University Research Board grant sponsored project: Affine-Algebraic Methods in Complex-Analytic Geometry

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This project is sponsored by a University Research Board grant. The following sub-projects are open for graduate students in Mathematics. Minimal requirement is complex analysis at the level of MATH 304, but knowledge of complex analysis in several variables is a plus.

1 The Koras–Russel cubic threefold

The Koras–Russell cubic threefold is a smooth complex hypersurface given by the equation $x^2y + x + z^2 + t^3 = 0$ for $(x, y, z, t) \in \mathbb{C}^4$. This is a famous example of a manifold that is diffeomorphic to $\mathbb{C}^3$, but not isomorphic to $\mathbb{C}^3$ as an algebraic variety. This diffeomorphism is not known explicitly. Moreover, it is still an open question whether the Koras–Russel cubic threefold is biholomorphic to $\mathbb{C}^3$.

1. Construct explicitly a diffeomorphism from the Koras–Russel cubic threefold to $\mathbb{C}^3$ using Morse theory.
2. Investigate the structure of the group of holomorphic automorphism of the Koras–Russel cubic threefold and conclude that Koras–Russel cubic threefold is not biholomorphic to $\mathbb{C}^3$.

2 Finitely generated Lie algebras of vector fields

The holomorphic vector fields on $\mathbb{C}^n$ form a so-called Lie algebra. We know that this Lie algebra is infinite dimensional, yet it is possible to find just three holomorphic vector fields that generate the whole Lie algebra, and in addition, we can choose these vector fields to be completely integrable. We want to extend this result to other complex manifolds.

1. Find finitely many generating, completely integrable vector fields on

$$D_p = \{(x, y, z) \in \mathbb{C}^3 \,:\, xy = p(z)\}$$

   where p is a complex polynomial.

2. Find finitely many generating, completely integrable vector fields on

$$\mathrm{SL}_2(\mathbb{C}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \,:\, a, b, c, d \in \mathbb{C},\ ad - bc = 1 \right\}$$