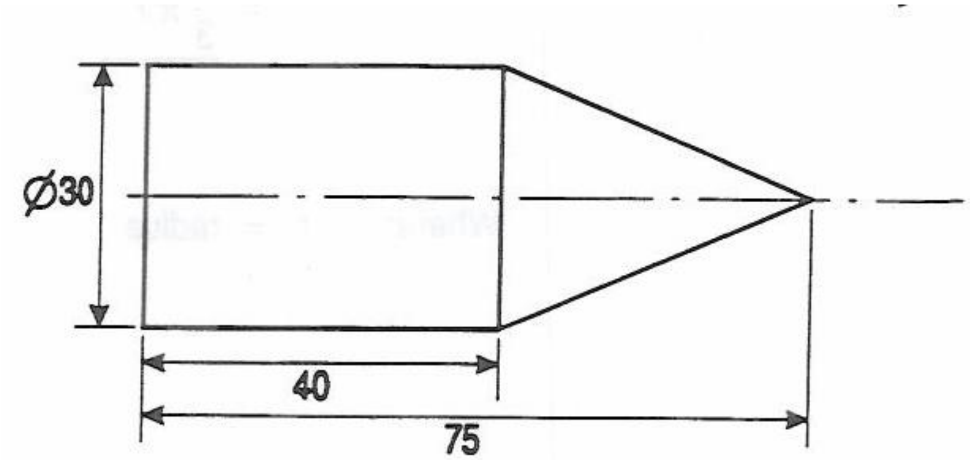
Exercise 3

Complete the table (π = 3.142)

	r	V
a)	25mm	
b)		4 m^3
c)		1500 mm^3

Now check your answers.

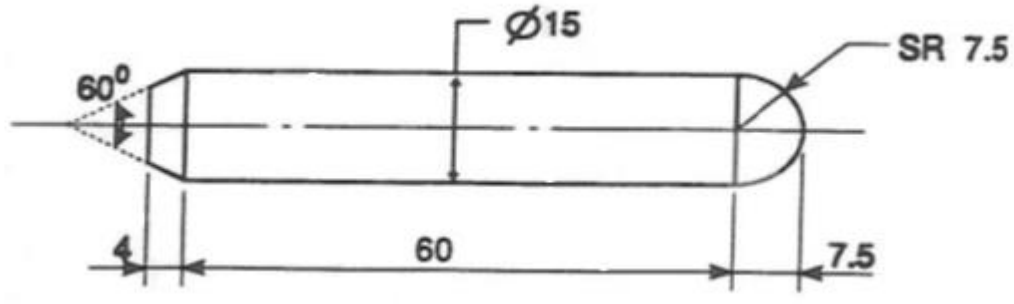
Exercise 4



Calculate the volume of the "plumb-bob" shown above. All dimensions are millimetres.

Now check your answers.

Exercise 5

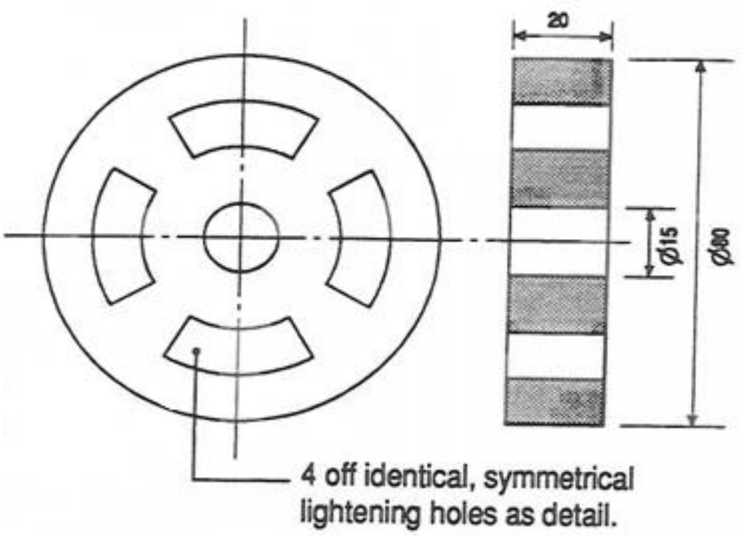


Calculate the volume of the “dowel” shown above.

All dimensions are in millimetres. (Note: SR 7.5 = spherical radius 7.5)

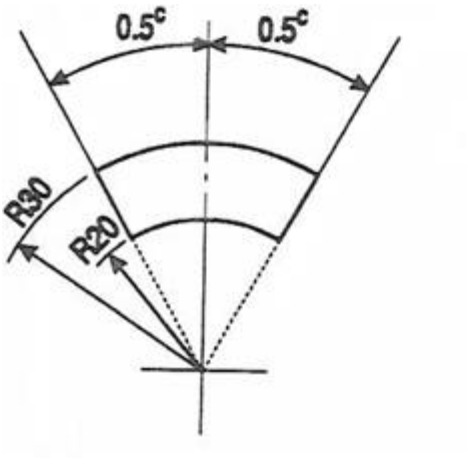
Now check your answers.

Exercise 6



Calculate the volume of the cast iron roller.

Linear dimensions are in millimetres = angular dimensions are in radians.



Now check your answers.

SUMMARY

a) Area of a triangle = $\frac{1}{2}$ (base x height)

(**Note**: the height is measured at right angles to the base).

b) Area of a sector = $\frac{\theta}{360}\left(\pi r^2\right)$

When the angle of θ is measured in degrees.

c) Area of a sector = $\frac{1}{2} r^2 \theta$

When the angle of θ is measured in radians.

d) Volume of a cylinder = $\pi r^2 h$

e) Volume of a cone = $\frac{1}{3} \pi r^2 h$

(**Note**: the height is measured at right-angles to the base).

f) Volume of a sphere = $\frac{4}{3} \pi r^3$

Please Note:

i) In all the above r = radius and
h = height (or length if the figure lies horizontally).

ii) You **must not mix** dimensional units in any of the above formula.

For example you **must not** work the radius in millimetres and the height in centimetres. **Both** radius and height must be in **either** millimetres **or** in centimetres.

ANSWERS

Exercise 1

	r	h	V
a)	10 mm	25 mm	**7855 mm^3**
b)	20 cm	12 mm	{ **1508.16 cm^3** / **1508160 mm^3** }
c)	**2 m**	5 m	62.84 m^3
d)	12 mm	{ **10 cm** / **100 mm** }	45.25 cm^3

The Answers are in **bold.**

a) $V = \pi r^2 h$

= 3.142 x 10^2 x 25

= 7855mm^3

b) The radius is in cm and the height is in millimetres. You must **not** mix them when substituting in the formula.

i) Working in cm

$V = \pi r^2 h$

= 3.142 x 20^2 x 1.2 (12mm = 1.2cm)

= 1508.16 cm^3

ii) Working in mm

$V = \pi r^2 h$

= 3.142 x 200^2 x 12 (20cm = 200mm)

= 1508160 mm^3

c) The formula has to be transposed to find r.

$$V = \pi r^2 h$$

So $$r^2 = \frac{V}{\pi h}$$

$$r = \sqrt{\frac{3V}{\pi h}}$$

$$r = \sqrt{\frac{62.84}{3.142 \times 5}}$$

$= \sqrt{4}$

= 2m

d) There is both transposition and mixed units at the same time.

$$V = \pi r^2 h$$

$$h = \frac{V}{r^2 \pi}$$

i) Working in cm

$$h = \frac{45.25}{1.2^2 x \times 3.142} = 10cm$$ (12mm = 1.2cm)

ii) Working in mm.

$$h = \frac{45250}{12^2 \times 3.142} = 100mm$$ (1 cm^3 = 1000 mm^3)

Now return to the text.

Exercise 2

	r	h	V
a)	20 mm	50 mm	**20946.67 mm^3**
b)	10 cm	0.5 m	{ **5236.67 cm^3** **0.0052367m^3**
c)	**2.51 m**	5 m	33 m^3
d)	25 mm	{ **10 cm** **100 mm**	65.46 cm^3

The Answers are in **bold.**

a) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.142 \times 20^2 \times 50$

= 20946.67 mm^3

b)

i) Working in cm

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.142 \times 10^2 \times 50$ (0.5m = 50 cm)

= 5236.67 cm^3

ii) Working in m

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.142 \times 0.1^2 \times 0.5$ (10cm = 0.1m)

= 0.0052367 m^3

c) The formula has to be transposed.

$$V = \frac{1}{3}\pi r^2 h$$

$$r = \sqrt{\frac{3V}{\pi h}}$$

$$= \sqrt{\frac{3x33}{3.142x5}}$$

$$= \sqrt{6.302}$$

= 2.51mm

d) There is both transposition and mixed units at the same time.

$$V = \frac{1}{3}\pi r^2 h$$

$$h = \frac{3V}{\pi r^2}$$

i) Working in cm $h = \frac{3\times 65.46}{3.142\times 25^2}$ (25mm = 2.5 cm)

= 10 cm

ii) Working in mm $h = \frac{3\times 65460}{3.142\times 25^2}$ (1 cm^3 = 1000 mm^3)

= 100mm

Now return to the text

Exercise 3

	r	V
a)	25mm	**65458.33 mm**
b)	**0.9847 m**	4 m^3
c)	**7.1 mm**	1500 mm^3

The Answers are in **bold.**

a) $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}$ x 3.142 x 25^3

= 65458.33 mm^3

b) This time you have to transpose the formula.

$V = \frac{4}{3}\pi r^3$

$r^3 = \frac{3V}{4\pi}$

$r = \sqrt[3]{\frac{3V}{4\pi}}$

$r = \sqrt[3]{\frac{3x4}{4x3.142}}$

$= \sqrt[3]{0.9548}$

= 0.9847m

c) Transpose the formula.

$r = 3\sqrt{\frac{3V}{4\pi}}$

$r = \sqrt[3]{\frac{3x1500}{4x3.142}}$

Now return to the text.

Exercise 4

The plumb-bob is made up from two geometrical shapes:

- A cylinder
- A cone

Cylinder

$$V = \pi r^2 h$$

$= 3.142 \times 15^2 \times 40$

Note: the diameter is 30mm
So the radius is 15mm

$= 8247.8\ mm^3$

Cone

$$V = \frac{1}{3} \pi r^2 h$$

$= \frac{1}{3} \times 3.142 \times 15^2 \times (75 - 40)$

$= \frac{1}{3} \times 3.142 \times 15^2 \times 35$

$= 8247.8mm^3$

Volume of plumb-bob = Volume of the cylinder plus
Volume of the cone

Volume of plumb-bob = 28278+ 8247.8

= $36525.8\ mm^3$

Now return to the text

Exercise 5

The dowel consists of three geometrical shapes.

- A cylinder
- A hemisphere (½ a sphere)
- A frustum of a cone (a cone with the top cut off)

Recognising the shape is half the battle.

Cylinder

$$V = \pi r^2 h$$

= 3.142 x 7.5^2 x 60

= 10604.25 mm^3

Hemisphere

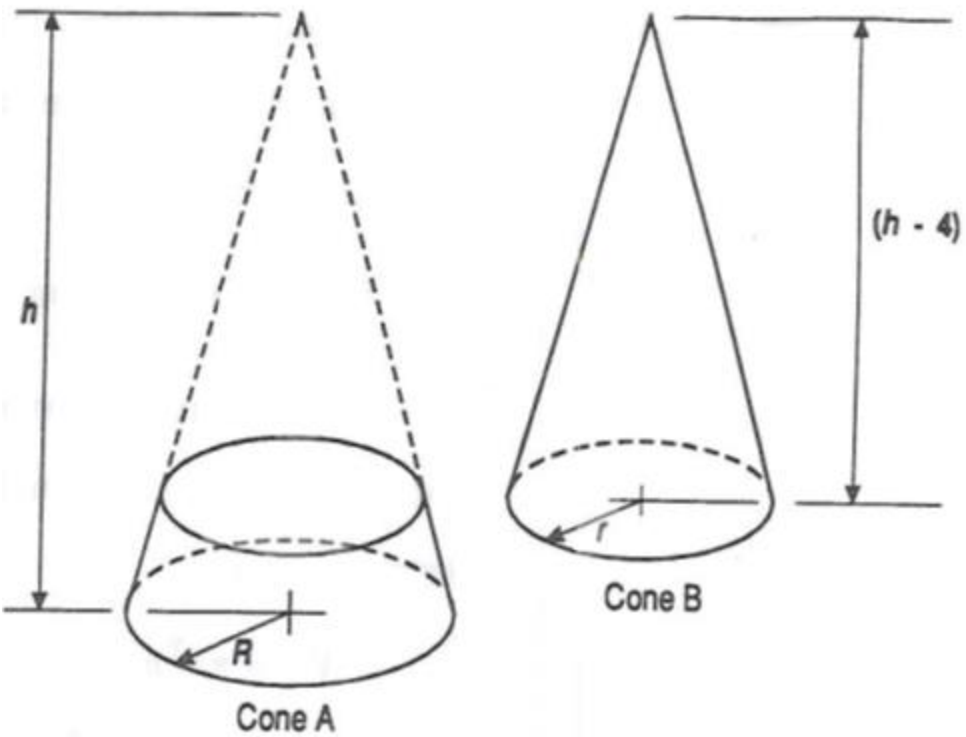
$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

= $\frac{1}{2}$ x $\frac{4}{3}$ x 3.142 x 7.5^3

= 883.69mm^3

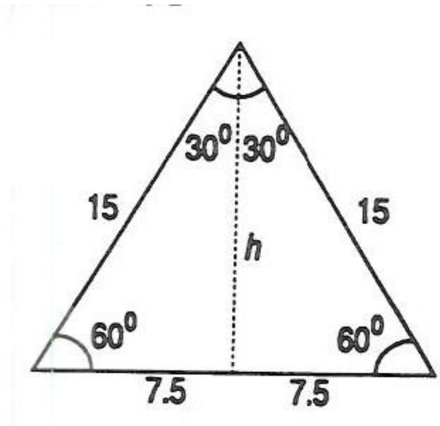
Frustum

The volume of the frustum is the difference between the volumes of the two cones. We also have to use some trigonometry to determine the dimensions of the cones.



Volume of frustum = volume cone A – volume cone B

Since the included angle of the nose of the cone is 60^0, and it is symmetrical, a slice through the cone on its centre line is an equilateral triangle. All sides 15mm, all angles 60^0. There are various ways of finding *h* using trigonometry or Pythagoras' – "you pays your money and takes your pick". Let's practice our trigonometry.



h = 15 cos 30

= 15 x 0.8660

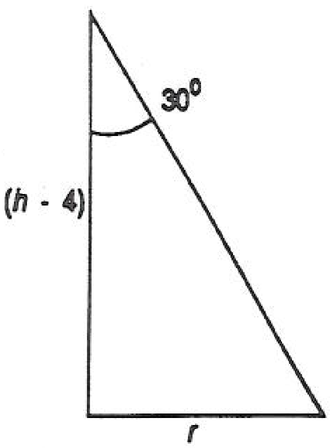
= 12.99 mm

Volume (cone A) $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}$ x 3.142 x 7.5^2 x 12.99

= 765.27 mm^3

Before we can find the volume of cone B, we have to find its base radius r.



r = (h-4) tan 30
= (12.99- 4) x 0.5774
= 5.19 mm

Volume (cone B) $= \frac{1}{3} \pi$ r^2

$= \frac{1}{3}$ x 3.142 x 5.19^2 x 8.99

= 253.62 mm^2

Therefore the volume of the frustum = 765.27 mm^2 – 253.62 mm^2
= 511.65 mm^3

Volume of cylinder = 10604.25 mm^3

Volume of hemisphere = 883.69 mm^3

Volume of frustum = 511.65mm^3

The total volume of the dowel = 11999.59 mm^3

For all practical purposes 12000 mm^3

The most likely pit falls are:

- Using the diameter (15mm) instead of the radius (7.5mm);
- Failing to plan your operation sequence so that each step produces the data needed in the next step;
- Not recognising the basic geometrical figures which combine together to make the dowel.

Now return to the text

Exercise 6

To answer this you have to find the volume of the whole roller and then subtract the volume of the centre hole and the lightening holes.

Roller blank

Volume = $\pi\ r^2 h$ (diameter = 80 mm, so radius = 40 mm)

= 3.142 x 40^2 x 20

= 100544 mm^3

Centre hole

Volume = $\pi\ r^2 h$ (diameter = 15mm, so radius = 7.5 mm)

= 3.142 x 7.5^2 x 20

= 3534.75 mm^3

Lightening hole

To find the volume of one of the lightening holes multiply the profile area by the thickness (20mm).

The profile area is the difference between two sectors.

Profile area $= \left(\frac{1}{2}R^2 0\right) - \left(\frac{1}{2}R^2 0\right)$

$= (\frac{1}{2}$ x 30^2 x 1.0) – ($\frac{1}{2}$ x 20^2 x 1.0)

= 450 – 200

= 250 mm^2

Volume of lightening hole = 250 mm^2 x 20 mm = 5000 mm^3

Total volume of the four lightening holes
= 5000 x 4 = 2000 mm^3

Therefore, the volume of the roller is:

100544 mm^3 – 3534.75 mm^3 – 2000 mm^3

Volume = 77009.25 mm^3

Now return to the text.